Power Allocation for Tri-node Cooperative Communication of Gaussian Channel Gains

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Abstract—Channel capacity is usually undetermined in cooperative communication because of the various power allocations. In order to improve the channel capacity, power allocation for tri-terminal cooperative communication is discussed for Gaussian channel gains in EGC model and MRC model. The optimum power allocation algorithm, viewing channel capacity as the destination function, is presented. The paper obtains the results that when the gain variance of the source-node-to-destination-node channel is larger than that of the source-node-to-relay-node channel, source node can transmit information to the destination node directly. When the product of the power from the source node and the variance of the channel gain from source node to destination node is much smaller than the other products, source node can transmit information via the relay nodes completely. In other cases, source node can partly transmit to the destination node via relay nodes and partly transmit information to the destination node directly. And experimental results show that the scheme is effective.

Index Terms—cooperative communication, power allocation, channel capacity, relay node, variable gain

I. INTRODUCTION

In wireless communication system, multipath fading degrades the system transmission performance severely. Multiple-input multiple-output (MIMO) system is an efficient way to suppress multipath fading phenomenon via space diversity by maintaining spectral efficiency. However, due to the size, power and cost limitations of the mobile devices, multiple antennae cannot be conveniently installed to most of mobile terminals. Cooperative communication technology can create the virtual multi-antenna system in multi-terminal environment by sharing their cooperative users’ antennae. Then virtual MIMO systems can thereby be set up.

Amplify-and-Forward (AF) and Decode-and-Forward (DF) [1, 2] are elementary ways for cooperative communication technology and their applications are reported by a substantial number of previous studies. Power allocation and channel capacity become the key technologies in cooperative communication technology. Those issues also emerge in two-user cooperative communication model and have been referred in many references. For example, reference [3] proposed a power allocation scheme in a specific channel gain model for the purpose of reducing the interruption probability. Reference [4] introduced a method of improving system performance with power allocation in half duplex and high signal-noise-ratios (SNRs) scenarios. Reference [5] analyzed the way of increasing channel capacity by how to allocate power in full duplex mode. For the condition that channel gains are all fixed or are all unfixed, and that the SNRs in all the channels are very high or very low, reference [5] was discussed. Reference [6] optimized the transmission power in both source node and relay node to reduce Bit Error Rate (BER) in the cases of fixed total power and high SNR. A power allocation algorithm aiming at improving MIMO channel capacity is discussed in [7] under a variety of SNRs, where the channel conditions are unknown to the source node. The performances of elementary channel capacity are analyzed in [8]. Reference [9] researched the channel capacity performances in two-cooperative-node model.

Considering Gaussian channel gains in this study, power allocation and channel capacity problems will be thoroughly discussed in three-cooperative-node model.

II. SYSTEM MODEL OF THREE COOPERATIVE NODES

The communication system model of three-cooperative-node is shown in Fig. 1. In the figure, node 1, node 2 and node 3 are cooperative nodes; node 4 represents destination node (also called base station). $c_{ji}$ represents the channel gain from node $i$ to node $j$. In these cooperative nodes, node 1 is source node which can send signals; node 2 and node 3 are relay nodes which can send and receive signals.

Equal Gain Combination (EGC) model and Maximum Ratio Combination (MRC) model will be discussed in this paper. In EGC model, the received signals at the relay nodes and the destination node are given by
The signal which is transmitted from the source node to relay node 2. \( X_{12} \) represents the signal which is transmitted from the source node to relay node 3. \( X_{14} \) represents the signal which is transmitted from the source node to the destination node. Therefore, the relay nodes not only can forward the information from source node to destination node but also transmit their own information to the destination node.

