An Improved Localization Algorithm for Wireless Sensor Network Based on the Selection of Benchmark Anchor Node

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Abstract—To improve the localization precision of nodes in Wireless sensor network (WSN), this paper analyses the source of the localization error in the least square localization algorithm (LSL), and it can be concluded that the minimum cumulative relative distance error of the benchmark anchor node directly influence the localization precision. Based on the analyses, this paper proposes an improved least square algorithm, which is the minimum cumulative related distance error in LSL (LSL-MCR) algorithm. In the improved algorithm, we give a new principle to choose the benchmark anchor node. The simulation results indicate that the proposed algorithm can improve the localization precision of the unknown nodes.

Index Terms—WSN, localization precision, least square method, benchmark anchor node

I. INTRODUCTION

The Wireless sensor network (WSN) is an emerging technology, and it is widely applied in many fields. The WSN consists of a collection of wireless networked low-power sensor devices, with each node integrating an embedded microprocessor, radio, and a limited amount of storage [1]. It comprehensively uses the microelectronics technology, embedded computation technique, wireless communication technology, distributed information processing technology. Its aim is to perceive, collect and process the information of the perceived objects by the distributed sensors in the region, and sends the information to the observers. With the recent advances in WSN research, applications employing WSN have become quite popular [2]. Today, WSN is widely used in industrial automation, traffic control, environmental monitoring, and military reconnaissance and so on. To make the information obtained from the sensor nodes effectively, we should also precisely know the location of sensor nodes. Therefore, the basis and key technology is the precise localization of the nodes in WSN, and the information which lacks precise localization is meaningless. The localization of nodes in WSN can be achieved by the perceived information and information processing of nodes.

To achieve more accurate localization, great deals of localization algorithms have already been developed in WSN. Savarese proposed two localization algorithms: cycle accuracy-Cooperative ranging [3] and Two-Phase localization [4] that can decrease the influence of distance error to localization; Avvides [5] proposed n-hop multilateration primitive localization algorithm, and he used Kalman filtering technique to calculate the accurate coordinates circularly, it reduced the accumulation error; The existent localization refinement algorithms are based on the circulatory method [6]-[10]. At present, the refinement methods that already existed can reduce localization errors in different degree, but there are still many deficiencies:

1) The existent refinement methods focus on the improvement of algorithms, and obtain high precision with the method of circulation, but its data operations are excessive, and it increases the energy of the nodes.
2) The measures of the refinement algorithm are multiple, and the information exchange between the nodes is frequent, so it increases the cost of the networks’ energy.

The least square localization algorithm (LSL) is simple and has small calculating amount, so it is beneficial to save the energy of the nodes in WSN. In consideration of these, it draws the researchers’ extensive attention [11]-[15]. The localization precision in LSL is mainly affected by the distance measurement error, and the distance measurement error in RSSI is mainly due to the influence of multipath propagation and reflection, so the distance measurement error in RSSI is large. The weighted least square localization algorithm can decrease the influence of localization error with the method that decreases the...
impact of the anchor nodes that have bigger errors [12]-[14]. The weighted matrix is the main factor to affect the localization precision in the weighted least square localization algorithm.

Based on the above issues, this paper aims at the influence of the anchor nodes in the least square algorithm, and then optimizes the selection of the benchmark anchor node. So the goal to improve the localization precision can be achieved.

The main work of this paper includes some aspects as follows:

1) Analyses the source of the localization error in the least square localization algorithm (LSL).

2) Analyses the influence of localization precision caused by the relationship between the distance measurement error of each anchor node and the distance measurement error of the benchmark anchor node, and then proposes the selection principle of the benchmark anchor node. So the goal to improve the algorithm, and then optimizes the selection of the localization algorithm.

3) Designs and emulates the improved LSL algorithms.

4) Summarizes the performance of the improved LSL algorithms.

