Coordinates and Bearing of Submerged Sensors Using a Single Mobile Beacon (CSMB)

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Abstract—This paper investigates the problem of localizing submerged sensors and provides a new mechanism to determine the coordinates and bearing of those sensors using only one beacon node. In underwater wireless sensor networks (UWSN), precise coordinates of the sensors that actuate or collect data is vital, as data without the knowledge of its actual origin has limited value. Mostly, the multilateration technique is used to determine the location of the sensors with respect to three or more known beacon nodes, besides incorporated nonlinear distance equations are solved in conventional method whereby degree-of-freedom does not guarantee a unique solution. In this study, a new method of determining the coordinates of submerged sensors with a single mobile beacon has been devised which requires no preinstalled infrastructure or reference point. Moreover, Cayley-Menger determinant and linearized trilateration are used to determine the coordinates of the nodes where none of the nodes have a priori knowledge about its location. Simulation results validate the proposed mathematical model by computing coordinates of sensor nodes with bearing information generating negligible errors.

Index Terms—Cayley-Menger determinant, underwater localization, linearization, bearing, mobile beacon

I. INTRODUCTION

Wireless communication applications have become more prominent in terrestrial environment than underwater. However, in recent years, researchers have shown fervent interest to explore and fulfill the needs of a multitude of underwater applications. The underwater wireless sensor network (UWSN) is envisioned to enable application for oceanographic data collection and offshorescopicfor the profusion of wealth underwater world possesses. It is not only the wealth; marine life helps determine the very nature of our planet at a fundamental level. Therefore, for its sustenance, it becomes crucial to obtain accurate environmental data using underwater sensors to provide and maintain sanctuary for the marine life. Moreover, advanced robot navigation, autonomous underwater vehicles control and surveillance, locating lost objects, estuary monitoring and pollutant tracking also require accurate localization [1].

In addition to underwater sensors, UWSN may also comprise of surface stations and autonomous underwater vehicles. Regardless of the type of deployment and configuration, the location of sensors needs to be determined for meaningful interpretation of the sensed data [2]. So, a pragmatic dynamic approach is obligatory to localize submerged sensors precisely with minimal logistics, which will require no preinstalled infrastructure and reference points.

Figure 1. Subset composed of a mobile beacon and three sensors

Localization has widely been explored in terrestrial wireless sensor networks (WSN) and various mechanisms have been proposed. Generally these methods can be classified into two categories: range-based and range-free schemes. The former apply inter node distances to multilateration or triangulation whereas the latter rely on profiling. Range-based scheme can provide more accurate position estimation with additional distance measurement hardware - a provision most contemporary sensors possess. Generally multiple beacons or preinstalled infrastructures are used as reference points for underwater localization; conversely our research problem domain consists of a single mobile surfaced beacon (boat or buoy) for localizing submerged sensors dynamically - a very pragmatic approach and usual configuration as depicted in Figure 1. Moreover, in
UWSNs, acoustic channels are naturally employed for range measurements as acoustic signals are more accurate than radio signals [1, 3].

For the coordinates of such dynamic configuration that has no preinstalled reference point, the proposed mathematical model incorporates the use of Cayley-Menger determinant followed by linearization to solve a system of non-linear equations. It is assumed that the plane of the mobile beacon and the plane of the deployed sensors are in parallel state. As of now the involuntary mobility of the sensors due to external factors has not been considered in the model yet. The model computes the coordinates with respect to beacon node that alleviates a number of problems in the domain of localization. Simulation results suggest that if the distances between beacon and sensors are true Euclidean then the positional errors are negligible. For a problem domain of 150m depth, positional errors found are in $10^{-12}$ to $10^{-14}$ m range. For a sensor of 0.5m to several meters in length, the generated error is quite negligible, which in turn validates the proposed mathematical model. As the model generates negligible positional error with Euclidean distances, it is conspicuous that distance measurements are the limiting factor for pin pointing the sensors.

