Dynamic Spectrum Management with Competitive Market Equilibrium

Ming Zhao\textsuperscript{1,2}, Tao Luo\textsuperscript{1,2}, Changchuan Yin\textsuperscript{1}, Guangxin Yue\textsuperscript{1}

\textsuperscript{1}Beijing University of Posts and Telecommunications, Beijing, China
\textsuperscript{2}The State Key Laboratory of Integrated Services Networks, Xidian University, Xi’an, China
wrtr2009@gmail.com

Xiaojun Wang \textsuperscript{3}

\textsuperscript{3}Dublin City University, Dublin, Ireland
xiaojun.wang@dcu.ie

Abstract—In an effort to improve the efficiency of spectrum usage, the cognitive radio (CR) \cite{1} technology was proposed, bringing the possibility of dynamic spectrum management (DSM) \cite{2}. Generally speaking, the multiuser communication systems in the background of CR are interference-limited. Power control is one of the central issues in the design of these systems. This problem can be modeled as a competitive market equilibrium (CME) system. In this paper, we reveal the differences between the CME problem and the standard Linear Complementary Problem (LCP), and then derive the closed form solution to a simplified CME problem. We also propose a hybrid algorithm as an alternative to the tâtonnement process \cite{3} in case the latter diverges. When the convergence conditions are not tenable, it is shown that the proposed algorithm can reduce the processing delay compared with the tâtonnement process.

Index Terms—dynamic spectrum management, competitive market equilibrium, multiuser power control

I. INTRODUCTION

The traditional spectrum allocation policy, which assigns a fixed portion of the spectrum to a specific license holder for exclusive use, is unable to manage the spectrum efficiently. In order to improve the efficiency of spectrum usage, CR technology is proposed to share the spectral resources between the primary users (PUs) and secondary users (SUs) \cite{1}, bringing the possibility of DSM \cite{2}. Unless otherwise specified, we use “user” to refer to secondary user (SU) in this paper.

Basically, the multiuser communication systems in the background of CR are interference-limited. Thus, power control is one of the central issues in the design of these systems. This problem can be modeled as a non-cooperative game where the users selfishly optimize their utility functions. Nash equilibrium (NE) in this game is a stationary point, to which the traditional iterative water-filling (IWF) algorithm can converge under suitable conditions \cite{4}. Sufficient conditions under which the IWF algorithm converges to a unique NE were derived and the closed form solution to the power allocation problem with NE was obtained in some special scenarios \cite{4-6}.

However, the power allocation scheme with NE may not be socially optimal \cite{2}. Because of the non-cooperative nature of the NE, users tend to compete for the superior channels regardless of the interferences to other users. This is an instance of the well-known “tragedy of the commons” from economics \cite{3}.

Therefore, many existing works turned to other models for the multiuser power control problem. Most of them in the non-cooperative setting often assume that the competitive users act only based on their private information and neglect information about their opponents. Reference \cite{7} first investigated the scenario where a fore-sighted user plays the Stackelberg Equilibrium \cite{8}. The fore-sighted player with perfect knowledge of its competitors’ responses to its actions can improve both its performance and the performance of other users. This conclusion can still hold even though such a priori knowledge is not available to the fore-sighted user \cite{9}. Moreover, there have been a number of works studying spectrum sharing in the setting of cooperative games \cite{10-12}. Several algorithms were proposed to converge to different operating points on the region of achievable rates. In these works, players are assumed to maximize a common objective function and communicate with each other for cooperation.

Taking a different approach to this problem, we focus on the solution to the competitive market equilibrium (CME) problem. In the spectrum market, each channel sells some amount of power allocation to the users with a fictitious price and each user maximizes her total data rate (Shannon utility function) under her budget constraint. The definition of CME will be presented in the section III.

Based on the existing results of \cite{2} and other works, this paper makes three main contributions. We first show that although the CME problem is equivalent to a set of equations with a structure similar to that of linear complementary problem (LCP), it doesn't strictly satisfy the definition of standard LCP and thus can not apply the
various algorithms designed for LCP directly. This conclusion is different from that in [2]. Second, the closed form solution to the simplified CME problem is derived. In centralized networks, the central point (CP) can solve CME problem with this result and no iterative operation is needed. To our best knowledge, closed form result about CME problem didn’t appear in previous works. Lastly, inspired by the idea of tâtonnement process [3], we propose an algorithm that converges to the CME point under suitable conditions [2] and generates a proportional allocation scheme when the tâtonnement process diverges.