### III. POWER ALLOCATION

In this model, \( P_1 \) represents total power of source node; \( P_2 \) and \( P_3 \) represent the power transmitted from relay node 2 and 3, respectively. \( b_1 \) represents the proportion of the power transmitted from source node to relay node accounting for total power. \( b_2 \) represents the proportion of the power that transmitted from source node to relay node 3 accounting for total power. Due to \( X_1 \sim N\left(0, P_1\right), X_2 \sim N\left(0, P_2\right), X_3 \sim N\left(0, P_3\right) \),

\[
X_{12} = X_1 + X_{13} + X_{14}
\]

\[
X_{14} = N\left(0, \left(1-b_1-b_2\right)P_3\right)
\]

where \( X_{12}, X_{13}, X_2, X_3 \) are independent Gaussian random variables. Because the channel gains are variable, the channel capacities are also variable. The performance can be described with ergodic channel capacity. It can be supposed that the transmission powers from all the nodes are fixed, and the channel gains are independent and identically distributed. The means and variances of the channel gains are fixed. Thus, the ergodic channel capacity is given by

\[
C = \max_{0<\sigma_{11},\sigma_{21},\sigma_{31}} \min \left\{ \frac{1}{2} \log \frac{1+h_1 \sigma_{11}^2 + h_2 \sigma_{21}^2 + \left(1-h_1-h_2\right) \sigma_{31}^2}{1+h_1 \sigma_{11}^2 + h_2 \sigma_{21}^2 + \left(1-h_1-h_2\right) \sigma_{31}^2} \right\}
\]

where

\[
R_i = \frac{1}{2} \log \frac{1+h_1 \sigma_{11}^2 + h_2 \sigma_{21}^2 + \left(1-h_1-h_2\right) \sigma_{31}^2}{1+h_1 \sigma_{11}^2 + h_2 \sigma_{21}^2 + \left(1-h_1-h_2\right) \sigma_{31}^2}
\]

It can be supposed that all the channels are narrowband frequency, non-selective and slow fading channels. Therefore, channel gains \( c_{ji} (i \in \{1, 2, 3\}, j \in \{2, 3, 4\}) \) are independent Gaussian distribution with the mean of zero and the variance of \( \sigma_{ji}^2 \). The formula from (5) to (8) can be rewritten as

\[
R_i = \frac{1}{2} \log \frac{1+h_1 \sigma_{11}^2 + h_2 \sigma_{21}^2 + \left(1-h_1-h_2\right) \sigma_{31}^2}{1+h_1 \sigma_{11}^2 + h_2 \sigma_{21}^2 + \left(1-h_1-h_2\right) \sigma_{31}^2}
\]

\[
R_i = \frac{1}{2} \log \frac{1+h_1 \sigma_{11}^2 + h_2 \sigma_{21}^2 + \left(1-h_1-h_2\right) \sigma_{31}^2}{1+h_1 \sigma_{11}^2 + h_2 \sigma_{21}^2 + \left(1-h_1-h_2\right) \sigma_{31}^2}
\]

\[
R_i = \frac{1}{2} \log \frac{1+h_1 \sigma_{11}^2 + h_2 \sigma_{21}^2 + \left(1-h_1-h_2\right) \sigma_{31}^2}{1+h_1 \sigma_{11}^2 + h_2 \sigma_{21}^2 + \left(1-h_1-h_2\right) \sigma_{31}^2}
\]

\[
R_i = \frac{1}{2} \log \frac{1+h_1 \sigma_{11}^2 + h_2 \sigma_{21}^2 + \left(1-h_1-h_2\right) \sigma_{31}^2}{1+h_1 \sigma_{11}^2 + h_2 \sigma_{21}^2 + \left(1-h_1-h_2\right) \sigma_{31}^2}
\]

The aim of the paper is that the ergodic channel capacity of system reaches to the maximum by allocating power. Then focus on how to get the value of \( b_1, b_2 \). It would be discussed as follows:

1. If \( \sigma_{41}^2 > \sigma_{31}^2, \sigma_{41}^2 > \sigma_{31}^2 \), \( R_i, R_2, R_3, R_4 \) decrease when \( b_1, b_2 \) increase. Therefore, if \( h_1 = h_2 = 0 \), that is when source node transmits information to the destination node directly, the channel capacity of system reaches maximum.