II. THE MODEL OF NODES’ LOCALIZATION

The trilateration is a classical deterministic localization method. We adopt the trilateration to calculate coordinate of the unknown node. It respectively uses the three anchor nodes as the center of three circles, and similarly uses the distance between unknown nodes and anchor nodes as the radius of three circles, so the three circles can be intersect at one point. If we have the known points \(A(x_1, y_1), B(x_2, y_2), C(x_3, y_3)\) and the unknown point \(D(x, y)\) (which position we are looking for) is \(d_1, d_2, d_3\) from the these points respectively. According to the neighboring nodes’ localization information and the distance of the nodes, the unknown node coordinates can be calculated. The model of the localization figure is shown in Fig.1.

![Figure 1. The model of Nodes' localization.](image)

Then we can get the following equations:

\[
\begin{align*}
(x_1 - x)^2 + (y_1 - y)^2 &= d_1^2 \\
(x_2 - x)^2 + (y_2 - y)^2 &= d_2^2 \\
(x_3 - x)^2 + (y_3 - y)^2 &= d_3^2.
\end{align*}
\] (1)

Subtracting the third equation from the first and the second equation in (1), then we can get the following expression.

\[
\begin{align*}
a_1x + b_1y &= c_1 \\
a_2x + b_2y &= c_2.
\end{align*}
\] (2)

Where

\[
\begin{align*}
a_1 &= 2x_i - x \\
b_1 &= 2y_i - y \\
c_i &= (x_i - x)^2 + (y_i - y)^2 - d_i^2
\end{align*}
\]

With the simultaneous equations, we can calculate the coordinate of the unknown node \(D\).

\[
\begin{align*}
x &= \frac{b_1c_2 - b_2c_1}{a_1b_2 - a_2b_1} \\
y &= \frac{a_1c_2 - a_2c_1}{a_1b_2 - a_2b_1}.
\end{align*}
\]

III. LEAST-SQUARE LOCALIZATION ALGORITHM (LSL)

In the localization of wireless sensor network, the wireless signal transmission will cause attenuation mainly due to the influence of transmission medium, multipath propagation, reflection, non-line of sight, antenna gain, so it will produce error when measure the distance. When the error appears, these three circles will not have one common intersection.

Therefore, we cannot get the coordinates of the nodes. In order to solve this problem, the least square localization algorithm (LSL) has been adopted. The LSL algorithm is a mathematical optimization technique, and it is concise and practical for the WSN which attaches importance to energy consumption. The specific steps are shown as follows:

The coordinate of the unknown node is \((x, y)\), and there are \(n\) anchor nodes within the communication distance of the unknown node. The anchor node coordinates are \((x_1, y_1), (x_2, y_2), (x_3, y_3), \ldots, (x_n, y_n)\) respectively, then we can get the following expressions.

\[
\begin{align*}
(x_1 - x)^2 + (y_1 - y)^2 &= d_1^2 \\
(x_2 - x)^2 + (y_2 - y)^2 &= d_2^2 \\
&\hspace{1cm} \ldots \\
(x_n - x)^2 + (y_n - y)^2 &= d_n^2
\end{align*}
\] (3)

The other equations subtract the \(N\)th equation respectively, then we can get the following expressions.

\[
\begin{align*}
2(x_{1h} - x)x + 2(y_{1h} - y)y &= (x_1 - x)^2 + (y_1 - y)^2 - d_1^2 \\
2(x_{2h} - x)x + 2(y_{2h} - y)y &= (x_2 - x)^2 + (y_2 - y)^2 - d_2^2 \\
&\hspace{1cm} \ldots \\
2(x_{nh} - x)x + 2(y_{nh} - y)y &= (x_n - x)^2 + (y_n - y)^2 - d_n^2
\end{align*}
\]

And we can use (4) to instead of it.

\[AX^* = B.\] (4)

Where
respectively.

\[ A = 2 \times \begin{bmatrix}
    x_1 - x_a & y_1 - y_a \\
    x_2 - x_a & y_2 - y_a \\
    \vdots & \vdots \\
    x_n - x_a & y_n - y_a \\
\end{bmatrix}. \]

\[ B^* = \begin{bmatrix}
    (x_1 - x_{j#})^2 + (y_1 - y_{j#})^2 - (d_{j#}^2)
    \\
    (x_2 - x_{j#})^2 + (y_2 - y_{j#})^2 - (d_{j#}^2)
    \\
    \vdots
    \\
    (x_n - x_{j#})^2 + (y_n - y_{j#})^2 - (d_{j#}^2)
\end{bmatrix}. \]

The \( X^* \) can be solved.

\[ X^* = (A^T A)^{-1} A^T B^*. \]  

(5)

So we can calculate the accurate location of the unknown node.

