Key Technology of Predicting Dynamic Surface Subsidence Based on Knothe Time Function

Qing Feng Hu*, Xi Min Cui, Guo Wang, Meng Ru Wang, Yin Xiao Ji, Wei Xue
College of Geoscience and Surveying Engineering, China University of Mining & Technology, Beijing 100083, P.R.China.
E-mail: huqingfengxy@qq.com

Abstract—A method for calculating the parameter of Knothe time function was presented. The principle of predicting dynamic mining subsidence was discussed based to the relationship between the periodic fractures of main roof and surface subsidence. Pointed out there might be a section whose width was shorter than a mining unit when divided a workface into n equal mining elements, the section should be calculated specially in the algorithm. A model of predicting dynamic mining subsidence was made according to the Knothe time function and probability integral method. A model how to divide a workface and two coordinate transformation models among three coordinates also were made. An algorithm step of the dynamic prediction model for multi-face mining was designed and implemented. And the algorithm was tested by nine measured subsidence data in different times of some mine 1176 east face, the result showed that the prediction results agreed well with the observation data, and the feasibility and effectiveness of the algorithm were demonstrated. Finally, the process of surface subsidence and deformation of 1176 east face was back analyzed.

Index Terms—mining subsidence, dynamic prediction algorithm, Knothe time function, multi-faces mining

I. INTRODUCTION

Ref. [1] The ground subsidence process induced by underground longwall mining is a very complicated process, so is the process of subsidence induced damages to the surface and subsurface structures. With the working face advance, because the relative position between the working face and surface point is different in different time, the effect of mining to surface is also different. Ref. [2] The process of surface subsidence is from slowing to violence, and finally stops. Traditionally, many prediction algorithms are made based on the final subsidence, so the dynamic surface subsidence and deformation can’t be reflected. Ref. [3] However, in fact some final compression regions might was ever stretched in the process of surface movement and vice versa; or some areas of which final states have litter movement, but they have moved acutely in the process of mining. Ref. [1] As far as supercritical condition is concerned, the bottom of the final subsidence basin is flat; there the surface slope and curvature induced by ground subsidence are zero. This means that there is no permanent surface deformation and thus no damages to those structures located in the subsidence basin bottom. However, during the formation of the subsidence basin, the damage potential of dynamic deformation must not be overlooked; proper precautions should be considered accordingly. So it is necessary to further study the key technology of predicting dynamic surface subsidence.

Ref. [3,4,5,6] Many data show that mining subsidence is a complicated four-dimensional space problem has to do with time. Thus select a proper time function and how to design a dynamic prediction algorithm according to a frequently-used subsidence prediction model are the key technology of predicting dynamic mining subsidence. Ref. [7,8,9,10] Knothe time function and double parameters time function and generalized time function are common time function, Knothe time function is used most widely among them. For subsidence prediction models, there are mainly the probability integral method and curve prediction method and profile function and so on, among them the prediction parameters of probability integral method have concise physical meanings, the prediction accuracy is very good if the values of prediction parameters are set properly, and the method can be implemented easily and is frequently-used in china. So this paper will take the Knothe time function and probability integral method as the base, combine the inner relationship and law between overburden and surface subsidence, to further study the key technology of predicting dynamic mining subsidence.

II. KNOTHE TIME FUNCTION

A. Principle of Knothe Time Function

After the underground resource exploitation, the immediate roof is damaged first, with the expansion of mining areas, destruction is gradually passed to the overlying rocks, when the working face move forward the distance equal to 0.25 0.5H0 (H0 is the average mining
depth), the mining depth will spread to the ground surface; as the scope of the local mining continues to increase, the range of subsidence basin and the maximum subsidence also increases, when the mining area reaches to the sub-critical size of mining. Maximum subsidence reached its limit, but the range of subsidence basin will still increase with the increase of mining areas. So assume the speed of surface subsidence \( dW(t) / dt \) is proportional to the difference between the final value of surface subsidence \( W_0 \) and Dynamic subsidence \( W(t) \) of some moment \( t \), that is

\[
\frac{dW(t)}{dt} = c(W_0 - W(t))
\]  

(1)

Where \( c \) is time influence coefficient which is relative to mechanical properties of overlying strata, whose dimension is 1/a.