The following notations are used throughout this paper. \( I \) is the column vector with all entries equal to 1. \( I_n \) denotes the \( n \)-by-\( n \) identity matrix. \((.)^t\) stands for transpose operation. \( N \) is the set of all positive integers.

II. SYSTEM MODEL

In this paper, we consider a communication system consisting of \( n \) SUs and \( m \) frequency-orthogonal channels. Each channel can be shared by all the SUs. The power allocated for user \( i \) on a channel \( j \) is denoted by \( p_{i,j} \). What’s more, the total power allocated on channel \( j \) is limited by the channel mask \( b_j \), which is set by the spectrum seller to protect the PUs from interferences of SUs:

\[
\sum_{i=1}^{n} p_{i,j} \leq b_j, \quad \forall j = 1,2,\ldots,m. \quad (1)
\]

As a feature of the competitive market model, we define \( e_j \) as the price of unit power allocation on channel \( j \). The budget constraints can then be described by the following inequations:

\[
\sum_{j=1}^{m} e_j p_{i,j} \leq w_i, \quad \forall i = 1,2,\ldots,n. \quad (2)
\]

where \( w_i \) is the budget of user \( i \).

It is natural to define the utility function of user \( i \) as the total data rate of this user on all the channels. As the data rates of user \( i \) on individual channel are independent, we can obtain the utility function of user \( i \) as follows.

\[
u_i = \sum_{j=1}^{m} \log \left(1 + \frac{p_{i,j}}{\sigma_{i,j} + \sum_{j=1}^{m} c_{i,j}^t p_{i,j}}\right) \quad (3)
\]

In (3), \( \sigma_{i,j} \) is the noise power experienced by user \( i \) on channel \( j \); and \( c_{i,j}^t \) stands for the coefficient of interference to user \( i \) from user \( l \) on channel \( j \). For simplicity reason, when \( l \) is just equal to \( i \), the interference coefficient is defined to be one.

Given the prices of unit power allocation over \( m \) channels and the power allocations of other users, user \( i \) can determine her power to maximize the utility function defined in (3) subject to her budget constraint:

\[
p_i = \arg \max_{p_i} u_i \quad s.t. \quad \sum_{j=1}^{m} e_j p_{i,j} \leq w_i, \quad p_{i,j} \geq 0 \quad (4)
\]

where \( p_i \) is a vector consisting of all the power allocations of user \( i \) across \( m \) channels and \( p_i \) is the optimal power allocation vector maximizing the total data rate of user \( i \). It can be easily shown that the solution to problem (4) is the water-filling power allocation scheme:

\[
p_{i,j} = p_{i,j}^* = \left( L_j / e_j - \sigma_{i,j} - \sum_{l \neq i} c_{i,l}^t p_{i,l} \right) \quad (5)
\]

where the water level \( L_j \) is implicitly determined by the budget constraint (2). The symbol ‘\(^*\)’ is defined by the equation \( (x)^* := \max \{0,x\} \), ensuring that the power allocation is always non-negative.

III. THE CME PROBLEM

A. CME vs LCP

Generally speaking, the CME is an operating point, on which the system allocates all the power resources so that each user’s utility function is maximized and at the same time all user budgets and channel masks are achieved as well. According to this definition, we can derive the conditions that the CME point satisfies. On the CME point, as the utility function (3) is maximized, equation (5) is tenable. By applying the fact that \( y = \max \{0,x\} \) is the same as \((y \geq x) \& (y(y-x) = 0) \& (y \geq 0)\), it can be shown that (5) is equivalent to (6), (7) and (8).

\[
p_{i,j} \geq \frac{L_j}{e_j} - \sigma_{i,j} - \sum_{l \neq i} c_{i,l}^t p_{i,l} \quad (6)
\]

\[
p_{i,j} \left( p_{i,j} - \frac{L_j}{e_j} + \sigma_{i,j} + \sum_{l \neq i} c_{i,l}^t p_{i,l} \right) = 0 \quad (7)
\]

\[
p_{i,j} \geq 0 \quad (8)
\]