2. If \( \sigma_{41}^2 < \sigma_{31}^2, b_1 = 0 \). It can conclude that \( R_i < R_4, R_i < R_2 \), \( R_1 \) increases when \( b_2 \) increases. While \( R_3 \) decreases when \( b_2 \) increases. If \( P_1 \sigma_{31}^2 \geq P_2 \sigma_{31}^2 \), and if \( h_1 = 0, b_2 = 1 \), that is when source node transmits the
whole information via relay node 3, the channel capacity of system reaches maximum. If \( P_2\sigma^2_{42} < P_1\sigma^2_{31} \), and if \( R_1=R_3 \), the channel capacity of system reaches maximum. Therefore,

\[
b_1 = \frac{P_1\sigma^2_{31}}{P_2\sigma^2_{42}}
\]

(13)

(3) If \( \sigma^2_{31} < \sigma^2_{12} < \sigma^2_{21} \), \( b_1 = 0 \). It can conclude that \( R_1-R_3 < R_4 < R_2 \), \( R_1 \) increases when \( b_1 \) increases, while \( R_4 \) decreases when \( b_1 \) increases. If \( P_2\sigma^2_{42} \geq P_1\sigma^2_{21} \), and if \( b_1 = 1 \), \( b_2 = 0 \), source node transmits the whole information via relay node 2 and the channel capacity of system reaches maximum. If \( P_2\sigma^2_{42} < P_1\sigma^2_{21} \), and if \( R_1=R_4 \), the channel capacity of system reaches maximum. Therefore,

\[
b_2 = \frac{P_1\sigma^2_{21}}{P_2\sigma^2_{42}}
\]

(14)

(4) If \( \sigma^2_{41} < \sigma^2_{31} \), \( \sigma^2_{41} < \sigma^2_{21} \), the formulae from (9) to (12) can be rewritten as

\[
R_{11} = \frac{1}{2} \log_2 (1 + b_1 P_1\sigma^2_{31} + b_2 P_2\sigma^2_{42})
\]

(15)

\[
R_{21} = \frac{1}{2} \log_2 (1 + \sigma^2_{42} P_2 + \sigma^2_{31} P_1)
\]

(16)

\[
R_{31} = \frac{1}{2} \log_2 (1 + b_1 P_1\sigma^2_{31} + \sigma^2_{21} P_1)
\]

(17)

\[
R_{41} = \frac{1}{2} \log_2 (1 + b_1 P_1\sigma^2_{31} + \sigma^2_{21} P_1)
\]

(18)

\( R_{11}, R_{21}, R_{31}, R_{41} \) in the formula from (15) to (18) are equivalent to \( R_1, R_2, R_3, R_4 \) in the formulae from (19) to (12). Then it would be discussed as follows:

1) If \( P_2\sigma^2_{42} \geq P_1\sigma^2_{31} \), \( P_3\sigma^2_{43} \geq P_1\sigma^2_{31} \), no matter what the values of \( b_1, b_2 \) are, \( R_{11} \leq \min \{ R_{21}, R_{31}, R_{41} \} \). Therefore, if \( b_1 + b_2 = 1 \), source node transmits the whole information via relay node 2 and relay node 3. If \( \sigma^2_{31} > \sigma^2_{31} \), \( b_1 = 1 \), source node transmits the whole information via relay node 2. If \( \sigma^2_{31} < \sigma^2_{31} \), \( b_1 = 1 \), source node transmits the whole information via relay node 3. If \( \sigma^2_{31} = \sigma^2_{31} \), no matter what the values of \( b_1, b_2 \) are, the channel capacity of the system cannot be affected.

2) If \( P_2\sigma^2_{42} \geq P_1\sigma^2_{31} \), \( P_3\sigma^2_{43} < P_1\sigma^2_{31} \), no matter what the values of \( b_1, b_2 \) are, \( R_{11} > R_{11} \), \( R_{21} > R_{31} \). If \( \sigma^2_{31} \geq \sigma^2_{31} \), \( b_2=0, b_1=1 \). If \( \sigma^2_{31} < \sigma^2_{31} \),

\[
b_2 = \frac{P_2\sigma^2_{42}}{P_3\sigma^2_{43}}
\]

(19)

\[
b_1 = 1 - b_2.
\]

3) If \( P_2\sigma^2_{42} < P_1\sigma^2_{31} \), \( P_3\sigma^2_{43} \geq P_1\sigma^2_{31} \), it is similar with 2).

