Minimal K-Covering Set Algorithm based on Particle Swarm Optimizer

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Abstract—For random high density distribution in wireless sensor networks in this article have serious redundancy problems. In order to maximize the cost savings network resources for wireless sensor networks, extend the life network, this paper proposed a algorithm for the minimal k-covering set based on particle swarm optimizer. Firstly, the network monitoring area is divided into a number of grid points. Utilization rate and the node minimum are used as optional objective, and a combinatorial optimization mathematical model is established. Then using Particle Swarm Optimizer to solve optimization model, thus the optimal network coverage and the utilization of sensor nodes are obtained. Simulation results that algorithm has reduced node redundancy and the energy consumption, and improved the network coverage effectively.

Index Terms—Wireless Sensor Networks (WSN); Particle Swarm Optimizer (PSO); The Minimal K-Covering Set; Network Lifetime

I. INTRODUCTION

Coverage and control problem is one of the basic problems in application of wireless sensor network [1] that often adopts the approach of random deployment. Owing to high density of sensor node, redundancy of a large quantity of nodes will be caused if all sensor nodes of the wireless sensor network are in operation. Moreover, energy of the sensor node is limited, so node redundancy will undoubtedly reduce the service efficiency of network. Node control and density control can save energy and extend lifetime of the entire wireless sensor network by reducing network redundancy. Literature [2] adopts the node regulation scheme of alternating “active” nodes with “dormant” nodes. It can effectively save energy and increase network lifetime, but it is a problem of NP difficulty to put the fewest sensor nodes into active working condition [3].

Artificial intelligent group optimization algorithm shows quite superior characteristics when solving the problem of NP difficulty; such algorithms have parallelism, so they can adapt to wireless sensor network well. In recent years, intelligent group optimization algorithm has been applied to the coverage problems of wireless sensor network in many studies. Literature [4] raises the optimized coverage mechanism based on genetic algorithm and constrained genetic algorithm which can work out quasi-optimal sensor node set required by the region fully covered with sensor network.

In order to extend working time of wireless sensor networks, the node density is controlled [5], which means that under the precondition of maintaining coverage property of the entire wireless sensor network, some sensor nodes are transferred into the sleep state of low-power dissipation. In this way, the requirement for coverage property of the network has been satisfied; meanwhile, the work node density has been reduced, sensor node redundancy of wireless sensor network has been decreased, and interference in wireless communication has been lessened. The ultimate purpose of node density control is to cover the target region with the fewest sensor nodes. It is a NP-complete problem to seek the minimal covering set of wireless sensor network.

A distributed density control algorithm based on detection (PEAS) [6] [7] can control switchover between active state and dormancy of the node via the sensor perception radius, and thus reach the purpose of minimal node covering set. Literature [8] provides a self-adaption and self-organization algorithm (ASCENT) to allocate the topological structure of network node organization. Both of the above algorithms cannot guarantee effective coverage rate for the target region; meanwhile, number of nodes in the minimal covering set cannot be determined.

Particle swarm optimizer was adopted to solve NP-complete problem in the above, and an algorithm of optimization selection for minimal node covering set was provided. This algorithm can solve the minimal sensor node set by aiming at different coverage rates.

II. PARTICLE SWARM OPTIMIZER

Particle swarm optimizer (PSO) [9] is an optimization method based on population and it was raised by J. Kennedy and R. C. Eberhart [10, 11] for the first time. It has advantages of simple structure, easy application and feasibility without gradient information, so it has been widely used in various optimization problems. With development of particle swarm, Literature [12] raises the improved algorithm of inertia factor \( \omega \) linear decrease, and particle swarm optimizer with inertia factor has been called standard particle swarm optimizer internationally.

Literature [13] provides a modified particle swarm optimizer which has effectively overcome the shortcoming of poor optimization effect of high dimensional function in standard particle swarm optimizer. Literature [14] raises a clustering particle swarm optimizer based on collision theory which has...
solved the premature problem in standard particle swarm optimizer.