As each user budget is achieved, we have

\[
\sum_{j=1}^{m} e_j p_{i,j} = w_i \quad (9)
\]

Similarly, as the channel masks are achieved, the following equation can be derived.

\[
\sum_{j=1}^{m} p_{i,j} = b_j \quad (10)
\]

In one word, the power allocation \( p_{i,j} \) achieves the CME point when and only when \( p_{i,j} \) satisfies equation (6)-(10) for any user \( i \) and channel \( j \). After introducing the slack variables \( s_{i,j} \geq 0 \) and the revenue of spectrum seller \( r_{i,j} = p_{i,j} e_j \), we can obtain the following equations from (6), (7), (9) and (10):

\[
\sum_{j=1}^{m} c_{i,j}^t r_{i,j} + \sigma_{i,j} e_j - L_j = s_{i,j} \quad (11)
\]

\[
r_{i,j} \left( \sum_{j=1}^{m} c_{i,j}^t r_{i,j} + \sigma_{i,j} e_j - L_j \right) = 0 \quad (12)
\]

\[
\sum_{j=1}^{m} r_{i,j} = w_i \quad (13)
\]

\[
\sum_{j=1}^{m} r_{i,j} + b_j = e_j \quad (14)
\]

Substitute equation (14) into equation (11), we get equation (15). This is to eliminate the price variable \( e_j \). Substitute equation (11) into equation (12), we get equation (16). This is to simplify the form of equation (12).

\[
\sum_{j=1}^{m} \left( c_{i,j}^t + \sigma_{i,j} b_j \right) r_{i,j} - L_i - s_{i,j} \quad (15)
\]
\[ r_{i,j} s_{i,j} = 0 \]  \hspace{1cm} (16)

To simplify the above expressions, we first define the following vectors: \[ r_j = (r_{1,j}, r_{2,j}, \ldots, r_{n,j})^T, \]
\[ \sigma_j = (\sigma_{1,j}, \sigma_{2,j}, \ldots, \sigma_{n,j})^T, \]
\[ s_j = (s_{1,j}, s_{2,j}, \ldots, s_{n,j})^T, \]
\[ w_j = (w_1, w_2, \ldots, w_p)^T. \]
\[ L = (L_1, L_2, \ldots, L_n)^T. \]

After some derivations, (13), (15) and (16) can be converted into equation (17)-(19).
\[ M_j r_j - L - s_j = 0 \] \hspace{1cm} (17)
\[ \sum_{j=1}^{m} s_j^T r_j = 0 \] \hspace{1cm} (18)
\[ \sum_{j=1}^{m} r_j = w \] \hspace{1cm} (19)
\[ s_j, r_j \geq 0 \] \hspace{1cm} (20)

where the matrix \( M_j \) is defined as follows:
\[ M_j = C_j^T + \sigma_j T / b_j, \quad [C_j],_{ij} = c_{ij}^T. \]

We can rewrite equations (17)-(20) as follows:
\[ \begin{pmatrix} M & -T^T \\ T & 0 \end{pmatrix} \begin{pmatrix} r \\ L \end{pmatrix} = \begin{pmatrix} s \\ w \end{pmatrix} \] \hspace{1cm} (21)
\[ s^T r = 0 \] \hspace{1cm} (22)
\[ s, r \geq 0 \] \hspace{1cm} (23)

where
\[ M = \text{diag}(M_1, \ldots, M_m) \]
\[ r = (r_{1,1}, \ldots, r_{m,1})^T, \quad s = (s_{1,1}, \ldots, s_{m,1})^T \]
\[ T = (T_{1,1}, \ldots, T_{m,1}), \quad T_{1} = T_{2} = \ldots = T_{m} = I_n. \]

It is well known that the standard LCP can be defined as follows.
\[ y = Mx + q \] \hspace{1cm} (24)
\[ y^T x = 0 \] \hspace{1cm} (25)
\[ x, y \geq 0 \] \hspace{1cm} (26)

The CME problem ((21)-(23)) has a similar structure to that of the LCP ((24)-(26)), still there are two obvious differences between them. First, the vector \( y \) in LCP is an unknown vector while in the CME problem, its counterpart \( (s, w)^T \) is in part known by us (vector \( w \) is given). Second, \( w^T L = 0 \) is not tenable in CME problem. Otherwise, all the allocated power values will equal to zero.