According to boundary conditions of initial moment \( t = 0, W(t) = 0 \), integral formula(1),then

\[
W(t) = W_0 (1 - e^{-ct})
\]

(2)

Order

\[
\varphi(t) = 1 - e^{-ct}
\]

(3)

Formula (2) is the expression of Knothe dynamic process of surface movement, formula (3) is the expression of time function of subsidence, and \( c \) is time influence coefficient of surface dynamic subsidence.

B. The Method of Obtaining Parameters of Knothe Time Function

Methods of obtaining parameters of Knothe time function includes graphic method, contradistinction method, approximation method and measuring method, but a large number of measured data are required in the above four methods, if there is no measured data, there is no way to determine time influence coefficient. So using some experience value Knothe time function can be obtained, that is interval estimation method.

Interval estimation method is based on the fact: when the working face move forward to \( (1.2-1.4)H_0 \) (\( H_0 \) is the average mining depth), surface movement achieve full extraction, according to the theory of probability integration method, this moment maximum subsidence of surface approximately equal to 0.98\( W_0 \)(\( W_0 \) is the maximum surface subsidence value of the final state). If the speed of face advance is \( v \), the surface movement to achieve full extraction time is between \( 1.2H_0 / v \sim 1.4H_0 / v \); the following formula can be established according to the expression of predicting dynamic surface subsidence based Knothe time function.

\[
W_0 (1 - e^{-\frac{1.2H_0}{v}}) \geq 0.98W_0
\]

(4)

\[
W_0 (1 - e^{-\frac{1.4H_0}{v}}) \leq 0.98W_0
\]

(5)

It is available by calculation:

\[
-\frac{v \ln 0.02}{1.4H_0} \leq c \leq -\frac{v \ln 0.02}{1.2H_0}
\]

(6)

So, if the average mining depth \( H_0 \) and the average rate of face advance \( v \) are known, it is convenient to estimate the parameter \( c \). From (6) we can see that \( c \) is directly proportional to rate of face advance but mining depth is otherwise. Moreover, when a full extraction mining area of critical size is known, there is further:

\[
c = -\frac{v}{L_1} \ln 0.02
\]

(7)

III. DYNAMIC PREDICTION ALGORITHM

A. Principle of Dynamic Prediction

The basic principles of predicting dynamic surface subsidence are: the working face is divided into \( m \) mining units along the strike length; surface deformation of the working face can be seen as a superposition of the \( m \) mining units dynamic deformation. Assuming that the initial time of mining is 0, and the mining duration of the \( i \) th element is \( t_i \) and its advancing rate is \( v_i \), then the \( i \) th mining length is \( v_i t_i (i = 1, 2, \ldots, m) \). For moment \( t \), the mining time which the \( i \) th experienced is \( t - \sum_{j=1}^{i-1} t_j \), then the Knothe time function of the first and the \( i \) th mining units can be expressed in formula(8):

\[
\begin{align*}
\varphi(t) &= 1 - e^{-ct} \\
\varphi(t - \sum_{j=1}^{i-1} t_j) &= 1 - e^{-c(t - \sum_{j=1}^{i-1} t_j)}
\end{align*}
\]

(8)

The dynamic subsidence due to the first and the \( i \) th mining unit on the major cross section of trend can be expressed in formula(9):

\[
\begin{align*}
W_t(x_i, t) &= W^0(x_i) \varphi(t) \\
W_t(x_i, t - \sum_{j=1}^{i-1} t_j) &= W^0(x_i) \varphi(t - \sum_{j=1}^{i-1} t_j) \quad (i \geq 2)
\end{align*}
\]

(9)

Where \( x_i (i = 1, 2, \ldots, m) \) is the independent coordinate of the \( i \) th mining unit.

\[
W^0(x_i) = W(x_i) - W(x_i - v_i t_i)
\]

(10)

\[
W(x_i) = \frac{W_0}{2} [erf \left( \frac{\sqrt{\pi} x_i}{r} \right) + 1]
\]

(11)

Formula (10) and (11) are used to calculate the surface subsidence due to finite mining and semi-infinite mining respectively, the meaning of the symbols are the same as the probability integral method.

