Abstract—Performance of cooperative spectrum sensing with multiple antennas at each cognitive radio is discussed in this paper. A new algorithm based on auto-correlation is proposed in which the optimal weights are obtained for each antenna in case little priori knowledge of channel characteristics as well as noise is known. In multiple antennas spectrum sensing, as long as the antenna characteristics are similar, the detection probability can be improved if more antennas are involved. However, if the antenna characteristics are quite different or the number of poorly performed antenna is large the detection probability deteriorates. Therefore, the well-performed antennas are selected in order to improve the detection probability. The performance of an antenna is obtained to determine whether it is deployed to sense the spectrum. A criterion is proposed to select the well-performed antennas to sense spectrum. Simulations are used to verify the method. The results indicate that the proposed antenna weighting and selection algorithm can be able to optimize network performance.

Index Terms—spectrum sensing, cognitive radio, antenna selection, detection performance, optimization

I. INTRODUCTION

According to the recent report published by Spectrum Policy Task Force within Federal Communications Commission (FCC), most of the spectrum is under-utilized for significant periods of time [1]. It indicates that the scarcity of spectrum is mainly due to inefficient spectrum allocation rather than physical spectrum inadequacy. Therefore, the technology of cognitive radio (CR) was proposed in order to implement efficient spectrum utilization [2]. This technology allows an unlicensed user (secondary user) to access a spectrum unoccupied by licensed user (primary user). The fundamental requirement for secondary user is to avoid interference with potential primary users in their vicinity. One of the most critical tasks of cognitive radio is spectrum sensing. Spectrum sensing is currently one of the most challenging tasks in CR design and implementation.

As described in the deployment scenario of the IEEE 802.22 wireless regional area network (WRAN), secondary systems should be located sufficiently far from primary systems to protect primary receivers from occasional interference caused by secondary transmitters. Under that scenario, of course, the signal-to-noise ratio (SNR) of the primary signal is low at the secondary sensing node. Moreover, in a fading environment, spectrum sensing is challenged by the channel uncertainty such as deep fading or shadowing [3]. In such a low SNR region with the fading channel recent research has focused on overcoming this poor performance by utilizing spatial diversity employing multi-antenna techniques at the secondary sensing node [4-7]. With these techniques, multiple antennas are used to perform spectrum sensing simultaneously. The sensing performance gain achieved in this case, however, is the tradeoff of increased complexity [8], where all the radio frequency (RF) chains have to be used at the same time to exploit full spatial diversity. Different MTM-multi-antenna based techniques are proposed in [9]. [10] investigates a new spectrum sharing algorithm based on price in cognitive radio networks. [11] analyzes the performance of multi-hop relay cooperative spectrum sensing.

The idea of antenna selection, which uses a subset of antennas selected from all of the available antennas, has been discussed extensively for the purpose of improving data transmission in MIMO systems [8][12]. Nevertheless, the application of antenna selection in spectrum sensing for a cognitive radio network has remained largely unexplored. An antenna selection based sensing scheme is proposed in [13]. However it didn’t give actual selection method for antenna. In this paper a new algorithm is proposed for selecting the antenna with the best detection performance so as to maximize the spectrum sensing sensitivity.

The proposed algorithm is able to select the antenna with the best performance in case channel coefficient and noise power are unknown. Meanwhile this algorithm can
distinguish the necessity for selecting antenna so as to optimize spectrum sensing performance.

The rest of this paper is organized as follows. In section two, general model for spectrum sensing is introduced. In section three, a multiple antenna spectrum sensing model based on cyclic auto-correlation (CA) function is described. Then the algorithm for selecting the antenna with the best detection performance is proposed. In section four, simulations are used to evaluate and compare the methods and finally we conclude the whole paper in section five.

II. GENERAL MODEL FOR SPECTRUM SENSING

In this section, we first present the general model for spectrum sensing, then review the cyclic auto-correlation detection scheme and analyze the relationship between the probability of detection and the probability of false alarm.

A. Sensing Model

Suppose that we are interested in the frequency band with carrier frequency $f_c$ and bandwidth $W$ and the received signal is sampled at sampling frequency. When the primary user is active, the discrete received signal at the secondary user can be represented as

$$x(n) = h(n)s(n) + w(n)$$

which is the output under hypothesis $H_1$. When the primary user is inactive, the received signal is given by

$$x(n) = w(n)$$

This case is referred to as hypothesis $H_0$. There are some assumptions.