\section{Solution to the Simplified CME Problem}

Consider an ideal scenario in which the total power of interference and noise signals is always beneath the water-level. With this assumption, we can find that (6) becomes an equation and (7) can be eliminated from the equation set. In other words, the slack vector \( s_j \) introduced by (17) is always zero vector under this circumstance. Assume the following inverses of matrices exist. In (17), if \( s_j = 0 \), we have
\[ M_j r_j - L = 0 \quad \text{or} \quad r_j = M_j^{-1} L. \] \hspace{1cm} (27)

By applying (27) on (19), we obtain the following conclusion:
\[ \left( \sum_{j=1}^{m} M_j^{-1} \right) L = w \quad \text{or} \quad L = \left( \sum_{j=1}^{m} M_j^{-1} \right)^{-1} w. \] \hspace{1cm} (28)

Substitute the right hand side item in (28) for the vector \( L \) in (27), we have
\[ r_k = M_k^{-1} \left( \sum_{j=1}^{m} M_j^{-1} \right)^{-1} w, \quad k = 1, \ldots, m. \] \hspace{1cm} (29)

According to the definition of \( r_{i,j} \) and the equations (9), we obtain the power allocation solution to the simplified CME problem as follows:
\[ p_{i,j} = r_{i,j} b_j / \sum_{i=1}^{m} r_{i,j}, \quad \forall i = 1, \ldots, n; j = 1, \ldots, m \] \hspace{1cm} (30)
where \( b_j \) is the channel mask and \( r_{i,j} \) is calculated by (29).

\section{Proposed Power Allocation Algorithms}

In traditional centralized network, the central point (CP) solves the power allocation problem and subsequently publishes the result to all users in the network. However, this kind of architecture leads to heavy load imposed on the CP. According to reference [3], the tâtonnement process is a prospective way to achieve the CME point in decentralized systems where both CP and users participate in the power allocation process. The users only send their power allocations to the CP and receive the prices of the channels from the CP. The users and the CP consecutively update the power allocations and channel prices respectively until the whole system achieves the CME point. For sake of completeness, the detailed procedure of tâtonnement process is in appendix A and B.

\subsection{Proposed Hybrid Algorithm}

It cannot be guaranteed that the tâtonnement processes converge to the CME point in all cases. In other words, when the sufficient conditions given by [2] are not satisfied, the tâtonnement process may diverge. To solve this problem, we propose a hybrid algorithm that achieves the CME point when the tâtonnement process converges and switches to an alternative allocation scheme otherwise. Specifically, we need to design a detection technique to stop iterations when the tâtonnement process diverges. Also, we have to find a simple allocation scheme as an alternative to CME scheme when the latter is unable to achieve the CME point.

The basic idea of the detection technique is to record several iteration samples of the relative excess demand that is defined by equation (34) in appendix A and subsequently calculate the judgement statistic according to the recorded samples. The design of the judgement statistic is inspired by the definition of sequence convergence. If sequence \( x(n) \geq 0, n \in \mathbb{N}^+ \), then
\[ \lim_{n \to \infty} x(n) = 0 \iff x(n) < \varepsilon , \quad \forall n \geq N \in \mathbb{N}^+ \]  
so that \( \forall n \geq N , \ x(n) < \varepsilon . \) If we assume a specific \( \varepsilon , \) for instance \( \varepsilon = x(1) , \) then \( x(n) < \varepsilon \) will be tenable for most possible values of \( n \) if the sequence converges to zero finally. For most cases, the detection result of the proposed algorithm is correct and unnecessary iterations can then be avoided. This conclusion will be verified in the next section through simulation.

To make full use of power resources provided by the PU and meet different demands of SU, we use a proportional allocation scheme (PAS) as an alternative to CME scheme. Although the PAS can not achieve optimal sum data rate, it can still achieve all user budgets and channel masks as the CME does. We derive the PAS as follows. As all the user budgets and channel masks are achieved by the PAS, the total value of the spectrum market and the total budget of users should be equal.

\[
\sum_{j=1}^{m} e_j b_j = \sum_{i=1}^{n} w_i \quad (31)
\]

If we multiply both sides of equation (31) by \( w_i \sum_{j=1}^{n} w_i \), it can be shown that (31) is equivalent to (32).