Assuming advancing rate is unchanged, namely \( v_1 = v_2 = \cdots = v_m \) or \( t_1 = t_2 = \cdots = t_m \), then the trend
mining length of working face is $m v_1 t_1$. However under the case that the working face can not be whole-divided by the length of a mining unit, namely, there is a section of which length is shorter than $v_1 t_1$, when to predict the surface subsidence the section should not omitted if the situation is met.

Synthesizing formula (9) and (10), the dynamic surface subsidence on the major cross section of the trend can be calculated as the following formula:

$$W(x,t) = \varphi(t)[W(x) - W(x - v_1 t_1)] + \sum_{i=1}^{l} \varphi(t - (i - 1)\tau)[W(x - iv_1 t_1) - W(x - (i + 1)v_1 t_1)] + W_i(x,t)$$

(12)

In the above formula, $x$ is the coordinate of working face coordinate system; $W_L(x,t)$ is the dynamic subsidence of the last mining unit of which length is shorter than $v_1 t_1$ on the major cross section of trend, it can be calculated using the following formula:

$$W_L(x,t) = \varphi(t-mt)[W(x - mt v_1 t_1) - W(x - v_1 t_1)]$$

(13)

According to reference [2] and the above analysis, the dynamic subsidence of some point of working face coordinate system can be calculated by (14):

$$W(x,y,t) = \varphi(t)W(x,y) + \sum_{i=1}^{l} \varphi(t - (i - 1)\tau)W_i(x,y) + W_i(x,y)$$

(14)

Where:

$$W_i(x,y) = (1/m_0 q \cos \alpha) [W(x_i) - W(x_i - v_1 t_1)[W(y) - W(y - l)]$$

(15)

$$W_L(x,y,t) = \varphi(t-mt)[W(x - mt v_1 t_1) - W(x - v_1 t_1)[W(y) - W(y - l)]$$

(16)

Both $x$ and $y$ are the coordinates of working face coordinate system; the final surface subsidence due to the $i$th mining element is $W_i(x_i,y_i)$; $W(y) - W(y - l)$ is the subsidence of the point $y$ on the major cross section of the tendency, when the trend is full mined and the tendency is finite mined, its calculation method is the same as probability integral method; $W_L(x,y,t)$ is the dynamic subsidence of the point $(x,y)$ due to the last mining unit. $m_0$, $q$ and $\alpha$ are seam thickness, subsidence coefficient and angle of seam; $l$ is the calculation length along tendency of working face. Similarly, the formulas of dynamic horizontal movement, dynamic slope, dynamic curvature and dynamic horizontal deformation of some point of working face coordinate system can be available.

B. How to Divide the Face

While mining, with the balance of stress in rock mass damaged, stress would be redistributed, and during this period the immediate and main roof would be bending failure. Reference [12] As the face advanced, caving zone developed continuously, the suspension area of the main roof was increased continuously, when the span exceeds the condition of limiting equilibrium or the stresses exceed the ultimate strength, the main roof will be fractured and falls down and the further deformation will transform upward to overburden rock strata also. The theoretical and site investigations demonstrate that if the mining area reaches the critical mining, the height of caved zone will no longer increasing with the face advance. The overburden rock movement will accompany with the periodic fractured length, shown in Figure 1.

According to the above analysis, it can be known that the periodic weighting of scope is due to the periodic fractures of main roof, and the overlying strata fracture is due to the expanding of the main roof periodic fractures upwards, finally a subsidence basin is form at the ground; the subsidence basin will spread along with the overlying strata fracture periodically. So a conclusion is educed that the rational size of mining unit mainly depends on the periodic fractured length of scope. Thus the method of periodic fractured length will be used in this article. According to Reference [11,13], the periodic fractured length has the form

$$L = h\sqrt{\frac{R_f}{3q}}$$

(17)

Where: $h$ is the thickness of main roof; $R_f$ is the tensile strength of rock; $q$ is the uniform load of overlying strata.

C. Construction and Conversion of Coordinate system

Based on above analysis, there are three kinds of coordinate systems involved, namely the coordinate system of mining area, working face and mining element.

