Multi-Commodity Flow and Multi-Period Equilibrium Model of Supply Chain Network with Postponement Strategy

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Abstract—With the constant changes of the market demand trends and modern information technology which greatly influence supply chain operation and management, postponement has been developed to be an important strategy in some of the supply chain processes. This paper studies the multi-period and multi-commodity flow supply chain network equilibrium problem with postponement strategy under a type of supply chain management pattern. Taking the influences of the relationship between orders and inventory management of supply chain into consideration, and employing variational inequality to establish multi-period equilibrium model at each level and network equilibrium model, this paper eventually works out the conditions to make the system achieve equilibrium, and also offers economic interpretation with concrete examples to verify them.

Index Terms—supply chain network, equilibrium, multi-commodity, multi-period, postponement strategy

I. INTRODUCTION

The major supply chain issues can be briefly described as this: if a company is required to timely provide a quantity of adequate goods to its customers, it should offer satisfying service and make sound decision to guarantee appropriate supply, manufacturing, and transportation in this network and the right chain, appropriate behavior to satisfy the members in this chain. Generally speaking, supply chain network mainly has the following characteristics:

(1) It is dynamic as the supply chain changes with the change of the objectives. Supply and demand also varies when the way of service differs.

(2) It is complex as there is a number of multi-level entities with multi-objective in a supply chain.

(3) Time-lag exists universally. As when pull-type production appears, the information of demand needs to be transformed along the chain from downstream to upstream, which leads to the born of inventory in each supply chain level.

It is noteworthy that the supply chain network equilibrium analysis is an effective method to solve complex supply chain issues. Nagurney [1]-[4] establishes static equilibrium model of supply chain network using the concept of balanced network and variational inequality and taking the independent behavior and interaction into consideration. However, they haven’t take inventory which is of great importance in a supply chain into consideration. Although reference [5] does a research on the dynamic equilibrium issues between manufacturers and wholesalers, its models don’t study specifically the competition among the members in the network but only do a research on the segmental equilibrium between price and inventory. Reference [6] extends the model to a finite-dimensional variational inequality governing the equilibrium of the multi-period competitive supply chain network. Reference [7] proposes a new concept of purchasing strategy to model the strategic behavior of retailers and consumers at demand markets in a capacitated supply chain network.

It is noteworthy that most of the literature above assumes that stock will never run out, which in most cases is very impossible. Besides, in the procurement, there are certain advantages when the appropriate stock or postponed orders happen. Postponement has been developed to be an important strategy in some of the supply chain processes. For the buyer, although the cost of shortage and customer waiting increases, the times of booking can be reduced and the time for inventory custody is shortened. Therefore, the total order disposal costs and inventory storage costs are reduced. For the supplier, it not only reduces reduces logistics operating costs, but also increases the flexibility of the production
operation. Besides, the market risk can be passed on to the buyer.

Based on the study above, taking the time factor into consideration, and use tools as network equilibrium and variational inequality, we will use a popular and representative Echelon Operational Autonomy (EOA) and study multi-period equilibrium issues in multi-level and multi-commodity supply chain network with postponement strategy where there are the competitions among members.

II. TYPES OF SUPPLY CHAIN AND THE PROBLEMS INVOLVED

The traditional supply chain involved a multi-level inventory where manufacturing, distribution and retail have their own inventory control strategy. However, management errors and the mutual impact of each node easily result in bullwhip effect in the supply chain, that is, the expansion of the inventory issues in the supply chain. With the development of the researches on supply chain, patterns as JMI, CPFR, and VMI are put forward in the inventory management among which VMI is the most popular one as in this pattern not the downstream but the upstream (usually the manufacturer) takes charge of the inventory management. And this pattern could improve the performance of its time variables. Taking this pattern as a research object is in line with trends in the supply chain and close to reality, and it also makes the network equilibrium models established more focused.