- The noise $w(n)$ is a Gaussian, independent and identically distributed (i.i.d) random process with mean zero and variance $E\{w(n)w(n)^H\} = \sigma_w^2$.
- The primary signal $s(n)$ is an i.i.d random process with mean zero and variance $E\{s(n)s(n)^H\} = \sigma_s^2$.
- The primary signal $s(n)$ is independent of the noise $w(n)$.

Two probabilities are of interest for spectrum sensing: probability of detection, which defines, under hypothesis $H_1$, the probability of the algorithm correctly detecting the presence of primary signal, and probability of false alarm, which defines, under hypothesis $H_0$, the probability of the algorithm falsely declaring the presence of primary signal. From the primary user’s perspective, the higher the probability of detection is, the better protection it receives. From the secondary user’s perspective, however, the lower the probability of false alarm is, there are more chances for which the secondary users can use the frequency bands when they are available. Obviously, for a good detection algorithm, the probability of detection should be as high as possible while the probability of false alarm should be as low as possible.

B. Cyclic Auto-correlation (CA) Detector

The probability of successful detection of primary users in given frequency bands largely depends on the knowledge of signal & noise. Energy detection is a fundamental method which requires the knowledge of accurate noise power. However, it is very difficult to obtain the accurate noise power in practice, leading to the degradation of the detection quality.

Cyclostationarity feature detection is a method for detecting primary user transmissions by exploiting the cyclostationarity features of the received signals. These features can be used to discriminate the noise from modulated signal. In this paper, a spectrum sensing detector based on cyclic auto-correlation (CA) is used.

The numerical cyclic auto-correlation estimation of $x(n)$ is defined as [14]

$$\hat{R}_c(\alpha, \tau) = \frac{1}{N} \sum_{n=1}^{N} x(n)x^*(n+\tau)e^{-j\alpha\tau}$$

where $N$ is the number of observations and $\tau$ is time delay. $\alpha$ is called cycle frequency. There are many cyclic frequencies and cyclic frequencies can be assumed to be known or they can be extracted, which can be used as features for identifying transmitted signals. In the case of signal classification is not necessary while testing the presence of primary user is needed only, special cyclic frequency $\alpha = 0$ can be used to sense spectrum. So the test statistic $\gamma$ for spectrum sensing is written as

$$\gamma = \hat{R}_c(0, \tau) \quad \tau \neq 0$$

According to [14-15], under hypothesis $H_0$, the test statistic $\hat{R}_c(\alpha, \tau)$ is a random variable whose probability density function (PDF) is approximated by a complex Gaussian Normal distribution with mean $\mu_o = 0$ and variance

$$\sigma_o^2 = \frac{1}{N} \sigma_w^2$$

If we choose the detection threshold as $\lambda$, the probability of false alarm $P_{fa}$ is then given by

$$P_{fa} = P(\gamma > \lambda | H_0) = Q\left(\frac{\lambda}{\sigma_o^2 / \sqrt{N}}\right)$$

where $Q(\cdot)$ is the complementary distribution function of the standard Gaussian, i.e.,

$$Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^\infty e^{-\mu^2/2} d\mu$$

If the probability of false alarm of sensing system is given, according to (6), threshold $\lambda$ is set as

$$\lambda = \left(\frac{\sigma_o^2}{\sqrt{N}}\right)Q^{-1}(P_{fa})$$

where $Q^{-1}(\cdot)$ denotes the inverse function of $Q(x)$.

Under hypothesis $H_1$, the PDF of the test statistic $\gamma$ can be approximated by a Gaussian distribution with mean

$$\mu = \hat{R}_c(0, \alpha)$$

and variance

$$\sigma^2 = \frac{1}{N} \sigma_s^2$$
where $\hat{R}_i(\sigma, \tau)$ is the cyclic auto-correlation estimation of primary signal $s(n)$.  

For a chosen threshold $\lambda$, the probability of detection is given by

$$P_d = P(\gamma \geq \lambda | H_1)$$

$$= Q\left(\frac{\lambda - \hat{R}_i(0, \tau)}{\sigma^2_\gamma \sqrt{1 + 2\sigma^2 h^2 / \sigma^2_\gamma}}\right)$$

(11)

Usually, a constant false alarm rate (CFAR) method is used to verify the performance of the proposed sensing method. First, the threshold based on probability of false alarm $P_{fa}$ is fixed. Then the probability of detection $P_d$ can be given by

$$P_d = P(\gamma \geq \lambda | H_1)$$

$$= Q\left(\frac{\sigma^2_\gamma O(\tilde{P}_w) - \hat{R}_i(0, \tau)}{\sigma^2_\gamma \sqrt{1 + 2\sigma^2 h^2 / \sigma^2_\gamma}}\right)$$

(12)

### III. MULTIPLE ANTENNAS SENSING

A CR with multiple antennas at the receiver side is considered. It is assumed that there are $M$ antennas at the receiver. The channel between the primary user transmitter and the $i^{th}$ antenna of the CR receiver is modeled as a Rayleigh flat-fading channel with gain $h_i$ and $h_i$ is i.i.d random variables with unit variance.