Rest of the paper is organized as follows. Section II focused on related works with associated constraints. Section III describes the problem domain, environmental constraints and the scope of the paper, and Section IV explains the proposed theoretical mechanism to determine the sensors coordinates and bearing. Simulation results and discussions are reported in Section V and section VI respectively, finally conclusions in Section VII.

II. RELATED WORKS

Sensor localization algorithms estimate the locations of initially unknown sensors by using knowledge of the absolute positions of a few sensors and inter-sensor measurements such as distance and bearing. Although localization has been widely studied for terrestrial wireless sensor networks, existing techniques cannot be directly applied to UWSNs due to dissimilar nature and the challenges posed by the harsh underwater environment. Besides, different spectrum of signals propagation phenomena and model of varied environments do not permit the application of generic localization algorithm to underwater world. Chandrasekhar et al. [4] explore such schemes for UWSNs, as well as the challenges in meeting the requirements posed by UWSNs for off shore engineering applications. Since then, a multitude of localization schemes have been proposed specifically for UWSNs.

In [5], 3D Euclidean distance estimation method requires the need of a certain number of neighboring nodes to measure inter-node distances and where error is propagated through the system due to its recursive nature. In [6], the authors propose a localization scheme based on buoys moored to the waterbed and mobile nodes that need to communicate directly with these buoys to get their location. This method does not support dynamic environment because buoys need to be deployed in advance in known locations. Four different positions are used to obtain the beacon nodes positions of a 3D local positioning system in [7].

In [8], Duff and Muller proposed a method to solve the multilateration equations by means of nonlinear least square optimization when positions are not known. The algorithm is based on degree-of-freedom analysis – which says enough measurements from different positions will provide enough equations to solve the problem. In [9], same technique is used incorporating extended Kalman filter. However, the degree-of-freedom analysis does not guarantee a unique solution in a system of nonlinear equations, such as trilateration, when the only data available is the distance measured between the nodes [10]. In [11], Guevara et al. introduced a new closed-form solution where no position information of nodes is required to determine the positions of multiple static beacon nodes, the only information they used is the distance measurement between static beacons and mobile node. Recently the accuracy of the coordinates for the same configuration with Euclidean distance has been shown in [12].

Having analyzed the various studies discussed above, in this paper we propose a closed-form solution to determine the coordinates and bearing of the submerged sensors having only one beacon node at the surface. The precise conditions for obtaining initial subsets of nodes were justified using rigidity theory in [10].

III. PROPOSED CONFIGURATION

To determine the coordinates of the sensors, the proposed method assumes at least three submerged sensors and a floating beacon at the surface of the water column. Usual number of sensors deployed underwater varies from a few to thousands to collect data in UWSNs; in the proposed configuration a single beacon determines the coordinates of three sensors at a time. It is also assumed that the distance measurements between the beacon and sensors will be conducted by measuring the flight time of acoustic signals as devised in [13]. As the water column is considered to be homogeneous; the variation in the speed of acoustic signals due to the changes of temperature, depth and salinity is out of the scope of this paper - 1500m/s is considered in most underwater localization schemes; however to develop the mathematical model of coordinates determination and to perform simulation, average speed of acoustic signal is not a necessary parameter to be considered. It is worth noting here that the distance determination is out of the scope of this paper, signals propagation model and the multipath fading phenomenon of signals due to obstruction and other factors left undiscovered.

While taking multiple distance measurements as required by the proposed model we also assume that the plane on which beacon surfs and the plane created by the three submerged sensors are in parallel state. In reality not being in the parallel states would contribute to errors in coordinates, though the surfing area of the beacon...
while taking multiple measurements can be in close proximity to mitigate non-parallel effects. The proposed model in this paper does not incorporate non-parallel effect on coordinates and bearing determination. For simplicity, the model also assumes that the submerged sensors are stationary during distances measurements process. The general properties of a transducer or beacon have the capability of generating and receiving signals, whereas sensors may have the restricted capability. A solvable configuration of one beacon with three submerged sensors is denoted in Figure 2.