\[
\sum_{j=1}^{m} e_j \left( \frac{w_i b_j}{\sum_{j=1}^{n} w_i} \right) = w_i \quad (32)
\]

Let

\[
p_{i,j} = \frac{w_i b_j}{\sum_{j=1}^{n} w_i} , \quad i = 1, \ldots, n ; \quad j = 1, \ldots, m . \quad (33)
\]

Then it is obvious that (32) is actually the budget constraint. It is easy to verify that (33) satisfies the power limits on all channels. Thus the power allocation result (33) achieves the user budget and the channel mask as well. According to (33), the allocated power of user \( i \) on channel \( j \) is proportional to the normalized budget of user \( i \) or the power limit of channel \( j \). Thus, we call the equations (33) as the proportional allocation scheme. Although this scheme is not optimal for each user's utility function (3), its non-iterative nature eliminates the risk of divergence, which can not be avoided by the tâtonnement process. What's more, the users can determine the proportional allocation scheme without any information exchange for current channel prices and power allocations. The detailed procedure of the proposed algorithm is included in appendix C (algorithm III).

V. NUMERICAL EXAMPLES

Assume the power allocation system involves \( n \) users and \( m \) channels. We first describe two groups of examples to demonstrate how algorithm I converges to the CME point. The examples in the first group (shown in figure 1) have the same number of users \( (n=10) \) with fewer channels \( (m=2) \), an equal number of channels \( (m=10) \) and more channels \( (m=18) \), respectively. Meanwhile, the examples in the second group (shown in figure 2) have the same number of channels \( (m=10) \) with fewer users \( (n=2) \), an equal number of users \( (n=10) \) and more users \( (n=18) \), respectively. To make algorithm I converge finally, it is necessary to establish some assumptions.

Assume the interference coefficients are independent random variables with the uniform distribution on \([0, 1/(n-1)]\). The noise power levels \( \sigma_{ij} \) satisfy the equal noise condition, or \( \sigma_{ij} = \sigma_j \), for any SU \( i \), given the channel \( j \). \( \sigma_j \) are independent random variables with the uniform distribution on \([0, 1]\). All the user budgets and channel power limits are normalized to unit one. The typical values of the threshold \( T_1 \), \( T_2 \) and the step variable 'step' in algorithm I are \( 10^{-3} \), \( 10^{-2} \) and \( 10^{-1} \) respectively. Under these assumptions, both Figure 1 and 2 show the convergence of algorithm I. After about 60 iterations, the relative excess demand has gone down below \( 10^{-2} \). Comparisons of CME and NE can be found in reference [13].

The above examples are used to show the convergence of tâtonnement process. However, the scenario where the tâtonnement process diverges is also meaningful because we can use it to demonstrate the advantage of algorithm III over traditional tâtonnement process. We can generate these scenarios by violating the sufficient conditions.
given by [2]. The thresholds of iteration number, i.e., $N_1$ and $N_2$ in algorithm I and III are set to 20 and 100 respectively. The average iteration numbers of two algorithms when the convergence conditions are not satisfied are listed in table I. With the values of $N_1$ and $N_2$ configured in this paper, we can draw the conclusion that the proposed algorithm (algorithm III) can reduce 73% to 77% of the processing delay compared with that of tâtonnement process (algorithm I).

**TABLE I. AVERAGE ITERATION NUMBERS WHEN THE TÂTONNEMENT PROCESS DIVERGES ($N_1=20, N_2=100$).**

<table>
<thead>
<tr>
<th>Number of Users</th>
<th>Number of Channels</th>
<th>Average Iteration Number of Algorithm I</th>
<th>Average Iteration Number of Algorithm III</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>10</td>
<td>100</td>
<td>22.4</td>
</tr>
<tr>
<td>10</td>
<td>10</td>
<td>100</td>
<td>23.2</td>
</tr>
<tr>
<td>18</td>
<td>10</td>
<td>100</td>
<td>24.8</td>
</tr>
<tr>
<td>10</td>
<td>2</td>
<td>100</td>
<td>24</td>
</tr>
<tr>
<td>10</td>
<td>18</td>
<td>100</td>
<td>26.4</td>
</tr>
</tbody>
</table>