Nowadays, the demands of the market change rapidly, and information technology has greatly developed, compared with stocking-type, make-to-order will be more responsive to the demand of the market, where each level is not only the supplier of next level, but also the one who orders in the former level. Thus, the supply chain network demand this paper studies changes over time, considering inventory factors, and reasonable separation of trade flow and material flow with the consideration of order. It includes three different levels, the manufacturer, the retailer and the market of demand. Each node can be divided into different types according to its members: manufacturers, retailers and market of demand. The specific structure is: there are M manufacturers, n distributors j = 1, 2, ..., n ; G customers, k = 1, 2, ..., o ; L products h = 1, 2, ..., l . What the manufacturers produces, distributors sales, and customer needs are the same products with no difference with and there is non-cooperation and competition in the supply chain network of the same level. As the commodities each manufacturer produces and retailer sales are all the same ones the consumers need, so there is Non-cooperative competition in the same level of the supply chain network. The acts the manufacturer makes at t go as the following descriptions. Firstly, the manufacturer manufactures. Then, it gets the orders from the retailer. Finally, the manufacturer makes the decision of delivering after checking the inventory of t-1 and the ones produced at t. The commodities will be delivered to the retailer where the consumers can get them. After this, the manufacture checks the stock and makes clear of the commodities lacking to prepare for the following production. The acts the retailer makes at t go like this. First, it begins to organize the market display and promotional activities and receives the orders. The retailer responds to customers’ needs and sends orders to the manufacturer. Finally, it will prepare for the following marketing events.

Besides, we suppose that according to the changes of time, the inventory at the beginning and end of each period and the inventory delayed change at the same time and the cycle time we study includes many periods; the retailer is required to pay when it orders, and the manufacturer makes the decision to deliver the commodities to the retailer. Once the decision is made, the commodities-delivery will be finished at once. If the consumers’ needs are not satisfied, the manufacturer must offer the customer-waiting cost which is accumulated as the result of time for its delay.

Thus, under the EOA, the manufacturer and retailer can get the demand and logistics information without delay which is different from centralized supply chain whose goal is maximizing the profit. Different from the decentralized supply chain studied by the literature above, the problems involved in decision-making and models which each level faces change on the basis that mechanism and business model of logistics operations are widely recognized by the market. But the independent decision-making behavior of the member of the supply chain is still considered, and also the interactions among the members. So equilibrium also can be reached which could satisfy of all parties. Under multi-phase mode, the manufacturer could adjust its production and operation strategies by each phrase’ production, inventory management and order processing, while the retailer through adjusting order management strategies, and finally the equilibrium of the multi-commodity flow in the supply chain network could be achieved.

Variational inequalities equilibrium problems, a tool now widely used in the economic field and urban transportation network modeling. Originally, it was developed as a research tool, studying a class of mechanical problems’ partial differential equations which were defined in the infinite dimensional space. Finite-dimensional variational inequality problem (VIP) is to determine a vector \( x^* \in D \subseteq E^n \), making the formula:

\[
F(x^*)^T (x-x^*) \geq 0, \quad \forall x \in D, \quad F(X): D \rightarrow E^n
\]

is a given continuous vector-valued functions, \( x^* \in D \) is a non-empty closed convex set, and vector \( F(x^*) \) and the feasible set D is orthogonal, pointing to the inside of the feasible set when it is \( x^* \). The following are researches on every level of the supply chain network. It is aimed to describe the decision-making behaviors of every layer’s market member and to derive the variational inequality model which meets the equilibrium conditions of the decision-making behavior and convert it into a complementary form of a question. Finally, the economic interpretation for the equilibrium conditions will be given.

A. Manufacturing Level Equilibrium Model

Let us assume \( q_{it}^b \) is the non-negative output the manufacturer produces at time \( t \), \( q_{it} = (q_{i1t}, ..., q_{it}^1, ..., q_{it}^m) \) is the production vector the manufacturer produces at time \( t \) , \( q_i = (q_{i1}, ..., q_{it}, ..., q_{in}) \) is the production vector
all manufacturers produce at time $t$, $q = (q_1, ..., q_i, ..., q_J)$ is the production vector all manufacturers produce during each plan period, the rest of the three-dimensional vector is similar to definition mentioned above; $v_{ij}^h$ is the commodity $h$’s order quantity the manufacturer $i$ and retailer $j$ transact at time $t$, $V_{ij} = (V_{ij}^1, ..., V_{ij}^b, ..., V_{ij}^h)$ is the each commodity’s order vector of manufacturer $i$ and retailer $j$ at time $t$, $V_i = (V_{i1}, ..., V_{ij}^b, ..., V_{iJ})$ is the order vector of manufacturer $i$ and each retailer’s, $V = (V_{11}, ..., V_{ij}, ..., V_{JJ})$ is the order vector of each manufacturer and each retailer’s. The following four-dimensional vector is similar to definition mentioned above; $q_{ij}^b$ is the manufacturer $i$ and retailer $j$’s actual quantity of replenishment of commodity $h$ at time $t$, $Q_{ij}, Q_i, Q_j, Q$ are its four-dimensional vectors; $p_{ij}^b$ is the price of the product $h$ between manufacturer $i$ and retailer $j$; $c_{ij}^h$ is the manufacturer $i$’s unit production cost of $h$; $d_{ij}^h$ denotes the unit transport costs; $e_{ij}^h$ is each unit’s inventory holding cost of per unit time; $I_{ij}^t$ is the manufacturer $i$’s warehouse inventory levels at the time points $t$, $I_{ij}, I_i, I$ are its three-dimensional vectors; $w_{ij}^h$ denotes the manufacturer $i$’s the customer waiting cost per unit time; $\theta_{ij}$ is the manufacturer $i$ and retailer $j$’s actual quantity of delayed commodity $h$ at time $t$, $\theta_{ij}, \theta_i, \theta_j, \theta$ are its four-dimensional vectors. We could start from a rational economic man, each manufacturer seeks to maximize their profits. Their total revenue is equal to the total sales subtracts the sum of production costs, transportation and transaction costs, and adds the cost of inventory holding.