1) Coordinate Transformation between working face and mining area

For the rectangular working face, working face coordinate is defined that the coordinate origin is the lower left point of the working face, $x$ axis is along the strike direction, $y$ axis is inclined direction. Generally, the working face coordinate system is not the same as the mining area coordinate system, so the mining area coordinate should be converted into working face coordinate when calculating surface subsidence. The relationship between the mining area coordinate and working face coordinate as Figure 2 Conversion formula is as follows:
\[
x = (X - X_0)\cos \alpha + (Y - Y_0)\sin \alpha \\
y = -(X - X_0)\sin \alpha + (Y - Y_0)\cos \alpha
\]  
(18)

Where: \(x\) and \(y\) are the coordinates of working face coordinate system; \(X\) and \(Y\) are the coordinates of mining area coordinate system; \((X_0, Y_0)\) is the coordinate of mining area coordinate system, and it is the origin of the coordinate system of working face; when \(x_0 < x_1\), \(\alpha = \arctan[(Y_1 - Y_0)/(X_1 - X_0)]\), and when \(x_0 \geq x_1\), then \(\alpha = \pi + \arctan[(Y_1 - Y_0)/(X_1 - X_0)]\).

![Relationship between mine area and face coordinate.](image)

(2) Coordinate Transformation between working face and mining elements

According to the above analysis, the coordinate origins among independent units in the working face coordinate system have the following relationship:

The coordinate origin of the first unit is 0, the \(i\)th is \(\sum_{j=1}^{i-1} v_{ij}\), and thus the working face coordinate system can be transformed to coordinate system of the independent various units, so it is convenient to calculate the dynamic subsidence due to each unit.

C. Algorithmic Approaches

Take calculation subsidence as an example to illustrate the algorithmic approaches. Assuming \(n\) is working faces of the mining area, \(t_{ik}\) is the start mining date of the \(k\)th face, \(t_{ik}\) is its stopping mining date, \(v(k)\) is its advancing rate, then the algorithmic approaches of dynamic prediction are listed as follows:

1. Determine the estimation range of the mining area according to each origin of all working faces, and divide estimation range into grids based on a given grid size.

2. For the \(k\)th \((k = 1, 2, \cdots , n)\) working face, determine the relationship among calculation time \(t\) and \(t_{ik}\) and \(t_{ik}\) respectively. If \(t \leq t_{ik}\), the working face is not mined yet, and turn to step (7); else if \(t_1(k) < t < t_1(k)\), the working face has been mined the length \(v(k)(t - t_{ik})\), and turn to step (3); else if \(t \geq t_1(k)\), the face has been mined over, and turn to step (3).

3. Transform mining area coordinates to working face coordinates, and then transforms working face coordinates to each grid unit coordinate.

4. Calculate the superposition subsidence of all mining units of the working face. The calculation formula is as follows:

\[
W(x, y, t) = \frac{1}{W_0} \phi \left( \frac{x}{W(x - v_1 t_1)} \right) \\
+ \sum_{i=1}^{n} \phi \left( \frac{t - t_{ik}}{W(x - v_{ik} t_{ik} - W(x - (i + 1) v_{ik} t_{ik})} \right)
\]  
(19)

5. Calculate the subsidence of the remaining mining unit of whose length is shorter than \(v_1 t_1\). The calculation formula is as follows:

\[
W_{k2}(x, y, t) = \phi \left( t - mt_{ik} \right) W(x - mv_1 t_1) - W(x - v_1 t_1) W^0(y)
\]  
(20)

6. The superposition of (4) and (5) is the subsidence influence of mined area.

7. Do \(k = k + 1\), until \(k \leq n\), \(n\) is the number of working faces.

8. Superimpose and export the subsidence of each working face.

IV. ALGORITHM IMPLEMENTATION AND ITS APPLICATION

A. Algorithm Implementation

According to the above dynamic prediction algorithm of multi-working face, the author has developed the integrated System for Data Processing by using C# language based on .NET 2005; the database was Access2003 which was used to record the parameters of all working faces. The functions of the system main include predicting dynamic surface subsidence, calculating prediction parameters of probability integration method, calculating the influence coefficient of Knothe time function, and post-processing predicting results. The dynamic surface subsidence module can be used to predict the subsidence of any moment and any point during mining, it also can be used to predict surface slope, curvature, horizontal movements and horizontal deformation of some point at some moment, Figure 3 showed the master interface of the dynamic surface movement module; the module of calculating prediction parameters mainly according to the Measured data and prediction model of probability integration method to gain the prediction parameters; the module of calculating the influence coefficient of Knothe time function was mainly to calculate \(c\) according to the actual mining area conditions; post-processing module was mainly visualize the predicting result.
B. Research on Application

The system was tested to verify the feasibility and correctness of the algorithm by the measured subsidence values of 30 monitoring points located in observation east line of some mine 1176 east working face. Then predict surface subsidence of any moment, in order to inverse the dynamic process of surface subsidence.