IV. COORDINATES AND BEARING COMPUTATION

A. Coordinates with Respect to the Sensor

The objective of localization algorithms is to obtain the exact positions and bearing of all the submerged sensors by measuring distances between beacon and nodes. Only measurement available here to compute is the distance and typically it is considered as optimization problem where objective functions to be minimized have residuals of the distance equations. The variables of any localization problem are the 3D coordinates of the nodes, where in principle more number of distance equations is needed than number of variables to solve. However, this approach known as degree-of-freedom analysis may not guarantee the unique solution for a nonlinear system.

Trilateration or multilateration techniques are usually applied in nonlinear system to determine the locations or coordinates of the sensors in partial or full. According to Guevara et al. in [11], the convergence of optimization algorithms and Bayesian methods depend heavily on the initial conditions used; where they circumvent the convergence problem by linearizing the trilateration equations.

Figure 2 shows the initial subset composed of a mobile beacon node $S_j$, $j = 4,5,...,9$ and three sensor nodes $S_i, i = 1,2,3$. Without loss of generality, a coordinate system can be defined using one of the sensor nodes $S_i, i=1,2,3$ as the origin $(0,0,0)$ of the coordinate system. Now the trilateration equations can be written as a function of two groups of distance measurements. The distance between beacon and sensors $d_{12}, d_{13}, d_{14},...$, which are known (measured) data, and inter-sensor distances $d_{12}, d_{13}, d_{23}$ and the volume of tetrahedron $V_t$ (formed by the beacon and sensors) are unknown. Based on the local positioning system configuration of Figure 2, we need to write equations that will include all known and unknown distances. For that matter, we express the volume of tetrahedron $V_t$ using Cayley-Menger determinant as following:

$$V_t = \begin{vmatrix} 1 & 0 & 1 & 1 & 1 \\ 1 & d_{12}^2 & 0 & d_{23}^2 & d_{24}^2 \\ 1 & d_{13}^2 & d_{23}^2 & 0 & d_{24}^2 \\ 1 & d_{14}^2 & d_{24}^2 & d_{43}^2 & 0 \end{vmatrix}$$

By expanding (1), we obtain:

$$d_{14}^2d_{23}^2 - d_{14}^2d_{12}^2 + d_{14}^2d_{13}^2 - d_{14}^2d_{14}^2 + d_{14}^2d_{23}^2 - d_{14}^2d_{23}^2 - d_{14}^2d_{24}^2 - d_{14}^2d_{24}^2 = 0$$

Grouping known and unknown variables, we get

$$d_{14}^2(d_{12}^2 - d_{13}^2 - d_{14}^2) + d_{14}^2(d_{12}^2 - d_{13}^2 - d_{14}^2) + d_{14}^2(d_{12}^2 - d_{13}^2 - d_{14}^2) - d_{14}^2d_{14}^2 - d_{14}^2d_{14}^2 - d_{14}^2d_{14}^2$$

$$d_{14}^2 + 144 d_{14}^2 + d_{14}^2 = (d_{12}^2 - d_{13}^2 - d_{14}^2)(d_{12}^2 - d_{13}^2 - d_{14}^2)$$

Here,

$$\left( a_{i1}^2 - d_{12}^2 - d_{13}^2, a_{i2}^2 - d_{13}^2 - d_{14}^2, a_{i3}^2 - d_{14}^2 - d_{12}^2 \right)$$

are unknown terms.

So, the above expansion can be rewritten as follows:

$$d_{14}^2X_1 + d_{14}^2X_2 + d_{14}^2X_3 - (d_{14}^2 - d_{13}^2)(d_{14}^2 - d_{12}^2)X_4 - (d_{14}^2 - d_{13}^2)(d_{14}^2 - d_{12}^2)X_5 + X_6 = (d_{14}^2 - d_{13}^2)(d_{14}^2 - d_{12}^2)$$

Equation (2) in fact resembles the linear form of $a_1x_1 + a_2x_2 + \cdots + a_nx_n = b_1$, where we have six unknowns. So we need at least six measurements to solve this system of linear equations, which could be done following the same procedure steering the beacon node $S_j, j = 4,5,...,9$ to six different locations and measuring the distances in the vicinity of $S_i$ as in Figure 3.