For any manufacturer, the profit maximization objective function is:

$$\max \sum_{i=1}^{I} \sum_{j=1}^{J} \sum_{h=1}^{H} p_{ij}^b v_{ij}^h - \sum_{i=1}^{I} \sum_{j=1}^{J} c_{ij}^h (q_i^h) - \sum_{i=1}^{I} \sum_{j=1}^{J} d_{ij}^h (q_i^h) - \sum_{i=1}^{I} \sum_{j=1}^{J} e_{ij}^h (I_{ij}^t) - \sum_{i=1}^{I} \sum_{j=1}^{J} \sum_{h=1}^{H} w_{ij}^h (\theta_{ij})$$

(1)

We assume that all the cost functions above are continuous and convex. And consider equation (2) the production quantity, order quantity and flow quantity of non-negative constraints.

$$q_{ij}^h \geq 0, v_{ij}^h \geq 0, q_{ij}^b \geq 0, I_{ij}^t \geq 0, \theta_{ij} \geq 0, \forall j, h, t$$

(2)

In the manufacturing market, any manufacturer $i$’s quantity of product $q_{ij}^b$ during the period $t$ and the inventory of last period $I_{ij}^{t-1}$ will be offered to the retailers of the downstream. If excess supply exists in this period, at the end of the period the manufacturer will convert the spare products to the current retained stock $I_{ij}^t$. Equation (3) represents the constraints of the flow of inventory; the quantity of the manufacturer’s current production plus last period’s inventory minus the current inventory must be no less than the current compensation received by the retailer’s.

$$q_{ij}^b + I_{ij}^{t-1} - I_{ij}^t \geq \sum_{i=1}^{N} q_{ij}^b, \forall h, t$$

(3)

On the basis of the assumption (1), considering that $I_{ij}^t$’s indicator should coordinate with $q_{ij}^b$’s indicator, here we make $I_{ij}^0 = I_{ij}^t$, which means the inventory at the beginning of the cycle is the same to the one in the end. Besides, manufacturer will make the logistics operational decisions according to the remaining detailed order $Q_{ij}$ and the order in period $t$ and select current orders to replenish directly. If the current order is still excessive, the manufacturer make them the current period’s detailed order $I_{ij}^t$. Equation (4) represents the order processing flow constraints, which means the manufacturer’s current replenishment and the current pending orders minus last period’s pending order must be no less than the orders the retailer needs when purchasing.

$$q_{ij}^b + \theta_{ij} - \theta_{ij} \geq \sum_{j=1}^{J} v_{ij}^h, \forall j, h, t$$

(4)