Suppose that the particle swarm is composed of \( m \) particles, the particles are searching in the target space of \( D \) dimension, \( z_i = (z_{i1}, z_{i2}, \ldots, z_{id}) \) is the position vector of No. \( i \) particle, \( v_i = (v_{i1}, v_{i2}, \ldots, v_{id}) \) is the movement speed of No. \( i \) particle, \( p_i = (p_{i1}, p_{i2}, \ldots, p_{id}) \) is the optimal position searched by No. \( i \) particle, and \( p_s = (p_{s1}, p_{s2}, \ldots, p_{sd}) \) is the optimal position searched by entire particle swarm. In the iteration, the standard particle swarm will update velocity and position according to the following formula:

\[
v_{id}^{k+1} = \omega v_{id}^k + c_1 r_1 (p_{id} - z_{id}^k) + c_2 r_2 (p_{sid} - z_{id}^k) \quad (1)
\]

\[
z_{id}^{k+1} = z_{id}^k + v_{id}^{k+1} \quad (2)
\]

In which \( i = 1, 2, \ldots, m \); \( d = 1, 2, \ldots, D \), and \( d \) is the current dimensions; \( k \) is the current iteration time, and \( r_1 \) and \( r_2 \) are the random numbers in \([0,1]\).

The inertia weight \( w \) is:

\[
w = w_{\text{max}} - k \times (w_{\text{max}} - w_{\text{min}}) / \text{iter}_{\text{max}} \quad (3)
\]

In which \( w_{\text{max}} \) is the initial weight; \( w_{\text{min}} \) is the ultimate weight; \( \text{iter}_{\text{max}} \) is the maximum iteration time; \( k \) is the current iteration time.

### III. PROPOSED SCHEME

#### A. The Minimal Covering Set Problem Model

The sensor node in high density random distribution in the coverage region \( S = \{s_1, s_2, \ldots, s_s\} \), in which \( s_i \) is the sensor node by setting \((x_i, y_i)\) as coordinate and \( r_i \) as perception radius. A Boolean control vector \( X = (a_1, a_2, \ldots, a_s) \) was defined, and this control vector describes the state of sensor network nodes. In which \( a_i = 1 \) means that No. \( i \) sensor node is in working condition, and \( a_i = 0 \) indicates that No. \( i \) sensor node is in dormant state.

In terms of coverage optimization for wireless sensor network, on the one hand, network coverage rate \( f_1(X) \) should be maximized; on the other hand, node utilization rate \( f_2(X) \) should be minimized. Therefore, algorithm for minimal covering set of wireless sensor network is a multi-objective combinational optimization problem. Linear combination of objectives can be formed via weighting, and the original sub-objective optimization function can be transformed into single objective optimization function. The overall objective optimization function can be defined as:

\[
F(X) = \omega_1 f_1(X) + \omega_2 (1 - f_2(X)) \quad (4)
\]

In which \( 0 < \omega_1, \omega_2 < 1 \), \( \omega_1 + \omega_2 = 1 \), \( \omega_1 \) and \( \omega_2 \) are corresponding weights of sub-objective function, and their weights depend on the designer’s comprehensive requirement for this network index.

#### B. Objective Function Solution

In order to calculate \( k \)-coverage rate \( f(X) \), the coverage area was divided into \( m \times n \) small squares, and the barycentric coordinate of each small square expresses the entire region of small squares. Probability for the target node \( p(x, y) \) to be covered by node \( i \) is

\[
k_p = \left\{ \begin{array}{ll}
1, & k_p \geq k \\
0, & \text{others}
\end{array} \right.
\]

Then the multiple number \( k_i \) of the target node \( p(x, y) \) coverage is

\[
k_i = \sum_{p=1}^{N} k_p
\]

Judge whether node \( p(x, y) \) is under \( k \)-coverage of sensor node

\[
h_p = \left\{ \begin{array}{ll}
1, & k_p \geq k \\
0, & \text{others}
\end{array} \right.
\]