4) If \( P_2\sigma^2_{42} < P_1\sigma^2_{31} \), \( P_3\sigma^2_{43} < P_1\sigma^2_{31} \), and if \( R_{11}=R_{21}=R_{31}=R_{41} \),

\[
b_1 = \frac{P_2\sigma^2_{42}}{P_1\sigma^2_{31}}
\]

(20)

\[
b_2 = \frac{P_3\sigma^2_{43}}{P_1\sigma^2_{31}}
\]

(21)

Because \( b_1 + b_2 \leq 1 \),

if \( \frac{P_2\sigma^2_{42}}{P_1\sigma^2_{31}} \geq 1 \), \( b_1 + b_2 = 1 \). And if \( \sigma^2_{31} > \sigma^2_{31} \), \( b_1 \) is shown in formula (20), \( b_2 = 1 - b_1 \). If \( \sigma^2_{31} < \sigma^2_{31} \), \( b_2 \) is shown in formula (21), \( b_1 = 1 - b_2 \). If \( \sigma^2_{31} = \sigma^2_{31} \), no matter what the values of \( b_1, b_2 \) are, the channel capacity of the system cannot be affected. If \( \frac{P_2\sigma^2_{42}}{P_1\sigma^2_{31}} \geq \frac{P_3\sigma^2_{43}}{P_1\sigma^2_{31}} < 1 \), \( b_1, b_2 \) are shown in formula (20) and formula (21).

IV. MRC MODEL

In this model, the received signals at the relay nodes and the destination node are given by

\[
Y_5 = c_1X_{12} + N
\]

(22)

\[
Y_5 = c_3X_{13} + N
\]

(23)

\[
Y_4 = c_2X_{14} + N
\]

(24)

\[
Y_4 = c_4X_4 + N
\]

(25)

\[
Y_4 = c_4X_3 + N
\]

(26)

respectively, where \( Y_4 \) represents received signal at the destination node which is transmitted from the source node. \( Y_4 \) represents received signal at the destination node which is transmitted from the relay node 2. \( Y_3 \) represents received signal at the destination node from the relay node 3. It is assumed that

\[
X_2 = \beta_1 Y_2 = \beta_1 (c_1X_{12} + N)
\]

(27)

\[
X_3 = \beta_2 Y_3 = \beta_2 (c_3X_{13} + N)
\]

(28)

where \( \beta_1, \beta_2 \) are amplifying coefficients. Then formula (25) and formula (26) are rewritten by

\[
Y_4 = \beta_1 c_2 c_{12} X_{12} + (1 + \beta_1 c_2)N
\]

(29)

\[
Y_4 = \beta_1 c_4 c_{14} X_{14} + (1 + \beta_2 c_4)N
\]

(30)

Therefore, \( \beta_1, \beta_2 \) are given by

\[
\beta_1 = \left( \frac{P_1}{b_2 c_{12} P_1 + 1} \right)^{\frac{1}{2}}
\]

(31)

\[
\beta_2 = \left( \frac{P_1}{b_2 c_{14} P_1 + 1} \right)^{\frac{1}{2}}
\]

(32)

In this model, the system signal-noise-ratio (SNR) is given by
\[ \gamma = \gamma_1 + \gamma_2 + \gamma_3 \]  

(33)

where

\[ \gamma_1 = (1 - b_1 - b_2)P_c^2 \]  

(34)

\[ \gamma_2 = \frac{b_1P_c^2\sigma_{21}^2 + P_{c2}^2}{1 + b_1P_c^2\sigma_{21}^2 + P_{c2}^2} \]  