#### A. Multiple Antennas Sensing Model

Suppose there is a primary signal transmission $s(n)$, the signal $x_i(n)$ is received at the $i^{th}$ receiver antenna over channel $h_i$ with Additive White Gaussian Noise $w_i(n)$. The received signal at the $i^{th}$ antenna can be two hypotheses and written as:

$H_0$: $x_i(n) = w_i(n)$

$H_1$: $x_i(n) = h_i(n)s(n) + w_i(n)$

(13)

(14)

Hypothesis $H_1$ refers to the presence of a primary user and hypothesis $H_0$ refers to the absence of a primary user.

Basic detector on each antenna of the secondary receiver first carries out detection process for the corresponding received signal. We also use cyclic auto-correlation (CA) detector as the channel sensing scheme to present our results. The decision statistic of $i^{th}$ receiver antenna is $\gamma_i$ and it is the cyclic auto-correlation of $x_i(n)$.

$$\gamma_i = \hat{R}_i(0, \tau)$$

(15)

After all the antennas finish sensing, $M$ decision results from $M$ antennas are obtained. The sensing decision is made according to the following test statistic

$$\gamma_M = \sum_{i=1}^{M} \epsilon_i \gamma_i$$

(16)

where $\epsilon_i$ is the weight coefficient of $i^{th}$ receiver antenna used to control the global spectrum detector.

For a large $N$, the PDF of $\gamma_M$ under hypothesis $H_0$ can be approximated by a Gaussian distribution with mean

$$E[\gamma_M | H_0] = 0$$

(17)

and variance

$$Var[\gamma_M | H_0] = \frac{\sigma^4_\gamma}{N}$$

(18)

Under hypothesis $H_1$, the PDF of the test statistic $\gamma_M$ can be approximated by a Gaussian distribution with mean

$$E[\gamma_M | H_1] = \sum_{i=1}^{M} \hat{R}_i(0, \tau)\epsilon_i$$

(19)

and variance

$$Var[\gamma_M | H_1] = \frac{\sigma^4_\gamma}{N} + \frac{2\sigma^2 h^2 \sigma^2_\gamma}{N}$$

(20)

The algorithm uses $\gamma_M$ & $\lambda_M$ to determine whether a primary user exists or not. Note that this threshold is different from that given in (8) for the single antenna case.

If we choose the detection threshold as $\lambda_M$, the probability of false alarm is then given by

$$P_{fa} = P(\gamma_M > \lambda_M | H_0) = Q\left(\frac{\lambda_M}{\sqrt{\sum_{i=1}^{M} \sigma^4_\gamma / N} \epsilon^2_i}\right)$$

(21)

When $P_{fa}$ is given, threshold $\lambda_M$ is set as

$$\lambda_M = \left\lfloor \sqrt{\sum_{i=1}^{M} \sigma^4_\gamma / N} \epsilon^2_i \right\rfloor$$

(22)

#### B. Antenna Weighting

Suppose the channel coefficients from the primary user to each receiver are known. Using maximal ratio combining (MRC) scheme, the weighting factor for each antenna is defined as

$$\epsilon_i = \frac{h_i}{\sum_{i=1}^{M} |h_i|^2}$$

(23)

So

$$\gamma_M = \sum_{i=1}^{M} \frac{h_i^2}{\sum_{i=1}^{M} |h_i|^2} \gamma_i$$

(24)

When the channel coefficients are unknown, a simple way for weighting factor is to choose $\epsilon_i = \frac{1}{\sqrt{M}}$. This is equal gain combining (EGC) method. In this case, we obtain:
\[ \gamma_M = \sum_{i=1}^{M} \frac{1}{\sqrt{M}} \gamma_i \]  

Equal density of noise is assumed in the two methods above and thus only the channel gains are considered. A new algorithm based on auto-correlation is proposed in this paper in which the optimal weights are obtained for each antenna in case little priori knowledge of channel as well as noise is known.