The approximate \( k \)-coverage rate of the region is

\[
f_1(X) = \frac{\sum_{p=1}^{N} h_p}{m \times n}
\]

The computational formula of node utilization rate function \( f_2(X) \) is

\[
f_2(X) = \frac{\sum_{i=1}^{N} a_i}{N}
\]

Then according to Formula (4), the computational formula of the objective function is:

\[
F(X) = \omega_1 f_1(X) + \omega_2 (1 - f_2(X))
\]

#### C. Algorithm Flow

Suppose that the surveyed area is \( 20m \times 20m \) square, and it is divided into \( 20 \times 20 \) grid points with the same size and area of 1, D sensor nodes scatter randomly, there are \( m \) particulates in the particle, and each particulate has \( D \) dimensions. \( v_i \) is initialized randomly within \([v_{\text{min}}, v_{\text{max}}]\).

The basic process of the minimal \( k \)-covering set algorithm based on particle swarm optimizer is as follows:

1) Positions of D sensors are initialized, and \( m \) particles are initialized in the target region.

2) The corresponding objective function of each particle is worked out according to the formula, to assign
the maximum particle datum of the objective function to $p_g$.

3) Velocity and state of the particle are updated according to Formula (3) and (4).

4) The regional objective function of each particle is re-calculated and then compared with $p_i$; if the value of new objective function is greater, then $p_i$ should be reset. Then it is compared with $p_g$; if the value of new objective function is greater, then value of this particle should be assigned to $p_g$.

5) End condition is judged: (maximum iteration time or good objective function value), return to the best sensor state, or go back to Step 3) continue.

IV. SIMULATION EXPERIMENT

A. Experimental Simulation

Suppose that the region A is the given target region of $20m \times 20m$, 300 sensor nodes randomly lay out in A, and the perception radius of sensor node is $2m$. Single coverage rate and k-coverage rate were simulated respectively. Fig. 1(b-f) presents the node distribution situation of the minimal single covering set selected after 50, 100, 150, 200 and 250 iteration algorithms via particle swarm optimizer. Fig. 2(b-f) presents the node distribution situation of the minimal k-covering set selected after 50, 100, 150, 200 and 250 iteration algorithms via particle swarm optimizer.

B. Analysis on Algorithm Performance

In order to further illustrate the situation, the simulation results were further analyzed through the following table. Table 1 shows the coverage rate, node utilization rate and objective function of the minimal single covering node set selected after different iteration times. Table 2 presents the coverage rate, node utilization rate and objective function of the minimal k (k=3)-covering node set selected after different iteration times.

In order to prove validity of the algorithm, Fig. 3(a) and Fig. 3(b) present relation among different iteration times, single coverage rate, and node utilization rate respectively. Fig. 4(a) and Fig. 4(b) present relation...
among different iteration times, k-coverage rate, and node utilization rate respectively.

**TABLE I. SIMULATION RESULT OF THE MINIMAL NODE SET OF SINGLE COVERAGE**

<table>
<thead>
<tr>
<th>Iteration time</th>
<th>Coverage rate ( (f_c) )</th>
<th>Node utilization rate ( (f_u) )</th>
<th>Objective function ( (F) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>100.00%</td>
<td>100.00%</td>
<td>0.3</td>
</tr>
<tr>
<td>50</td>
<td>98.00%</td>
<td>70.50%</td>
<td>0.3397</td>
</tr>
<tr>
<td>100</td>
<td>97.75%</td>
<td>64.50%</td>
<td>0.58085</td>
</tr>
<tr>
<td>150</td>
<td>96.50%</td>
<td>61.50%</td>
<td>0.5976</td>
</tr>
<tr>
<td>200</td>
<td>93.75%</td>
<td>58.00%</td>
<td>0.61275</td>
</tr>
<tr>
<td>250</td>
<td>92.55%</td>
<td>57.00%</td>
<td>0.61567</td>
</tr>
</tbody>
</table>