(35)

\[ \gamma_3 = \frac{b_1P_c^2\sigma_{31}^2 + P_{c3}^2}{1 + b_1P_c^2\sigma_{31}^2 + P_{c3}^2} \]  

(36)

In these formulae, \( \gamma \) represents the system SNR. \( \gamma_1 \) represents the SNR which is transmitted from the source node to the destination node directly. \( \gamma_2 \) represents the SNR which is transmitted from the source node to the destination node via relay node 2. \( \gamma_3 \) represents the SNR which is transmitted from the source node to the destination node via relay node 3. When the SNRs are high ( \( P_j \gg 1, (j = 1, 2, 3) \) ), \( \gamma_1, \gamma_2, \gamma_3 \) are exponentially distributed with parameter \( \lambda_1, \lambda_2, \lambda_3 \) respectively. Then the parameters are given by

\[ \lambda_1 = (1 - b_1 - b_2)P\sigma_{21}^2 \]  

(37)

\[ \lambda_2 = \frac{b_1P\sigma_{21}^2 + P\sigma_{22}^2}{b_1P\sigma_{21}^2 + P\sigma_{22}^2} \]  

(38)

\[ \lambda_3 = \frac{b_1P\sigma_{31}^2 + P\sigma_{33}^2}{b_1P\sigma_{31}^2 + P\sigma_{33}^2} \]  

(39)

Therefore, the ergodic capacity is given by

\[ C = E[\log_2(1 + \gamma_1 + \gamma_2 + \gamma_3)] \]  

(40)

Because the formula of SNR is very complex, it is difficult to solve the expectation according to the defining formula. Because the function \( f(x) = \log_2(1 + x) \) is a convex monotonous increasing function related to \( x \). Therefore, the ergodic capacity is a monotonous increasing function related to the system SNR. According to Jensen inequality ( \( E[f(x)] \leq f(E[x]) \) ), it is given by

\[ C \leq \log_2(1 + E[\gamma_1] + E[\gamma_2] + E[\gamma_3]) \]  

(41)

When the SNRs are all high, the equals mark can be used. According to the distributions of \( \gamma_1, \gamma_2, \gamma_3 \), they are given by

\[ E[\gamma_1] = (1 - b_1 - b_2)P\sigma_{21}^2 \]  

(42)

\[ E[\gamma_2] = \frac{b_1P\sigma_{21}^2 + P\sigma_{22}^2}{b_1P\sigma_{21}^2 + P\sigma_{22}^2} \]  

(43)

\[ E[\gamma_3] = \frac{b_1P\sigma_{31}^2 + P\sigma_{33}^2}{b_1P\sigma_{31}^2 + P\sigma_{33}^2} \]  

(44)

Because \( E[\gamma_1] \leq b_1P\sigma_{21}^2, E[\gamma_3] \leq b_2P\sigma_{31}^2 \), it is given by

\[ E[\gamma] \leq b_1P\sigma_{21}^2 + b_2P\sigma_{31}^2 + (1 - b_1 - b_2)P\sigma_{41}^2 \]  

(45)

Then it would be discussed as follows:

1. If \( \sigma_{41}^2 \geq \max\{\sigma_{21}^2, \sigma_{31}^2\} \), \( b_1 = b_2 = 0 \), that is when source node transmits information to the destination node directly, the channel capacity of system reaches maximum.

2. If \( P\sigma_{41}^2 \leq \min\{P\sigma_{21}^2, P\sigma_{31}^2, P\sigma_{41}^2\} \), it is considered that the channel from the source node to the destination node doesn’t exist. Therefore, \( b_1 + b_2 = 1 \).