Correlation research is an important basic foundation for the study of mutual contact mode and the interrelated close degree of variable or variables group [16-18]. Usually the signal samples should be correlated due to that the signal is over sampled, while \( h_i(n) \) and \( w_i(n) \) are i.i.d, both will influence the correlation of the signal \( x_i(n) \). The more channel fades and the worse the noise becomes, the more they make influence on correlation of signal.

Define the sample auto-correlation of the received signal for the receiver antenna as
\[ \theta(l) = \frac{1}{N} \sum_{m=0}^{N-1} x(m)x(m-l) \quad l = 0, 1, \ldots, L - 1 \]  

where \( N \) is the number of available samples, \( L \) denotes time delay. Let
\[ G = \sum_{n=1}^{L} \theta(n) \]  

And define ratio B as
\[ B = \frac{G}{\theta(0)} \]  

where \( \sum_{n=1}^{L} \theta(n) \) reflects the correlation extent of signal and it is influenced by channel fading. \( \theta(0) \) reflects the correlation of additive white Gaussian noise and this value is bigger with noise becoming worse.

When the channel fading and the power of noise are comparatively small, B values are large and vice versa. B values reflects the synthetic impact of channel characteristics \( h_i(n) \) and noise \( w_i(n) \) on the signal \( s(n) \).

For a multi-antenna sensing system, it is assumed that channel gain \( h_i(n) \) for each antenna is different while the power of noise \( w_i(n) \) is identical. If channel gain is unknown for each antenna, weight equals
\[ \varepsilon_i = G_i / \sum_{i=1}^{M} G_i \]  

In reality, noise may lead to difference in \( w_i(n) \). If the channel characteristics and noise are considered simultaneously, the weights should be assigned adaptively according to B values. This method is termed as Auto Correlation (AU) method. The corresponding weights equal
\[ \varepsilon_i = B_i / \sum_{i=1}^{M} B_i \]

C. Antenna Selection Algorithm

In multi-antenna spectrum sensing system, threshold \( \lambda_M \) is set as
\[ \lambda_M = \left( \sqrt{\sum_{j=1}^{M} \sigma^4_i} \right) Q^{-1}(P_{fa}) \]

For a chosen threshold \( \lambda_M \), the probability of detection is given by
\[ P_d = \lambda_M \sum_{n=1}^{M} \varepsilon_i R_{11}(0, \tau) \]  

It is observed that the detection probability depends on the number of antennas \( M \), weights for each antennas \( \varepsilon_i \) as well as channel gains \( h_i \).

The optimization of detection probability \( P_d \) can be re-written as:
\[ \max_{P_d} \quad P_d \]  

s.t. \( P_f \leq P_{fa} \)

where \( P_f \) and \( P_{fa} \) represent the real false detection probability and target false detection probability respectively.

Solving (33) involves complicated computation. A frequently used method to obtain diversity gain is based on the assumption that both SNR and the number of antennas are large. As long as the antenna characteristics are similar, the detection probability can be improved if more antennas are involved. However, if the antenna characteristics are quite different or the number of poorly performed antenna is large the detection probability deteriorates. Therefore, the well-performed antennas are selected in order to improve the detection probability.

Which antennas are selected to sense spectrum is considered in this section. The goal is to optimum the detection sensitivity through antennas selection while meeting a given requirement on the probability of false alarm.

To better compare the performance of different antennas, define relative ratio \( \Delta B \) as
\[ \Delta B_i = 100 \times 1 / \max_{j \neq i} B_j \]  

\( \Delta B \) represents the difference ratio between \( i^{th} \) antenna and the best antenna. So ratio \( \Delta B \) can be used as criterion to select appropriate antenna to sense spectrum.

IV. NUMERICAL RESULTS

In this section Monte Carlo simulations are used to verify the method. A four-antenna system with four RF chains is considered. As a signal of interest, a BPSK time series is taken. The number of observations is \( N=200 \) and \( L=10 \). The computing results of B value are displayed in Fig.1 and Fig.2.
Fig. 1 shows different B values for various SNR. As seen, B values increase with the channel gains and vice versa when noise power is same. When noise power is large, e.g., SNR< -5dB, the difference of B values between three antenna are significant. As the SNR increases, the difference becomes marginal. Hence this method performs better in scenarios with low SNR values.

Fig. 2 shows the B value versus channel gain in case of given noise power. Three scenarios are simulated. SNRs are -5dB,-10dB and -15dB. It is observed that B values increase as the power of noise decrease and vice versa.