IV. THE ANALYSIS OF LOCALIZATION PRECISION IN LSL

A. Noun Definition

In order to achieve better effect, we define some nouns as follows:

Reference anchor node: The anchor node which participates in the localization of the unknown node.

Benchmark equation: In the (3), each distance equation based on the distance between the anchor node and the unknown node constitutes the localization equations; during the process of reduced-order, each equation subtracts an equation (usually it is the \( n \)th equation), so the \( n \)th equation is called the benchmark equation.

Benchmark anchor node: The anchor node that corresponds to the benchmark equation.

Cumulative relative distance error: The distance measurement error of other each anchor node subtracts the distance measurement error of the \( j\# \) anchor node, and then averages the sum of the absolute value of the difference, and it can be described as

\[ \sum_{j \neq \text{ref}} \left| e_j - e_{\text{ref}} \right| / (n - 1), \]

we call it the cumulative relative distance error of \( j\# \) anchor node.

B. The Analysis of Localization Error Caused by the Distance Measurement Error in the LSL Algorithm

The RSSI is usually used to measure the distance between the anchor nodes and the unknown node, and it will cause attenuation mainly due to the influence of transmission medium, multipath propagation, reflection and so on, so it will produce distance measurement error in the localization of wireless sensor network. Based on the (3), we introduce the distance error, then

\[
\begin{align*}
(x_1 - x_a)^2 + (y_1 - y_a)^2 &= d_{1#}^2 + e_1 \\
(x_2 - x_a)^2 + (y_2 - y_a)^2 &= d_{2#}^2 + e_2 \\
\vdots \\
(x_n - x_a)^2 + (y_n - y_a)^2 &= d_{n#}^2 + e_n
\end{align*}
\]

The other equations subtract the \( j\)th equation respectively, then

\[
\begin{align*}
&\left[ (x_1 - x_{j#})^2 + (y_1 - y_{j#})^2 - (d_{j#}^2) \right] + \left[ (x_2 - x_{j#})^2 + (y_2 - y_{j#})^2 - (d_{j#}^2) \right] + \ldots + \left[ (x_n - x_{j#})^2 + (y_n - y_{j#})^2 - (d_{j#}^2) \right] \\
&= (x_1 - x_{j#})^2 + (y_1 - y_{j#})^2 - (d_{j#}^2) \] 
\]

(6)

So

\[ X = (A^T A)^{-1} A^T (B^* + E) \] 

(8)

Where

\[ B^* = \begin{bmatrix}
    (x_1 - x_{j#})^2 + (y_1 - y_{j#})^2 - (d_{j#}^2)
    \\
    (x_2 - x_{j#})^2 + (y_2 - y_{j#})^2 - (d_{j#}^2)
    \\
    \vdots
    \\
    (x_n - x_{j#})^2 + (y_n - y_{j#})^2 - (d_{j#}^2)
\end{bmatrix}. \]

\[ E = \begin{bmatrix}
    (e_1 - e_{j#})
    \\
    (e_2 - e_{j#})
    \\
    \vdots
    \\
    (e_n - e_{j#})
\end{bmatrix}. \]

In order to expound clearly, we suppose the fellows:

\[ F = (A^T A)^{-1} A^T . \] 

Then

\[ X = F \cdot (B^* + E) = X^* + FE. \] 

(10)

Where

\[ X^* = (A^T A)^{-1} A^T B^* \]

By using the Cauchy-Schwarz inequality, we can get the identification error of \( X \)

\[ \left\| X - X^* \right\| \leq \left\| F \right\| \cdot \left\| E \right\|. \]

The \( x_a, y_a \), and \( d_i \) are known, so the matrix \( A \) and vector \( F \) can be obtained. When the Euclidean norm of error vector \( E \) is smaller, the Euclidean norm of vector \( FE \) can be smaller.