We also make $Q_{ij} = Q_{ij}^t$, which means the quantity of detailed orders at the beginning of the cycle is the same to the one in the end. Then we can put the constraints into the objective function using the Lagrange multipliers, the optimal model can be transformed into:

$$\max \sum_{i=1}^{I} \sum_{j=1}^{J} \sum_{h=1}^{H} p_{ij}^b v_{ij}^h - \sum_{i=1}^{I} \sum_{j=1}^{J} c_{ij}^h (q_i^h) - \sum_{i=1}^{I} \sum_{j=1}^{J} d_{ij}^h (q_i^h) - \sum_{i=1}^{I} \sum_{j=1}^{J} e_{ij}^h (I_{ij}^t) - \sum_{i=1}^{I} \sum_{j=1}^{J} \sum_{h=1}^{H} w_{ij}^h (\theta_{ij}) - \sum_{i=1}^{I} \sum_{j=1}^{J} c_{ij}^h (q_i^h) - \sum_{i=1}^{I} \sum_{j=1}^{J} d_{ij}^h (q_i^h) - \sum_{i=1}^{I} \sum_{j=1}^{J} e_{ij}^h (I_{ij}^t) - \sum_{i=1}^{I} \sum_{j=1}^{J} \sum_{h=1}^{H} w_{ij}^h (\theta_{ij})$$

(5)

When every manufacturer makes the best response to other manufacturer’s actions and has no motivation to change its strategy, the relatively stable point, that is the Nash equilibrium $(q^*, V^*, Q^*, I^*, u^*, \pi^*)$ can be reached. We can well know the optimization problem in literature [8], [9]. The conditions members of all manufacturing level can achieve the equilibrium can be equivalent to the following variational inequalities:

$$\sum_{i=1}^{I} \sum_{j=1}^{J} \sum_{h=1}^{H} \frac{\partial c_{ij}^h (q_{ij}^b)}{\partial q_{ij}^b} (q_{ij}^b - u_{ij}^b) \times (q_{ij}^b - q_{ij}^b) + \sum_{i=1}^{I} \sum_{j=1}^{J} \sum_{h=1}^{H} \frac{\partial d_{ij}^h (q_{ij}^b)}{\partial q_{ij}^b} (q_{ij}^b - q_{ij}^b) + \sum_{i=1}^{I} \sum_{j=1}^{J} \sum_{h=1}^{H} \frac{\partial e_{ij}^h (I_{ij}^t)}{\partial I_{ij}^t} (I_{ij}^t - I_{ij}^t) + \sum_{i=1}^{I} \sum_{j=1}^{J} \sum_{h=1}^{H} \frac{\partial w_{ij}^h (\theta_{ij})}{\partial \theta_{ij}} (\theta_{ij} - \theta_{ij}) + \sum_{i=1}^{I} \sum_{j=1}^{J} \sum_{h=1}^{H} \frac{\partial Q_{ij}}{\partial \theta_{ij}} (Q_{ij} - Q_{ij}) + \sum_{i=1}^{I} \sum_{j=1}^{J} \sum_{h=1}^{H} \frac{\partial \theta_{ij}}{\partial Q_{ij}} (\theta_{ij} - \theta_{ij}) + \sum_{i=1}^{I} \sum_{j=1}^{J} \sum_{h=1}^{H} \frac{\partial \theta_{ij}}{\partial \theta_{ij}} (\theta_{ij} - \theta_{ij})$$

(6)
\[ u^h_{ij} - u^*_{ij} \times (1 - t^*_{ij}) + \sum_{j=1}^{m} \sum_{k=1}^{n} \sum_{i=1}^{r} \sum_{t=1}^{s} \left( \frac{\partial g^h_{ij}}{\partial q^h_{ij}} (v^h_{ij}) - p^h_{ij} \right) \]

\[ + \pi^h_{ij} \times (v^h_{ij} - v^*_{ij}) + \sum_{j=1}^{m} \sum_{k=1}^{n} \sum_{i=1}^{r} \sum_{t=1}^{s} \left( \frac{\partial h^h_{ij}}{\partial \theta^h_{ij}} (\theta^h_{ij}) + \pi^h_{ij} \right) \]

\[ - \pi^h_{ij} \times (\theta^h_{ij} - \theta^*_{ij}) + \sum_{j=1}^{m} \sum_{k=1}^{n} \sum_{i=1}^{r} \sum_{t=1}^{s} (q^h_{ij} - \sum_{j=1}^{m} g^h_{ij} + l^h_{ij} - l^*_{ij}) \times (u^h_{ij} - u^*_{ij}) + \sum_{j=1}^{m} \sum_{k=1}^{n} \sum_{i=1}^{r} \sum_{t=1}^{s} (\theta^h_{ij} - \theta^*_{ij}) - \pi^h_{ij} \times (\theta^h_{ij} - \theta^*_{ij}) \]

\[ \times (\pi^h_{ij} - \pi^*_{ij}) \geq 0 \]

\[ \forall (q, V, Q, I, \theta, u, \pi) \in R^{nT + mT + nT + mT + nT} \]