Finally, we get m-linear equations of the form;
we calculate. To find and the and in between are known computed distances.

\[
\begin{align*}
& d_{i2} = \frac{X_1}{(1 - X_4 - X_5)} , \\
& d_{i3} = \frac{X_4X_5}{(1 - X_4 - X_5)} .
\end{align*}
\]

If we let the coordinates of the submerged sensors \(S_1, S_2, \) and \(S_3\) to be \((0,0,0), (0,y_2,0)\) and \((x_1,y_1,0)\) respectively, then the inter-sensor distances could be written with respect to coordinates of the sensors as follows:

\[
\begin{align*}
& d_{i2} = y_2^2, \\
& d_{i3} = x_1^2 + y_1^2, \\
& d_{23} = (y_1 - y_2)^2.
\end{align*}
\]

From the above values the unknown variables can be computed as:

\[
\begin{align*}
& y_3 = \frac{d_{i2}^2 + d_{i3}^2 - d_{23}^2}{2d_{i2}}, \\
& x_1 = \sqrt{d_{23}^2 - \frac{(d_{i2}^2 + d_{i3}^2 - d_{23}^2)}{2d_{i2}}}.
\end{align*}
\]

where \(d_{i2}, d_{i3}\) and \(d_{23}\) are known computed distances. Table 1 summarizes the coordinates of the sensors for the proposed problem domain.

**Table 1. Coordinates of the Sensors**

<table>
<thead>
<tr>
<th>Sensor</th>
<th>Coordinates</th>
</tr>
</thead>
<tbody>
<tr>
<td>(S_1)</td>
<td>((0,0,0))</td>
</tr>
<tr>
<td>(S_2)</td>
<td>((0,d_{i2},0))</td>
</tr>
<tr>
<td>(S_3)</td>
<td>(\sqrt{d_{23}^2 - \frac{(d_{i2}^2 + d_{i3}^2 - d_{23}^2)}{2d_{i2}}}, \frac{d_{i2}^2 + d_{i3}^2 - d_{23}^2}{2d_{i2}}, 0)</td>
</tr>
</tbody>
</table>

**B. Coordinates with Respect to the Beacon**

Up to now we have been able to determine the coordinates of the sensor nodes with respect to \(S_1\). In order to find the coordinate with respect to the beacon node we follow the following steps.

We assume that with the use of appropriate sensors, the depth \(h\) in Figure 2 can be measured as depicted in [14]. After measuring the vertical distance \(h\) in between the beacon node \(S_b(x_b, y_b, z_b)\) and the \(XY\) plane, we can assume the projected coordinate of the beacon node \(S_b(x_b, y_b, z_b)\) on the plane \(XY\) is \(P_b(x_b, y_b, 0)\) . To find \(x_4\) and \(y_4\), we can apply trilateration in the following manner assuming the distances between \(S_1, S_2, S_3\) and \(P_b\) are \(D_{14}, D_{24}\) and \(D_{34}\) respectively and devise following relations.

\[
\begin{align*}
& D_{14}^2 = x_4^2 + y_4^2, \\
& D_{24}^2 = x_4^2 + (y_4 - y_1)^2, \\
& D_{34}^2 = (x_4 - x_1)^2 + (y_4 - y_1)^2.
\end{align*}
\]
From (4), (5) and (6) we obtain the projected beacon’s coordinates \( P(x_1, y_1, 0) \), where

\[
x_1 = \frac{1}{2d_{12}} \left( 4d_{12}^2D_1^2 - (D_1^2 - D_3^2 + d_{12}^2) \right),
\]

\[
y_1 = \frac{1}{2d_{12}} \left( D_1^2 - D_3^2 + d_{12}^2 \right).
\]

As \( d_{14}, d_{24} \) and \( d_{34} \) are the hypotenuse of the \( \Delta S_1P_1S_4, \Delta S_2P_2S_4 \) and \( \Delta S_3P_3S_4 \) respectively, so it is possible to obtain \( D_{14}, D_{24} \) and \( D_{34} \) using Pythagorean Theorem. So the coordinate of the beacon node \( S_4(x_1, y_1, z_4) \) would be \( (x_i, y_i, h) \) where all the elements are known.