Based on the above conclusion, we can get:

\[ \left\| X - X^* \right\| \propto \left\| F \right\| \cdot \left\| E \right\|. \]

The Euclidean norm of vector \((X-X^*)\) can be transformed to (11).

\[ \left\| X - X^* \right\| = \sqrt{(x - x_1)^2 + (y - y_1)^2}. \]

(11)

According to (11), the Euclidean norm of vector \((X-X^*)\) is just the localization error of the unknown node.

Based on the above proof, a theorem can be concluded:

Theorem 1: If the Euclidean norm of \( E \) is smaller, the localization error of the unknown node can be smaller.

V. THE MINIMUM CUMULATIVE RELATED DISTANCE ERROR IN LSL (LSL-MCR)
A. The Analysis Based on the Selection of the Benchmark Anchor Node

In the traditional LSL algorithm, the selection of the benchmark anchor node is random. How to further select the benchmark anchor node from the anchor nodes reasonably? Some algorithms select the anchor node that has the minimum distance measurement error as the benchmark anchor node. These methods cannot improve the localization precision greatly, even decrease the localization precision.

Based on the theorem 1, we know that the Euclidean norm of E has profound effect on the localization error of the unknown node. We attempt to seek the condition that the \( \|E\|_2 \) get minimum. We can conclude that the cumulative relative distance error of the anchor node has great effect on the localization precision of the unknown node. Selecting the benchmark anchor node reasonably is beneficial to reduce the extra errors and improve the localization precision.

Based on (9), we can get the following expressions.

\[
\|E\| = \begin{bmatrix} e_1 - e_j \\ \vdots \\ e_n - e_j \end{bmatrix} = \sqrt{\sum_{i,j}(e_i - e_j)^2}.
\]

Then

\[
\|E\| = \sum_{i,j} (e_i - e_j) = \sum_{i,j} (e_i - e_j) \propto \sum_{i,j} |e_i - e_j|.
\]

Based on the Theorem 1, we choose the anchor node \( j^\# \) as the benchmark anchor node. Where

\[
j = \min_j \sum_{i \neq j} |e_i - e_j| / (n - 1).
\]

B. The simulation analysis based on the selection of the anchor node

The simulation analyses the impact on the localization precision caused by the cumulative relative distance error of the benchmark anchor node. Number the reference anchor node from 1#~n# according to the cumulative relative distance error in ascending order, and take the 1#~n# reference anchor node as the benchmark anchor node respectively, and then the LSL algorithm is adopted to calculate the location of the unknown node. The performance of the selection of the anchor node can be evaluated by a series of simulations. To make the analysis more general, 100 unknown nodes are selected to emulate the performance. Specific parameters are showing as follows:

100 unknown nodes are randomly deployed, and there are 7 random anchor nodes around each unknown node. The error is a random distribution and is proportional to the distance. The accurate distance between the unknown node and the anchor node is \( dc \). Set the measurement error of the distance is \( dc \times er \), in which \( er \) is the random noise of [-0.1, 0.1] and its mean value is 0. The localization error of each unknown node is the distance between the calculated coordinate and standard coordinate. The simulation is shown by the Fig.2.

We can conclude from the Fig.2 that the localization error increases with the increase of the cumulative relative distance error of the benchmark anchor node.

So theorem 2 can be concluded:

**Theorem 2:** If the anchor node that has smaller cumulative relative distance error is selected as the benchmark anchor node, the localization precision of the unknown node will be higher.