Equation (6) shows that the manufacturer will begin to manufacture only when the manufacturer's current period marginal production cost is equal to the minimum operating inventory holding cost. If it is more, the manufacturer won’t manufacture. When the manufacturer's current period logistics operations and management cost is equal to the minimum operating inventory holding cost, it will replenish the product. If it is more, the manufacturer won’t replenish. The manufacturer will accept orders when the minimum supply cost plus the minimum order cost is equal to the price. If it is more, the manufacturer won’t accept orders. The current inventory's minimum supply cost plus its marginal cost is equal to its next period's minimum supply costs to be paid to hold this inventory issue of holding, then a certain amount of inventory will be retained to the next period. However, if there is more, the result mentioned above won’t happen. Next period’s minimum supply cost plus the current period’s marginal customer-waiting cost is equal to the current period’s minimum supply cost, a certain amount of orders will be delayed to next period, but if there’s more, this won’t happen.

B. Retailing Level Equilibrium Model

Retailers located at the middle level are also seeking to maximize their own interests, which on one hand obtain orders from the demand market, on the other hand complete the ordering behavior of the manufacturer. We assume \( v^h_{ij} \) is order quantity retailer \( j \) obtain from the demand market \( k \) at time \( t \), \( V^h_{ij} \), \( V^*_{ij} \), \( \tilde{V} \), \( V \) are its four-dimensional vectors; \( p^h_{ij} \) is the price of the product \( h \) between retailer \( j \) and the consumers at demand market \( k \); \( g^h_{ij} = g^h_{ij}(V) \) is retailer \( j \)'s unit operating cost of retailer \( j \), decided by all orders of product \( h \).

For any retailer, the profit maximization objective function is:

\[ \max \sum_{k=1}^{n} \sum_{h=1}^{m} \sum_{i=1}^{r} \sum_{t=1}^{s} p^h_{ij} v^h_{ij} - \sum_{k=1}^{n} \sum_{h=1}^{m} \sum_{i=1}^{r} \sum_{t=1}^{s} \sum_{j=1}^{m} g^h_{ij}(V) + \sum_{i=1}^{m} \sum_{k=1}^{n} \sum_{j=1}^{m} \sum_{t=1}^{s} v^h_{ij} \]

\[ \forall i, k, h, t \]

\[ \forall i, k, h, t \]

This objective function shows every retailer wants to maximize its profits which come from the price difference between the manufacturer and the demand market minus the display and operating cost. Let’s assume \( g^h_{ij}(V) \) is the continuous convex function and consider all the transactions or its non-negative constraints in equation (8) and the transactions capacity constraints of (9).

\[ v^h_{ij} \geq 0, p^h_{ij} \geq 0, \forall i, k, h, t \]

\[ \sum_{i=1}^{m} \sum_{k=1}^{n} \sum_{h=1}^{m} \sum_{i=1}^{r} \sum_{t=1}^{s} v^h_{ij} \geq \sum_{i=1}^{m} \sum_{k=1}^{n} \sum_{h=1}^{m} \sum_{i=1}^{r} \sum_{t=1}^{s} v^h_{ij} \]

Then the optimal model can be transformed into:

\[ \max \sum_{k=1}^{n} \sum_{h=1}^{m} \sum_{i=1}^{r} \sum_{t=1}^{s} p^h_{ij} v^h_{ij} - \sum_{k=1}^{n} \sum_{h=1}^{m} \sum_{i=1}^{r} \sum_{t=1}^{s} \sum_{j=1}^{m} g^h_{ij}(V) + \sum_{i=1}^{m} \sum_{k=1}^{n} \sum_{j=1}^{m} \sum_{t=1}^{s} v^h_{ij} \]

\[ \forall i, k, h, t \]