\[
S_i(x_i, y_i, h) = \left( \frac{1}{2d_{12}} \left( 4d_{12}^2D_{1i}^2 - (D_{1i}^2 - D_{3i}^2 + d_{12}^2) \right), \frac{1}{2d_{12}} (D_{1i}^2 - D_{3i}^2 + d_{12}^2), h \right)
\]

The origin of the Cartesian system is transferred on to the coordinate of the beacon node and sensors coordinates are found with respect to the beacon node \( S_4 \).

![Figure 4. Calculated sensors positions with respect to actual coordinates](image)

V. SIMULATION RESULTS

The proposed method is simulated in Matlab for a problem domain depicted earlier with a depth of 150m where a single beacon node is capable of determining the coordinates and bearing of the submerged sensors. Figure 4 shows the computed coordinates of the sensors with Gaussian noise in distance measurements as the precision of distance measurements is one of the prime factors for accurate coordinate determination. In our proposed approach the number of beacons required is just one that floats on the surface of the water and minimum of three sensors - a recognized number in monitoring for analyzing environment with sensors. In case of numerous sensors, three at a time will be localized and so on. Besides, our method is capable of determining 3D coordinates with respect to the beacon node with bearing information which gives a better comprehension regarding the location of the sensors because coordinates of the beacon node could be known with the help of GPS.

In order to validate the mathematical model a group of three sensors are placed randomly on the XY plane and the mobile beacon is steered above, which is assumed to be in parallel state to the bottom plane where the sensors are deployed. While the coordinates of the sensors are chosen randomly, for computational simplicity one of the sensors is marked as the origin and the other one on the y-axis of the problem domain. Eventually the third sensor could be positioned in any point of XY plane as discussed in section 3. To get distance measurement from six different positions of the beacon, it has been randomly moved around to six different coordinates in different orientations. However, mobility of the submerged sensors is not considered in the proposed mathematical model. At first to prove the mathematical model, true Euclidean distances between the sensors and beacon are considered while computing the coordinates of the sensors \( S_2 \) and \( S_3 \) with respect to \( S_1 \); then Gaussian distribution with mean 0 and variance 1 is added with the Euclidean distances to see the effect on coordinates. Different orientations of the mobile beacon have been explored, straight line to angular fashion, circular to Archimedean spirals of different radius (5-50m) as well as in random fashions. The arclength \( l \) of Archimedean spiral is considered according to (7) and (8).

\[
r = a + b\theta
\]

\[
l = \int_{\theta_1}^{\theta_2} \sqrt{r^2 + \left( \frac{dr}{d\theta} \right)^2} \, d\theta
\]

Here, \( r \) is the distance from origin, \( \theta_1 \) to \( \theta_2 \) spans from inner and outer radius of the spiral respectively.

![Figure 5. Positional errors with 10m circular orientation of the beacon considering Euclidean distances](image)

A. Coordinates and Bearing with Euclidean Distances

Simulation results suggest that if the distances between beacon and sensors are true Euclidean then the positional errors are negligible. For a problem domain of 150m depth, positional errors are in \( 10^{-12} \) to \( 10^{-14} \) m range. For a sensor of 0.5m to several meters in length, the generated error is quite negligible, which in turn validates the proposed mathematical model. Figure 5 and Table II and III show scrupulous accuracy and precision in position detection. Positional and bearing errors are negligible for different orientation of the beacon; these
TABLE II. BEARING AND POSITIONAL ERRORS FOR $S_1$, $S_2$, AND $S_3$ WITH RESPECT TO BEACON FOR 36° (WITH EUCLIDEAN DISTANCES)