We further compare the proposed algorithm with the two power allocation algorithms proposed in [14], i.e., the basic power allocation (BPA) algorithm and the greedy spectrum sharing (GSS) algorithm. Both the BPA and GSS algorithms are based on the uniform power allocation and the graph representation of the secondary network. The BPA algorithm allocates one frequency band to at most one SU, while the GSS algorithm allows the spectrum sharing among all SUs on the same frequency band. We plot the sum rate performance of the three algorithms versus the number of total SUs in figure 3 and 4, according to the scenarios that the proposed algorithm converges to the CME point and uses the proportional allocation scheme, respectively. The number of channels is fixed to 10 in both figures. In figure 3, we use the same convergence assumptions taken by figure 1 and 2. Under the convergence assumptions, the proposed algorithm can converge to the CME point and thus achieve the optimal power allocation. The BPA and GSS algorithms use uniform power allocation instead. Consequently, the proposed algorithm achieves the highest sum rate performance among the three algorithms as it is shown in figure 3.

In figure 4, all the assumptions taken in figure 3 are still present except that the interference coefficients are modeled as independent random variables with the uniform distribution on $[0, 1]$, violating the weak interference condition. The proposed algorithm switches to the proportional allocation scheme as it can not converge to the CME point in this scenario. It can be seen in figure 4 that the proposed algorithm outperforms the GSS algorithms even when the convergence conditions are not satisfied. Moreover, it is illustrated in figure 4 that although the BPA algorithm achieves higher sum rate as compared with the proposed proportional algorithm, the difference between them is quite small. As the BPA algorithm does not allow spectrum sharing, some SUs have no frequency band to transmit data if the number of SUs is larger than the number of total channels. The proposed algorithm makes all SUs have the opportunity to transmit data and therefore it has the advantage over the BPA algorithm in terms of the allocation fairness.

**VI. LIMITATIONS OF THE PROPOSED ALGORITHM**

The proposed hybrid algorithm involves relatively high complexity as it uses the tâtonnement process to converge to the CME point. Nevertheless, a moderate increase in the complexity is worth the improvement of sum rate performance. Moreover, the proportional allocation scheme used by the proposed algorithm can not achieve optimal sum rate performance. But it at least avoids any unnecessary interruptions of SU transmissions due to the divergence of the tâtonnement process.

**VII. CONCLUSION**

This paper focuses on the power control problem in a CR-based multiple access communication system. We try to achieve the CME point, which has the potential to bring greater overall data rate as compared with the NE point [2], [13]. The first conclusion drawn in this paper is that although the problem of CME has a structure similar to that of LCP, it doesn't strictly satisfy the definition of standard LCP. Thus we can not utilize the various algorithms designed for LCP to solve the CME problem directly. Secondly, the closed form solution to the
simplified CME problem is obtained in this paper. Lastly, inspired by the idea of tâtonnement process [3], we propose an algorithm to converge to the CME point under suitable conditions and apply the proportional allocation scheme when the tâtonnement process diverges.

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APPENDIX A ALGORITHM I: TRADITIONAL TÂTONNEMENT PROCESS FOR CME

Assume the initial power allocation values and channel prices are known by both CP and users as predefined constants. Each user only knows the interference coefficients and noise power values that are related to itself. CP receives channel masks from PU.

1) CP holds and initializes the number of iterations: \( i = 1; \)

CP generates a random integer \( k \) from \( 1, 2, \ldots, n \) with equal possibilities and broadcasts \( k \) to all users;

2) CP receives the users' power values (It uses the initial power values directly for the first time) and calculates the relative excess demand:

\[
\text{excess} = 
\frac{\sum |p_{ij} - \sum p_{j} |}{\sum |b_{j}|},
\]

if \( \text{excess} < \text{threshold } T_{1} \)

\{ CP instructs the users to terminate iterations and use the allocated power; Algorithm terminates; \}

else

\{ if \( i \geq N_{i} \)

\{ CP instructs the users to terminate iterations and utilize the proportional scheme; Algorithm terminates; \}

\( i++; \}

3) CP adjusts the channel prices and transmits them to the users:

\[ e_{j} = e_{j} (1 + \text{step} \cdot (b_{j} - \sum p_{j})) , \forall j = 1, \ldots, m; \]

where \( \text{step} \) is a step variable;

4) The users receive the channel prices and adjust their power allocation values one by one:

while \( \{ \text{true} \} \)