Generally, it is difficult to get the exact measurement error, and the measurement error is proportional to the distance in RSSI, that is to say:

\[
\|X - X^*\| \propto \|E\| \propto \|D\|.
\]

Where

\[
D = \begin{bmatrix} (d_1 - d_{1,j}) \\ \vdots \\ (d_n - d_{n,j}) \end{bmatrix}
\]

So we can choose the anchor node \( j^\# \) which has the minimum cumulative relative distance error as the benchmark anchor node, that is to say

\[
j = \min_j \sum_{i \neq j} |d_i - d_j| / (n - 1)
\]

C. The Design of the Minimum Cumulative Related Distance Error in LSL (LSL-DMC)

Based on the above analysis, we select some nodes that the distance between each anchor node around the unknown node and unknown node is approximate as the corrected anchor nodes. So the distance measurement error between each anchor node around the unknown node and unknown node is approximate. Then we select the corrected anchor node which has the minimum cumulative relative distance error as the benchmark anchor node. That is to say, we select the central point from \( \{e_1, e_2, \ldots, e_s\} \) as the benchmark anchor node,

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and then the LSL algorithm is adopted to improve the localization precision of the unknown node.

The specific steps to select the benchmark anchor node from the corrected anchor nodes are shown as follows:
1) Obtain the distance between each anchor node and the unknown node.
2) Take a distance as a number.
3) Other each distance subtracts the distance of an anchor node, and then calculates the summation of the differences.

\[ D_{clus_j} = \sum_{i \in [1, n]} (d_i - d_j) \]  \tag{12}

4) Take the minimum \( i \) to \( D_{min} \), and take the maximum to \( D_{max} \), and then take the corresponding anchor node of \( D_{min} \) as the benchmark anchor node.

The steps of LSL-DMC are shown as follows:
- a) Aiming at the unknown node, obtain the distance between each anchor node around the unknown node and the unknown node, and then form the distance set.
- c) Selecting the corrected anchor nodes from the anchor nodes, and further select the benchmark anchor node from the corrected anchor nodes.
- d) The LSL algorithm is adopted to calculate accurate location of the unknown node.

VI. THE SIMULATION RESULTS

The performance of the proposed localization algorithms can be evaluated by a series of simulations. 100 unknown nodes are randomly deployed, and there are 7 random anchor nodes around each unknown node. The error is a random distribution and is proportional to the distance. The accurate distance between the unknown node and the anchor node is \( dc \). Set the measurement error of the distance is \( dc^* \), in which \( er \) is the random noise of \([-0.3, 0.3]\) and its mean value is 0.

The localization error of each unknown node is the distance between the calculated coordinate and standard coordinate. In consideration of the whole situation, we adopt the mean square deviation of the all nodes’ localization error.

We compare the estimated locations of a number of nodes in the two algorithms. The simulations also compared the localization precision of the unknown nodes in the two proposed algorithms. The simulations have been conducted to show the effectiveness of the improved localization algorithm. This paper mainly aims at the simulation in 2d layout environment. For future study, we will do more researches on the 3d physical model of the nodes.

TABLE I.

<table>
<thead>
<tr>
<th>Algorithm</th>
<th>LSL</th>
<th>LSL_MCR</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average error</td>
<td>1.6075</td>
<td>1.2319</td>
</tr>
</tbody>
</table>

Based on the simulation results, we can conclude that the localization precision of the most nodes in the LSL-MCR algorithm has greater improvement than the LSL algorithm, and the phenomenon of big errors is eliminated partly. As shown in Fig3 and Table1, it is clear that the LSL-MCR has better localization precision than the LSL algorithm.

VII. CONCLUSION

This paper analyzes the source of the localization error in the least square localization (LSL) algorithm, and proposes an improved least square localization algorithm. The key novelty of the improved algorithm (LSL-MCR) is the selection of the benchmark anchor node. We have also compared the localization precision of the unknown nodes in the two proposed algorithms. The simulations have been conducted to show the effectiveness of the improved localization algorithm. This paper mainly aims at the simulation in 2d layout environment. For future study, we will do more researches on the 3d physical model of the nodes.

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