When every manufacturer makes the best response to other manufacturer’s actions and has no motivation to change its strategy, the relatively stable point can be reached, that is the Nash equilibrium \((V^*, \tilde{V}^*, \tilde{u}^*)\). The conditions members of all manufacturing level can achieve the equilibrium can be equivalent to the following variational inequalities:

\[ \sum_{k=1}^{n} \sum_{h=1}^{m} \sum_{i=1}^{r} \sum_{t=1}^{s} \left( \frac{\partial g^h_{ij}}{\partial q^h_{ij}} (V^*) + p^h_{ij} - \pi^h_{ij} \right) \times (v^h_{ij} - v^*_{ij}) + \sum_{k=1}^{n} \sum_{h=1}^{m} \sum_{i=1}^{r} \sum_{t=1}^{s} \left( \frac{\partial h^h_{ij}}{\partial \theta^h_{ij}} (\theta^*_{ij}) + \pi^h_{ij} \right) - \pi^h_{ij} \times (\theta^h_{ij} - \theta^*_{ij}) \geq 0 \]

\[ \forall (V, \tilde{V}, \tilde{u}) \in R^{nT + mT + nT + mT + nT} \]

\( \tilde{u}^h_{ij} \) is constraints (9)'s Lagrange multiplier, which is the minimum supply cost the retailer \( j \) sell the product, and \( h \), \( \tilde{u}^h_{ij} \), \( \tilde{u}^h_{ij} \), \( \tilde{u}^h_{ij} \) respectively represent their corresponding vector in their dimension. The retailer will sell the product, only when every retailer’s current period’s minimum supply cost is no more than the price when it sells the product, or it won’t. The retailer won’t order when its current period’s marginal operating cost and the subscription price of the product is more than the minimum supply cost. Only when its current period’s marginal operating cost and the subscription price of the product is equal to the minimum supply cost, the retailer will order. When product \( h \) of the retailer \( j \)'s down stream and upper stream ‘market clears’, the minimum cost of supply appears.
C. Demanding Level Equilibrium Model

Let us assume $e_{jk}^h = c_{jk}^h (Q^h)$ is the transaction cost of demanding market $k$ from retailer $j$ at time $t$; $q_{jk}^h$ is the demanding market $k$ and retailer $j$’s actual quantity of replenishment of commodity $h$ at time $t$; $p_{jk}^h$ is the price of the product $h$; the consumers are willing to pay at the demand market $k$; $p = (p_1, ..., p_t, ..., p_T)$ is the price vector the consumers are willing to pay at all demand market at time $t$. $z_{jk}^h = z_{jk}^h (p)$ is demand quantity at demand market $k$ at time $t$, as competitions exist between market and other market, which decided by their own market and other market price.

\[
p_{jk}^h + c_{jk}^h (v_{jk}^h) \geq 0 \quad \text{and} \quad v_{jk}^h = 0
\]

(12)

(12) shows that when reaching an equilibrium, the price of product $h$ and the cost needed in the demand market will be more than $p_{jk}^h$ which is the cost the demand market will afford, or the cannot be achieved. (13) shows when reaching the equilibrium the price of the product the demand market will pay must be more than 0, or when supplies are more than demands, the products will be free. The condition for the equilibrium of all the members of the demand level is:

\[
\sum_{j=1}^{n} \sum_{k=1}^{d} \sum_{t=1}^{T} (p_{jk}^h + c_{jk}^h (v_{jk}^h) - p_{jk}^h (v_{jk}^h - v_{jk}^h)) + \sum_{j=1}^{n} \sum_{k=1}^{d} \sum_{t=1}^{T} \sum_{k=1}^{d} \sum_{t=1}^{T} (v_{jk}^h - z_{jk}^h (p^*)) \geq 0
\]

(13)

D. Establishment of Network Equilibrium Model

EOA type of supply chain competition and coordination in its mechanism, the supply chain network will eventually reach equilibrium, the level of inter-linked with the inflow of orders out the same, but the manufacturer’s production, fill volume, inventory, deferred amount and the price of the order must meet the variational inequality (6), (11), (14) and. Accordingly, we give the proposed structure of the EOA such a competitive supply chain network equilibrium definition. We make

\[
\Omega = \{(q, v, Q, \lambda, \theta, u, \pi, \mu, \pi, F, p) | \quad \text{such that} \quad q, v, Q, L, \theta, u, \pi, \mu, \pi, F, p \}
\]

Definition 1. The equilibrium condition governing the whole supply chain network is given by: determine $(q^*, v^*, Q^*, L^*, \theta^*, u^*, \pi^*, \mu^*, F^*, p^*) \in \Omega$ satisfying:

\[
\sum_{j=1}^{n} \sum_{k=1}^{d} \sum_{t=1}^{T} (q_{jk}^h - q_{jk}^h) + \sum_{j=1}^{n} \sum_{k=1}^{d} \sum_{t=1}^{T} (v_{jk}^h - v_{jk}^h) \times (u_{jk}^h - u_{jk}^h) + \sum_{j=1}^{n} \sum_{k=1}^{d} \sum_{t=1}^{T} \sum_{k=1}^{d} \sum_{t=1}^{T} (p_{jk}^h + c_{jk}^h (v_{jk}^h) - p_{jk}^h v_{jk}^h) \times (p_{jk}^h - p_{jk}^h) \geq 0
\]

(15)

Proof: add (6), (11), (14) together and simplify it, then we can obtain the above variational inequality (15).