Relations between coverage rate and node utilization rate reflected in Fig. 3 and Fig. 4 are contradictory. When the iteration increases, number of the sensor nodes in operation decreases, which will inevitably reduce energy consumption in wireless sensor network and increase network lifetime. However, reduction of node quantity will also cause decrease of effective coverage rate in the coverage region. In practical application, when the requirement of effective coverage rate is high for wireless sensor network, weight of effective coverage rate should be increased properly, which can increase number of nodes in operation and thus enhance effective coverage rate of wireless sensor network. When there is strict demand on the lifetime of wireless sensor network, weight of effective coverage rate should be decreased, to use fewer sensor nodes in operation to satisfy the coverage requirement of the target region and save network energy. In order to adapt to requirements of
practical environment, limiters can be added into particle swarm optimizer, and the algorithm can be terminated when it meets the requirement of coverage rate or the node quantity of the minimal covering set.

<table>
<thead>
<tr>
<th>Iteration time</th>
<th>Coverage rate ($f_1$)</th>
<th>Node utilization rate ($f_2$)</th>
<th>Objective function ($F$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>100.00%</td>
<td>100.00%</td>
<td>0.3</td>
</tr>
<tr>
<td>50</td>
<td>79.75%</td>
<td>66.00%</td>
<td>0.50915</td>
</tr>
<tr>
<td>100</td>
<td>79.00%</td>
<td>63.00%</td>
<td>0.5276</td>
</tr>
<tr>
<td>150</td>
<td>78.25%</td>
<td>61.25%</td>
<td>0.5373</td>
</tr>
<tr>
<td>200</td>
<td>77.50%</td>
<td>61.00%</td>
<td>0.5365</td>
</tr>
<tr>
<td>250</td>
<td>76.75%</td>
<td>60.00%</td>
<td>0.54095</td>
</tr>
</tbody>
</table>

There is difference between the objective function obtained after calculation and the optimal objective function. Firstly, sensor nodes lay out randomly in the target region, while optimal solution is the active node distribution under ideal condition; secondly, particle swarm optimizer can only give quasi-optimal solution; finally, solution of calculating objective function adopts the estimation method of discretizing coverage area in the algorithm, so there exist some errors. However, the higher the discretization degree is, the fewer the errors are. But this will inevitably increase complexity of the algorithm.

C. Comparison Among Various Algorithms

In order to measure superiority of the algorithm proposed by this paper, experiment was conducted to compare this algorithm with ant colony algorithm and genetic algorithm on matlab simulation platform. Refer to Fig. 5 and Fig. 6 for simulation results of the three algorithms.

Figure 5. Comparison among three algorithms in sensor node utilization rate

Figure 6. Comparison among three algorithms in effective coverage rate
Fig. 5 presents relation between iteration times and sensor node utilization rate of three algorithms for the minimal k (k=3)-covering set of wireless sensor network. Fig. 5 shows that under specific algorithm iteration times, the algorithm raised in this paper can obtain higher effective coverage rate, and 3-coverage rate reaches about 75%. The experiment shows that performance of this algorithm is better than other two algorithms. When effective coverage rate is increased greatly, fewer nodes are required, less energy is consumed, and lifetime of wireless sensor network is extended effectively.

V. CONCLUSION

By aiming at the problem of huge redundancy in wireless sensor network, this paper raised an algorithm for the minimal k-covering set based on particle swarm optimizer. In this algorithm, the monitoring area is divided into a number of grid points firstly. Then the composite function of effective coverage rate and node utilization rate of the network is used as objective function; optimal solution of the network is searched via particle swarm optimizer; redundant nodes are reduced to the greatest extent on the condition that a certain k-coverage rate is guaranteed; thus extra energy consumption is reduced and network lifetime is extended. Finally, simulation experiment was conducted for the algorithm. The experimental result shows that such algorithm can obtain the highest coverage rate with the fewest wireless sensor network nodes. This has provided reliable evidence for node layout and adjustment, greatly reduced energy consumption, and effectively improved coverage property of the network. Moreover, it was compared with other two algorithms and the result fully proves superiority of this algorithm.

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REFERENCES


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