3. If \( P\sigma_{41}^2 \geq \min\{P\sigma_{21}^2, P\sigma_{31}^2\} \), \( P\sigma_{41}^2 \leq \min\{P\sigma_{21}^2, P\sigma_{31}^2\} \), \( b_1 = 0 \). It is that the source node cannot transmit the information via relay node 2. Then

\[ E[\gamma] = (1 - b_2)P\sigma_{41}^2 + \frac{b_1P\sigma_{21}^2P\sigma_{41}^2}{b_1P\sigma_{21}^2 + P\sigma_{41}^2} \]  

(46)

Therefore,

\[ b_2 = \min\left\{ \frac{P\sigma_{41}^2}{\sqrt{P\sigma_{21}^2/P\sigma_{41}^2 - 1}}, 1 \right\} \]  

(47)

4. If \( P\sigma_{41}^2 \leq \min\{P\sigma_{21}^2, P\sigma_{31}^2\} \), \( P\sigma_{41}^2 \geq \min\{P\sigma_{21}^2, P\sigma_{31}^2\} \), \( b_2 = 0 \). It is that the source node cannot transmit the information via relay node 3. Then

\[ E[\gamma] = (1 - b_1)P\sigma_{41}^2 + \frac{b_1P\sigma_{21}^2P\sigma_{41}^2}{b_1P\sigma_{21}^2 + P\sigma_{41}^2} \]  

(46)

Therefore,

\[ b_1 = \min\left\{ \frac{P\sigma_{41}^2}{\sqrt{P\sigma_{21}^2/P\sigma_{41}^2 - 1}}, 1 \right\} \]  

(47)

V. SIMULATION RESULTS

Because the number of the parameters is large and the range of their values covers widely, in order to reduce the quantity of experiment, \( P\sigma_{ji}^2 \) ( \( i \in \{1, 2, 3\} \), \( j \in \{2, 3, 4\} \) ) can be considered as an integral variable. In EGC model, the simulation result is shown in Fig. 2 when \( P\sigma_{21}^2 = 0, P\sigma_{31}^2 = 12, P\sigma_{41}^2 = 4, P\sigma_{42}^2 = 3 \).

When the variance of the channel gain from source node to destination node is larger than the variances of the channel gains from source node to relay nodes, source node can partly transmit information to the destination node via relay nodes completely. In other general situation, source node can transmit information via the relay nodes and partly transmit information to the destination node directly. \( b_1, b_2 \in [0, 1] \) does exist and make the channel capacity of system to reach maximum.
When $P_1\sigma_{21}^2 = 12$, $P_1\sigma_{31}^2 = 2$, $P_2\sigma_{21}^2 = 8$, $P_2\sigma_{42}^2 = 4$, $P_3\sigma_{43}^2 = 6$, the simulation result is shown in Fig. 3.

When $P_1\sigma_{21}^2 = 6$, $P_1\sigma_{31}^2 = 8$, $P_2\sigma_{42}^2 = 1$, $P_3\sigma_{43}^2 = 18$, the simulation result is shown in Fig. 4.

In MRC model, when $P_1\sigma_{21}^2 = 10$, $P_2\sigma_{21}^2 = 20$, $P_1\sigma_{31}^2 = 100$, $P_2\sigma_{42}^2 = 200$, $P_3\sigma_{43}^2 = 300$, the simulation result is shown in Fig. 5.

When $P_1\sigma_{21}^2 = 80$, $P_1\sigma_{31}^2 = 50$, $P_1\sigma_{41}^2 = 10$, $P_2\sigma_{42}^2 = 60$, $P_3\sigma_{43}^2 = 80$, the simulation result is shown in Fig. 6.

When $P_1\sigma_{21}^2 = 200$, $P_1\sigma_{31}^2 = 10$, $P_1\sigma_{41}^2 = 100$, $P_2\sigma_{42}^2 = 300$, $P_3\sigma_{43}^2 = 200$, the simulation result is shown in Fig. 7.
VI. CONCLUSION

In this paper, power allocation and ergodic channel capacity for tri-terminal cooperative communication has been discussed when the channel gains are Gaussian distributed. In order to reach the maximum channel capacity, if the channel gain variance of the source-node-to-destination-node is larger than the source-node-to-relay-node, it can be shown that source node can transmit information to the destination node directly without relay. In other cases, source node can partly transmit to the destination node via relay nodes and partly transmit information to the destination node directly. And experimental results show that the scheme is effective.

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