Figs. 1 and 2, show the impact of B value as well as noise on the signal s(n).

Fig. 3 gives the detection probability for SNRs when system false detection probability $P_{fa} = 0.1$ and M denotes the number of sensing antennas. It is observed that diversity gain improves significantly as the number of antennas increases. Meanwhile, the detection probability increases.

Fig. 4 compares three weights assigning methods for different sensing systems. Equation (29) is employed in the AU method for same background noise and antennas. MRC and EGC methods adopt (24) and (25) respectively. As seen, AU and MRC have similar performance while EGC performs worse.

Fig. 5 shows the AU methods in (30) with different antenna characteristics $w_i(n)$. In this case, both the background noise and channel characteristics are incorporated. However, only channel gain is assumed in MRC method. It is observed that AU behaves better comparing with MRC. EGC performs the worst.
Four antennas with random fading realizations and random noise variances in the environment are simulated. The $B$ values of four antennas can be calculated respectively as $B_1=5.39$, $B_2=5.25$, $B_3=4.98$, and $B_4=4.64$, and $\Delta B=0$, $\Delta B_2=3\%$, $\Delta B_3=8\%$, $\Delta B_4=16\%$. The values of $\Delta B$ show that only one antenna is good, while other three antennas are bad ($\Delta B_i>15\%$). It can be found that if comparing the whole four antennas to sense, the $P_d$ is the worst, while selecting one antenna to sense, the $P_d$ is the best. Fig.8 shows that selecting one antenna to sensing achieves the optimum $P_d$.

The optimum $P_d$ is usually achieved by cooperating parts of antennas that have higher $B$ values. Usually if the $\Delta B$ of one antenna is less than $10\%$, this antenna can be selected to sense, so as to improve sensing performance by utilizing spatial diversity. If the $\Delta B$ of one antenna is higher than $15\%$, this antenna cannot be selected to cooperate sensing, which makes performance worse. This shows that the $P_d$ can be improved through selecting antennas.

In Fig.8 the $B$ values of four antennas are calculated as $B_1=5.39$, $B_2=4.62$, $B_3=4.32$, and $B_4=4.32$, and $\Delta B=0$, $\Delta B_2=17\%$, $\Delta B_3=17\%$, $\Delta B_4=25\%$. The values of $\Delta B$ show that only one antenna is good, while other three antennas are bad ($\Delta B_i>15\%$). It can be found that if comparing the whole four antennas to sense, the $P_d$ is the worst, while selecting one antenna to sense, the $P_d$ is the best. Fig.8 shows that selecting one antenna to sensing achieves the optimum $P_d$.

In this paper, a spectrum sensing optimization algorithm based on antenna selection is proposed. In the case where the channel coefficient and noise power are not known, cooperating all antennas in the network does not achieve the best sensing performance. The numbers of selected antennas have certain influence on probability of detection. One algorithm for selecting the antenna with the best detection performance is proposed. Based on this algorithm, it can be distinguished whether it is necessary for antennas selection so as to optimize spectrum sensing performance. The results indicate that the proposed antenna selection algorithm is able to optimize network performance.

V. CONCLUSIONS

In this paper, a spectrum sensing optimization algorithm based on antenna selection is proposed. In the case where the channel coefficient and noise power are not known, cooperating all antennas in the network does not achieve the best sensing performance. The numbers of selected antennas have certain influence on probability of detection. One algorithm for selecting the antenna with the best detection performance is proposed. Based on this algorithm, it can be distinguished whether it is necessary for antennas selection so as to optimize spectrum sensing performance. The results indicate that the proposed antenna selection algorithm is able to optimize network performance.
ACKNOWLEDGEMENT

The work is supported by National Natural Science Foundation of China under Grant No. 60872003, application Basic Research Plans of Suzhou China under Grant No.SYJG0925, and Doctoral found of Ministry of Education of China under Grant No.20093201110005 from Soochow University.

REFERENCES


Yang Ou received the M.S. degree in Electrical Engineering from North University of China in 1996. She works as an associate professor in Electronic Engineering at University of Science and Technology of Suzhou China. She is currently pursuing her PhD in Electronics and Information Engineering at Soochow University. Her research interests include cognitive radio, spectrum sensing, signal processing for communications, wireless networking and statistical signal processing.

Yi-Ming Wang is professor at Dept. of Electronics and Information Engineering, Soochow University, China. The main research direction includes multimedia communications and wireless communications. Currently her academic research focuses on communication signal processing, cognitive radio, broadband wireless communications technology and the source channel coding.