III. ALGORITHM AND EXAMPLE

In this section, we propose the algorithm to compute the variational inequality problem. The algorithm we use is the modified projection method presented by Nagurney. Variational inequality problem (15) can be transformed into the standard variational inequality problem, to find $X^* \in \Omega$ to make it satisfy the following variational inequality problem.

\[
(F(X), X - X^*) \geq 0 \quad \forall X \in \Omega
\]

(16)

Here, the elements of the variational inequality (15) in front of the multiplication function type, n-dimensional Euclidean space the inner product. Algorithm procedure is as follows:

Step 0: initialization. Construct the initial solution $X^0 \in \Omega$, denote the current iteration step $m = 1$, choose $\alpha : 0 < \alpha < 1/L$ (L is Lipschitz constant);

Step 1: compute. Compute $\bar{X}^m$ by solving:

\[
(\bar{X}^m + \alpha F(X^m - X^m)) \geq 0, \quad X \in \Omega
\]

Step 2: adaptation. Compute $X^m$ by solving:

\[
(X^m + \alpha F(X^m - X^m)) \geq 0, \quad X \in \Omega
\]

Step 3: Convergence verification. If $|X^m - X^m| \leq \varepsilon$ with $\varepsilon > 0$ (a pre-specified tolerance), then stop; otherwise set $m = m + 1$ and back to step 1.

As long as the function $F(X)$ is monotone and Lipschitz continuous, then the above Algorithm is convergent. We refer to Reference[5]-[6] and construct a simple supply chain networks where there are two manufacturers, two retailers, two demand markets and two kinds of products. Thus, there are two nodes in each level and four paths between each two levels. We also assume that the planning period is five unit times. And the process of cost is symmetry. In order to simplify the calculation, variables units are not considered here.

Decision variables are defined as follows:
TABLE I.
COST AND DEMAND FUNCTION PARAMETERS

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<td>1 1 1 1 1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\delta^o$</td>
<td>1 1 1 1 1</td>
<td>1 1 1 1 1</td>
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</tr>
<tr>
<td>1 1 1 1 1</td>
<td>1 1 1 1 1</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Manufacturer’s production cost:

$\delta^i = \delta^i(q^i) + \delta^o(w^i_{q_i} + v^i_{p^i})$  

Manufacturer’s logistics operating cost:

$\delta^i = \delta^i(q^i) + \delta^o(w^i_{q_i})$  

Manufacturer’s inventory holding cost:

$\delta^i = \delta^i(q^i) + \delta^o(v^i_{p^i})$  

Manufacturer’s transaction cost:

$\delta^i = \delta^i(v^i_{p^i} + v^i_{p^i})$  

Manufacturer’s customer waiting cost:

$\delta^i = \delta^i(w^i_{q_i} + v^i_{p^i})$  

Retailer’s operating cost:

$\delta^i = \delta^i(w^i_{q_i} + v^i_{p^i})$  

Customer’s transaction cost:

$\delta^i = \delta^i(v^i_{p^i} + v^i_{p^i})$  

Demand market’s demand quantity:

$\delta^i = \delta^i(p^i_{q_i})$  

The specific settings of cost and demand function parameters are shown in Table I. Based on the models and the modified projection algorithm established above, we simulate the numerical problem by matlab7.0 simulation software ($\alpha = 0.005$, $\epsilon = 0.0005$) and get the results which are shown in Table II.

As can be seen from Table I, the two kinds of commodity of the supply chain network have different requirements in their cost and needs. For example, the obvious difference that commodity 1 has a lower requirement of time but higher one as to inventory-holding cost. Requirements of the timeliness of goods a lower unit cost of higher inventory, lower their price sensitivity.