The data characterizing the process of 1176 east working face:

- Reach 990m,
- Length 160m,
- Thickness of seam up to 2.9m,
- Dip of the seam being $8^\circ$,
- Depth of mining below ground level 464m on average,
- Day advance was 2.6m/d,
- Periodic fractured length was 15m,
- Mining commenced at March 1999, and was ended in March, 2000.

According to geological data and measured data, using the relative module of the system, prediction parameters were shown in TABLE I, the time parameter $c$ was between 9.03 and 10.54, here $c=9.4$.

According to geological data and measured data, using the relative module of the system, prediction parameters were shown in TABLE I, the time parameter $c$ was between 9.03 and 10.54, here $c=9.4$.

According to prediction parameters which were shown in TABLE I and Knothe time function influence coefficient, dynamic surface movement of any moment could be predicted using the system. Fig.4 and Fig.5 were respectively measured subsidence curves and predicted results of surface subsidence along the inclined face of each moment. According to Fig.4 and Fig.5, it can be seen that:

1. With the continuous advance of working face, surface subsidence curve was growing.
2. When the working face reached to a certain extent, namely reached or exceeded the maximum surface subsidence of full extraction, surface subsidence of working face would not increase.
3. Contrast measured data and predicted data, results were shown in TABLE II, according to the analysis in TABLE II it could be seen that the predicted maximum mean square error was $\pm 70$mm, the maximum relative error was 5.3%, and the average relative error was 4%.
4. Based on TABLE II, it can be concluded that the predicted results was credible. Based on the probability integral method and above analysis, formulas of surface slope, curvature, horizontal movements and horizontal deformation are derived by surface subsidence, so movement and deformation prediction of them should also be credible.

![Figure 3. Main interface of predicting dynamic mining subsidence.](image1)

![Figure 4. Measured surface subsidence profiles as the face advances.](image2)

![Figure 5. Predicted surface subsidence profiles as the face advances.](image3)

<table>
<thead>
<tr>
<th>TABLE I. PARAMETERS OF PREDICTING DYNAMIC SURFACE MOVEMENT</th>
</tr>
</thead>
<tbody>
<tr>
<td>Parameter</td>
</tr>
<tr>
<td>------------</td>
</tr>
<tr>
<td>Subsidence coefficient</td>
</tr>
<tr>
<td>Tangent of major influence angle</td>
</tr>
<tr>
<td>Coefficient of mining influence propagation angle</td>
</tr>
<tr>
<td>Offset of inflection point</td>
</tr>
<tr>
<td>Coefficient of horizontal movement</td>
</tr>
</tbody>
</table>
Then 1176 east working face surface movement subsidence will be inversed and analyzed. Surface subsidence, slope, curvature, horizontal displace and horizontal deformation are shown separately in Fig.6.

<table>
<thead>
<tr>
<th>Date</th>
<th>Subsidence (mm)</th>
<th>Relative Error (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1999.10.05</td>
<td>±14mm</td>
<td>2.7%</td>
</tr>
<tr>
<td>1999.11.02</td>
<td>±30mm</td>
<td>3.4%</td>
</tr>
<tr>
<td>1999.12.07</td>
<td>±66mm</td>
<td>5.3%</td>
</tr>
<tr>
<td>2000.01.17</td>
<td>±70mm</td>
<td>4.9%</td>
</tr>
<tr>
<td>2000.02.15</td>
<td>±65mm</td>
<td>4.3%</td>
</tr>
<tr>
<td>2000.03.21</td>
<td>±65mm</td>
<td>4.3%</td>
</tr>
<tr>
<td>2000.05.08</td>
<td>±63mm</td>
<td>4.2%</td>
</tr>
<tr>
<td>2000.06.12</td>
<td>±54mm</td>
<td>3.5%</td>
</tr>
<tr>
<td>2000.09.14</td>
<td>±48mm</td>
<td>3.1%</td>
</tr>
</tbody>
</table>

Some conclusions can be drawn according to Figure 6.