<table>
<thead>
<tr>
<th>Circular orientation (radius)</th>
<th>Bearing (originally)</th>
<th>Bearing (computed)</th>
<th>Positional Error $S_1$(m)</th>
<th>$S_2$(m)</th>
<th>$S_3$(m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>5m</td>
<td>36°</td>
<td>35.976°</td>
<td>$1.10 \times 10^{-4}$</td>
<td>$1.98 \times 10^{-4}$</td>
<td>$3.47 \times 10^{-4}$</td>
</tr>
<tr>
<td>10m</td>
<td>36°</td>
<td>35.976°</td>
<td>$4.38 \times 10^{-5}$</td>
<td>$8.95 \times 10^{-5}$</td>
<td>$1.60 \times 10^{-4}$</td>
</tr>
<tr>
<td>15m</td>
<td>36°</td>
<td>35.999°</td>
<td>$1.45 \times 10^{-5}$</td>
<td>$2.13 \times 10^{-5}$</td>
<td>$1.13 \times 10^{-5}$</td>
</tr>
<tr>
<td>20m</td>
<td>36°</td>
<td>35.993°</td>
<td>$7.10 \times 10^{-5}$</td>
<td>$5.68 \times 10^{-5}$</td>
<td>$7.32 \times 10^{-5}$</td>
</tr>
<tr>
<td>30m</td>
<td>36°</td>
<td>35.994°</td>
<td>$7.10 \times 10^{-5}$</td>
<td>$4.26 \times 10^{-5}$</td>
<td>$1.03 \times 10^{-4}$</td>
</tr>
</tbody>
</table>

TABLE III. BEARING AND POSITIONAL ERRORS FOR $S_1$, $S_2$, AND $S_3$ WITH RESPECT TO BEACON FOR 72° (WITH EUCLIDEAN DISTANCES)

<table>
<thead>
<tr>
<th>Archimedean Spiral (single turn increase)</th>
<th>Bearing (originally)</th>
<th>Bearing (computed)</th>
<th>Positional Error $S_1$(m)</th>
<th>$S_2$(m)</th>
<th>$S_3$(m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>5m</td>
<td>72°</td>
<td>72.068°</td>
<td>$6.04 \times 10^{-3}$</td>
<td>$1.17 \times 10^{-2}$</td>
<td>$3.20 \times 10^{-2}$</td>
</tr>
<tr>
<td>10m</td>
<td>72°</td>
<td>72.038°</td>
<td>$4.57 \times 10^{-3}$</td>
<td>$9.23 \times 10^{-3}$</td>
<td>$2.59 \times 10^{-2}$</td>
</tr>
<tr>
<td>15m</td>
<td>72°</td>
<td>72.012°</td>
<td>$8.55 \times 10^{-3}$</td>
<td>$1.66 \times 10^{-2}$</td>
<td>$3.12 \times 10^{-2}$</td>
</tr>
<tr>
<td>20m</td>
<td>72°</td>
<td>72.031°</td>
<td>$5.74 \times 10^{-3}$</td>
<td>$1.13 \times 10^{-2}$</td>
<td>$7.73 \times 10^{-2}$</td>
</tr>
<tr>
<td>30m</td>
<td>72°</td>
<td>72.007°</td>
<td>$1.01 \times 10^{-2}$</td>
<td>$2.55 \times 10^{-3}$</td>
<td>$8.55 \times 10^{-3}$</td>
</tr>
</tbody>
</table>

Conspicuous negligible errors have been generated due to linearization of a system of nonlinear equations. Simulation also suggests that the orientation of the beacon while measuring distances can be any form except in straight line; measurements in straight line push the matrix to be singular and unsolvable. Whereas, the arbitrariness of the orientation of a mobile beacon on the surface of the water is very natural, hence Archimedean spiral and circles of different radius have also been explored to see the validity of the proposition.

B. Coordinates and Bearing with Gaussian Noise

Coordinates of the sensors have also been determined after incorporating Gaussian noise into distance measurements with a distribution of mean 0 and variance 1. Positional errors for sensors $S_1$, $S_2$, and $S_3$ are shown in Figure 6 with respect to beacon at the surface; produced mean positional errors of the submerged sensors are around 3m range without even omitting outliers, which is pretty good comparing the sizes of the submerged sensors as sensors can vary from 1-5m range in cases of automated unmanned vehicles (AUVs) or unmanned underwater vehicles (UUVs). Table IV and V summarize bearing and positional errors for different orientations of the beacon, which shows bearing errors also remain within $0.21-3.38$ variation.