TABLE II.
RESULTS OF SUPPLY CHAIN NETWORK EQUILIBRIUM MODE

<table>
<thead>
<tr>
<th>Parameter</th>
<th>time</th>
</tr>
</thead>
<tbody>
<tr>
<td>$q^i_{t+1}$</td>
<td>1 2 3 4 5</td>
</tr>
<tr>
<td>$h = 1$</td>
<td>41.34 41.39 22.33 41.29 41.30</td>
</tr>
<tr>
<td>$h = 2$</td>
<td>40.36 38.81 36.04 33.65 30.59</td>
</tr>
<tr>
<td>$I^i_{t+1}$</td>
<td>1 2 3 4 5</td>
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<tr>
<td>$h = 1$</td>
<td>9.39 13.67 0.00 2.79 5.84</td>
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<tr>
<td>$h = 2$</td>
<td>4.45 7.36 7.51 5.28 0.00</td>
</tr>
<tr>
<td>$v^i_{p^i}$</td>
<td>1 2 3 4 5</td>
</tr>
<tr>
<td>$h = 1$</td>
<td>18.77 18.76 18.76 18.76 18.76</td>
</tr>
<tr>
<td>$h = 2$</td>
<td>17.95 17.79 17.95 17.95 17.95</td>
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<tr>
<td>$q^i_{t+1}$</td>
<td>1 2 3 4 5</td>
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<tr>
<td>$h = 1$</td>
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<tr>
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<td>17.96 17.95 17.95 17.95 17.95</td>
</tr>
<tr>
<td>$p^i_{q^i}$</td>
<td>1 2 3 4 5</td>
</tr>
<tr>
<td>$h = 1$</td>
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</tr>
<tr>
<td>$h = 2$</td>
<td>246.3 246.3 246.4 246.4 246.4</td>
</tr>
</tbody>
</table>

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Obtained from the above examples can be seen that the equilibrium outcome: for goods 1, the inventory level is generally low, while demand was stable, but at time 3 a sudden increase in production costs, manufacturers produce more retain some inventory for current use, and this period will not leave any inventory, for goods 2, the production cost by increasing the rate of equilibrium by reducing the amount of a corresponding, but delayed due to its significantly higher than the cost of goods a high, although lower inventory costs, delayed orders for basic commodities is zero, just as the first five corresponding increase in operating costs of logistics, there very small part of the order of the extension. In addition, because the cost of the supply chain network and the demand function parameters for the kind of products is symmetrical, so after a period and the period of adjustment between the balance of production, inventory, traffic volume and price extension is the same, from the results validate the model is correct and reasonable.

We now further discuss and compare the results with other factors. Since We are concerned about the customer waiting cost, we compute it by the same data as the mentioned above, except that the manufacturer’s customer waiting cost function parameter $\theta^{w}$ for the commodity 1 is set to be 0.01, 0.05, 0.1, 0.5, 1.5. From Table III, we can find that the commodity 1’s warehouse inventory levels and actual delayed quantity at all of the time points have obvious changes. As the prices at the demand markets may have an effect on the performance of supply chain network, we simulate it by the same data, except that the demand quantity function parameter $\theta^{d}$ for the commodity 1 is assumed to be 0.8, 0.9, 1.0, 1.1, 1.2. From Table IV, we can see that the commodity 1 has a lower demand quantity, which lead to a dramatic change of the results.

### Table III.
**RESULTS OF SUPPLY CHAIN NETWORK EQUILIBRIUM MODEL**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Product</th>
<th>Time</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$q_{ij}$</td>
<td>h = 1</td>
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<td>7.51</td>
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<td></td>
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<tr>
<td></td>
<td></td>
<td>5</td>
<td>31.11</td>
</tr>
</tbody>
</table>

### Table IV.
**RESULTS OF SUPPLY CHAIN NETWORK EQUILIBRIUM MODEL**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Product</th>
<th>Time</th>
<th>Value</th>
</tr>
</thead>
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<td>18.76</td>
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### IV. Conclusion
Mechanism of supply chain operations in the company and business model has been experiencing profound changes. New features are emerging in the supply chain network. Equilibrium model is still a effective method in-depth study of optimization supply chain. This paper considers needs change with time, the influences of inventory, and the appropriate separation of commercial flow and material flow taking order into consideration. And it proposes a multi-commodity and multi-stage supply chain network equilibrium model which has different cost structure, thus it has practical significance. This paper eventually works out the conditions to make each level achieve equilibrium using equilibrium theory.
variational inequality theory, and also offers economic interpretation with concrete examples to verify them.

REFERENCES


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