1. With the continued exploitation of working face, the range of surface movement and deformation were growing.
2. Curves of the surface tilt were similar to horizontal movement curves; curves of the curvature were similar to the horizontal deformation curves.
3. Before the stop mining of working face, deformation of the initial exploit point was bigger than the region near the mining line, after a period of stop mining, the deformations at both ends were basically the same. Because ground deformation time of initial exploit
point was longer than the time of the region near the mining line; after a period of stop mining, deformation of the region near the stop line developed adequately, so it become the same as the initial exploit point.

In order to describe the deformation of surface movement intuitively, subsidence basin and horizontal deformation of three-dimensional surface map on September 14th, 2009 are given in this paper, the predicted angle is 30 degree, as shown in Fig. 7 and Fig. 8. From the map, it can be seen clearly the size of surface deformation of any point.

According to the analysis of any moment ground deformation inversion and three-dimensional surface map of 1176 east working face, it can be understood clearly the deformation situation of any point and any moment above the working face during the process of mining. Similarly, it also can be used to predict dynamically the ground deformation before exploit, which can be used to judge the deformation level to buildings caused by ground deformation, and then take some corresponding measures. So, the system could provide technical support for coal mining safety.

V. CONCLUSIONS

(1) Based on the Knothe time function and probability integration method, the principles of dynamic prediction of ground deformation are studied deeply, and methods to calculate the influence coefficient of the Knothe time function are given, the dynamic prediction model of ground deformation of any moment and any point is established.

(2) There might be a section of which width was shorter than a mining unit when dividing a working face using equal distance method, it should not be ignored, and the dynamic deformation of ground deformation should be the superposition of the mining units’ dynamic deformation and the dynamic deformation of less than a unit.

(3) The relationship between overburden roof fracture and ground deformation was discussed in this paper, it was indicated that dividing units should be according to the periodic fractures of main roof. It was also indicated that there were 3 coordinates in ground deformation prediction, during dynamic calculation, the mining area coordinates should be transformed to calculation coordinates.

(4) The algorithm steps of the model of predicting dynamic surface subsidence and deformation for multi-face mining were designed, and integrated Data Processing System for mining subsidence were developed; and the algorithm was tested by the measured subsidence values of 30 monitoring points located in observation east line of some mine 1176 east working face, the results showed that the prediction results agreed well with the observation data, the feasibility and effectiveness of the algorithm were demonstrated.

ACKNOWLEDGMENT

Financial supports from National Natural Science Foundation of China, National Basic Research Program of China (973 Program), Program for New Century Excellent Talents in University of China, and Key Laboratory of Mine Spatial Information Technologies (Henan Polytechnic University, Henan Bureau of Surveying & Mapping), State Bureau of Surveying and Mapping under Grant No.41071328, No.2007CB209400, No. NECT-07-07098 and KLM200816 are highly appreciated.

REFERENCES

[6] Qing feng Hu, Xi min Cui, Chun yi Li, et al. Algorithm of Predicting Dynamic Surface Subsidence and Deformation
Hu, Qing-Feng is presently studying for his doctorate in China University of Mining Technology in Beijing. He graduated from Henan Polytechnic University, China and received his master’s Degree in Jan, 2008. He has been engaged in integration of “3S” and its computer application, now he mainly engages in the research and application of mining subsidence and deformation surveying caused by coal mining.

Cui, Xi-Min is a professor who received his PhD in China University of Mining Technology in Beijing. He also pluralizes mining surveying standing committee commissioner of China’s the Ninth Generation Surveying Academy, security estimation expert of China Coal Industry Laboring Safeguard Institution in the aspect of coal mine construction item and commissioner of surveying academy of Beijing in the major of surveying education. He has been engaged in the research and application of mining subsidence, deformation surveying, industry precision surveying and integration of “3S”. He has published more than 50 journal papers and co-authored several works.