The orientation of the beacon’s mobility followed as fashioned in Figure 3 and the results suggest orientation of the beacon has no significant effect except theorization in a straight line. Besides, simulation also suggests spans of the beacon’s orientation do not affect the determination of coordinates, measurements can be taken in a close proximity - so that the errors generated from mobility of the sensors can be minimized. As the model generates negligible positional error with Euclidean distances, it is conspicuous that distance measurements are the limiting factor for pin pointing the sensors and distance measurements errors can be minimized using different signals and methods.

VI. ANALYSIS AND DISCUSSION

Proposed method is designed to determine coordinates and bearing of submerged sensors for a water column keeping a beacon floating at the water surface. This pragmatic configuration of the proposed method does not require any preinstalled infrastructure, whereas capable of determining coordinates in dynamic fashion with a single beacon. Simulation validates the mathematical model of coordinates determination from acquired distances from the beacon to the submerged sensors. In Matlab beacon and sensors are placed in Cartesian coordinates and distances are measured between those points to find the Euclidean distances. The simulation is performed without emulating the water column as we haven’t covered distance determination techniques in this paper. In reality distances between beacon and the deployed sensors are presumed to be done by determining the flight time of the acoustic signals; hence multipath phenomenon or propagation model of signals is out of the scope of this paper.

The positional error of the sensors with Euclidean distances between beacon and sensors is very negligible, which proves the validity of the mathematical model. Simulation also suggests the span of the surfing area has no effect on coordinates and bearing determination process of the sensors. However, the precise distances between beacon and the sensors are the determining factor of accurate coordinates. We have also found that straight line or right angle movement of the beacon while taking measurement pushes the matrix of the model to be singular without converging. In reality this formation is quite impossible to occur where movement of the beacon (boat/buoy) would be random by nature due to drift and steering.

The expanded Cayley-Menger determinant to solve volume of tetrahedron created by the single beacon and three submerged sensors is non-linear, as degree-of-freedom of non-linear equations does not guarantee solution; we tend to linearize the equation and get the number of six unknown variables, which is why we need six measurements to solve the system of linear equations.
While considering Euclidean distances with circular and Archimedean spiral orientations of the beacon with radius ranging from 5-50m, the positional error and bearing of the submerged sensors are very negligible, positional error in the picometer range and fraction of a degree in bearing is due to linearization of the determinant. However, the positional error increased to 3-4m range and bearing remains within couple of degrees once Gaussian noise in distance measurements is applied. Considering 150m simulated water column and the size of the sensors and AUV or UUV - achieved errors are in acceptablereange. Table VI shows the comparison and characteristics of different underwater localization algorithms for their own problem domain.

**VII. CONCLUSIONS**

Unswerving navigation and positioning are becoming imperative in more and more applications for safety-critical purposes and research. In this paper we presented a mathematical model to determine the coordinates and bearing of submerged sensors with a single beacon dynamically. The pragmatic orientation of the problem domain and the proposed solution has also been validated with simulated results. We also showed the effect of beacon’s mobility and span on determined coordinates and bearing of submerged nodes. Having a single mobile beacon at the surface without a preinstalled infrastructure is usual in nature for continuous localization of nodes. The method computes the coordinates and bearing with respect to the beacon and sensor nodes that alleviates a number of problems in the domain of localization. The proposed mathematical model generates negligible error when it considers distances between beacon and sensors as true Euclidean. It also shows that coordinates and bearing are within acceptable error range when Gaussian noise is applied to measured distances. Simulation also suggests that the beacon’s mobility and span do not affect the coordinates.
and bearing determination process as long as the submerged sensors are stationary for the duration of distance measurement process. However, as determined, some specific orientations of the beacon’s mobility need to be avoided to help converge.

In future work we plan to consider involuntary mobility of the submerged sensors due to currents and effect of parallel planes in the proposed model.

REFERENCES