\{ for \( \{ i=1; \text{false} \}; i++ \}

\{ User \( i \) adjusts its power according to the power allocations of the rest \( n-1 \) users(algorithm II); The rest \( n-1 \) users receive the updated power of user \( i \);

User \( k \) calculates the difference between the updated power allocation and the previous one:

\[ \text{difference} = \sum j \sum (p_{i,j}^{(\text{old})} - p_{i,j}^{(\text{new})}) \];

if \( \{ \text{difference} < \text{threshold } T_{2} \}

\{ User \( k \) informs the rest \( n-1 \) users that they should stop adjusting their power values; Go to step 5; \}

\}

5) The users transmit their current power allocations to CP, go to step 2;
APPENDIX B ALGORITHM II: WATER-FILLING FOR CME

Assume it is user $i$'s turn to adjust her power. The budget of user $i$ $w_i$, the power values of all users except user $i$ and the channel prices are input parameters. Define 'excess consumption' as follows.

$$excess\ consumption = \sum p_{ij} e_j - w_i$$ (35)

1) Initialize two water levels:
   
   $level1 = w_i / m, level2 = level1 / 2$;

   By applying (5), calculate the water-filling allocation results of user $i$ with 'level1' and 'level2', respectively; By applying (35), obtain corresponding excess consumption 'exc1' and 'exc2';

else if $\{ exc1==0 \}$

   Output the water-filling allocation result with the water level 'level1';
   Terminate the algorithm;

else if $\{ exc2==0 \}$

   Output the water-filling allocation result with the water level 'level2';
   Terminate the algorithm;

else

   Find an interval that contains the actual water level of user $i$:
   
   while $\{ true \}$

   if $\{ exc2>0 \}$

      $level1=level2$;
      $level2=level1 / 2$;
      Calculate the water-filling allocation result of user $i$ with 'level1' and 'level2' by (5) and then update 'exc1' and 'exc2' applying (35);

   else if $\{ exc1>0 \}$

      $level2=level1$;
      $level1=level1 * 2$;
      Calculate the water-filling allocation result of user $i$ with 'level1' and 'level2' by (5) and then update 'exc1' and 'exc2' applying (35);

   else

      break;

4) Apply bisection technique to estimate the actual water level of user $i$:

Set the accuracy threshold of the water level: 'accuracy';

if $\{ exc1 * exc2<0 \}$

   while $\{ true \}$

      newLevel = (level1 + level2) / 2;
      Calculate the water-filling allocation result of user $i$ with the water level 'newLevel' by (5) and then evaluate the corresponding excess consumption 'newExcess' by (35);

      if $\{ newExcess>0 \}$

         level1=newLevel;

      else if $\{ newExcess<0 \}$

         level2=newLevel;

      else

         break;

   if $\{(level1-level2)/ level1< accuracy\}$

      break;

Output the water-filling allocation result with the water level 'newLevel';
Terminate the algorithm;

else

   if $\{ exc1==0 \}$

      Output the water-filling allocation result with the water level 'level1';

   else

      Output the water-filling allocation result with the water level 'level2';

   Terminate the algorithm;

The typical value of the threshold 'accuracy' is $10^{-3}$.

APPENDIX C ALGORITHM III: PROPOSED HYBRID ALGORITHM

1) CP holds and initializes the number of iterations: $i=1$;
CP generates a random integer $k$ from 1, 2, ..., $n$ with equal possibilities and broadcasts $k$ to all users;
The users send their individual budget to the CP;
CP broadcasts the total user budget and channel masks to all users;

2) CP receives the users' power values (It uses the initial power values directly for the first time) and calculates the relative excess demand:

$$excess = \left( \sum_{j=1}^{N} p_{ij} e_j \right) / \sum_{j=1}^{N} b_j$$;

if $\{ excess < threshold T_i \}$

   CP instructs the users to terminate iterations and the users utilize the allocated power;
Algorithm terminates;

else

   $E[i]=excess$;

   if $\{ i==N_i \}$

      $Dtc = \left( \sum_{j=1}^{N_i} A[j] \right) / N_i$;

      if $\{ Dtc < 0.9 \}$

         CP instructs the users to terminate iterations and the users utilize the proportional scheme;
Algorithm terminates;

   else

      $\{ \}$

   $i++$;

Step 3) - 5) are the same as the counterparts in algorithm I.