Research on an Anti-Perturbation Kalman Filter Algorithm

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Abstract—An improved Kalman filter algorithm is proposed by two kinds of anti-perturbation method which is derived according to the perturbation theorem of inverse matrix. Furthermore, direction-correcting has been merged into the algorithm by using multiple hypothesis testing theory which can detect the current direction of a target. Finally, Both quantitative and qualitative analysis are given in detail. The measurements and experiments based on indoor positioning demonstrate that the improved algorithm(named IKF) has great performance.

Index Terms—Kalman Filter; Perturbation theorem; multiple hypotheses testing; indoor positioning; wireless sensor networks.

I. INTRODUCTION

Kalman filter is a linear unbiased minimum variance estimate algorithm, a practical introduction to the discrete Kalman filter was gave in reference [1], but it lacks of immunity to the gross errors of the measurement data and fault-tolerant to sudden failure of the sensor [2]. Dynamic noise is inaccuracy and it easily leads to model divergence in common Kalman filtering. Based on this shortcoming, simplified Sage-Husa and improved Sage-Husa filtering algorithm have been used in reference [2, 3]. Reference [2, 4] had proposed Kalman filter algorithm with immunity to the outliers through adding compression influence function and activation function respectively. And a method for estimating noise covariances from process data has been investigated in reference [5].

In this paper, an improved Kalman filter algorithm was proposed based on the perturbation theorem of inverse matrix. Furthermore, direction-correcting has been done for the improved algorithm by using multiple hypothesis testing theory. The verification of the validity of the algorithm had been done in the last.

II. IMPROVED KALMAN FILTER BASED ON PERTURBATION THEOREM OF INVERSE MATRIX

Consider the following linear discrete system:

\[
\begin{align*}
    x_{k+1} &= \Phi_{k+1,k} x_k + \Gamma_k n_k, \\
    z_k &= H_k x_k + w_k
\end{align*}
\]

(1)

where \( \Phi \) and \( \Gamma \) respectively represent the transition matrix and disturbance matrix of the system; \( H \) is the measurement matrix; and \( n \) is a \( p \times 1 \) vector of turbulent noise; \( w \) is a \( N \times 1 \) vector of measurement noise; \( x \) is an \( M \)-vector, representing the state of the system and \( z \) is an \( N \)-vector representing measurements.

The basic recursive equations are as follows:

\[
\begin{align*}
    P_{k/k-1} &= \Phi_{k/k-1} P_{k-1/k-1} \Phi_{k/k-1}^T + \Gamma_{k-1} Q_k \Gamma_{k-1}^T \\
    K_k &= P_{k/k-1} H_k^T (H_k P_{k/k-1} H_k^T + R_k)^{-1} \\
    \hat{z}_{k/k-1} &= z_k - H_k x_{k,k-1} \\
    \hat{x}_{k/k} &= x_{k,k-1} + K_k \hat{z}_{k,k-1} \\
    P_{k/k} &= (1 - K_k H_k) P_{k/k-1}
\end{align*}
\]

(2)

We can achieve adaptive algorithm by adjusting the error covariance matrix, measurement noise, system noise or innovations of the basic equations self-adaptively. The concept of the perturbation theorem and its use on Kalman filter was introduced in reference [6], and in this paper we gave both quantitative analysis and qualitative analysis with more specific details.

A. Quantitative Analysis of The Use of Perturbation Theorem in Kalman Filter

As for \( \forall A \), \( \text{cond}(A) = \|A\|_2 \|A^{-1}\|_2 \) is its condition number. And it is an important parameter in obtaining the perturbation of an inverse matrix. According to the perturbation theorem in reference [7], the greater the condition number of matrix \( A \), the greater the relative error of \( (A + \sigma A)^{-1} \) compared to \( A^{-1} \).
Assume that \((H_k P_{x/k-1} H_k^T + R_k)^{-1} = A^{-1}\), then
\[
(H_k P_{x/k-1} H_k^T + R_k + \sigma R_l)^{-1}
\]
can be wrote as
\[(A + \sigma A)^{-1} \]. Here, \(\sigma R\) represents the perturbation matrix of \(R\).

Based on the above analysis, we can adjust the parameters of the Kalman filter from two aspects to reduce the impact of noise which is not accurately measured. On one hand, the greater the cond(A), the more sensitive \(A^{-1}\) to \(\sigma A\), and the higher accuracy of \(A\) is needed. In turn, it can be understood as finding ways to adjust \(R\) or \(P\) to make \(\text{cond}(A)\) to a minimum.

According to the equivalence of matrix norm, we can use \[\| A \|_2 \| A^{-1} \|_2\] instead of \[\| A \| \| A^{-1} \|\]. Therefore, if
\[
A = (H_k P_{x/k-1} H_k^T + R_k),
\]
the question converted to the following optimization problem:

\[
\begin{cases}
\text{s.t.} & A = H_k P_{x/k-1} H_k^T + \lambda_R R_k \\
& \min \{ \varrho^{1/2}(A^T A) \rho^{1/2}((A^{-1})^T(A^{-1})) \}
\end{cases}
\]
\[
\begin{cases}
\text{s.t.} & A = H_k (\lambda_p P_{x/k-1}) H_k^T + R_k \\
& \min \{ \varrho^{1/2}(A^T A) \rho^{1/2}((A^{-1})^T(A^{-1})) \}
\end{cases}
\]
(3)

Where, \(\lambda_p\) is the correction factor of \(\sigma A\), and \(\lambda_R\) is the correction factor of \(P\). On the other hand, when \(\text{cond}(A)\) is large, the error of \(K\) caused by \(\text{cond}(A)\) is also large. According to the basic principles of Kalman filter, we need to reduce the innovation. According to reference [4], the innovation can be reduced by letting the observations multiplied by an activation function.

A is a symmetrical matrix, so type (3) is turned into:

\[
\begin{cases}
\text{s.t.} & A = H_k P_{x/k-1} H_k^T + \lambda_R R_k \\
& \min \{ \varrho(A) \rho(A^{-1}) \}
\end{cases}
\]
\[
\begin{cases}
\text{s.t.} & A = H_k (\lambda_p P_{x/k-1}) H_k^T + R_k \\
& \min \{ \varrho(A) \rho(A^{-1}) \}
\end{cases}
\]
(4)

Firstly we suppose that,

\[
\begin{bmatrix}
0,0,0,0 \\
0,1,0,0 \\
0,0,1,0 \\
0,0,0,1
\end{bmatrix}
\]

Considering the case to adjust \(R\),

\[
A_R = H P_{x/k-1} H^T + \lambda_R R_{k-1}
\]
\[
\begin{bmatrix}
p_{11} + \lambda_R R_{0}, p_{12} \\
p_{21}, p_{22} + \lambda_R R_{0} \\
p_{31}, p_{32}, p_{33}, p_{34} \\
p_{41}, p_{42}, p_{43}, p_{44}
\end{bmatrix}
\]
\[
H = \begin{bmatrix} 1,0,0,0 \\ 0,1,0,0 \\ 0,0,1,0 \\ 0,0,0,1 \end{bmatrix}, R_{k-1} = R_0
\]

C and the same time the inverse matrix of \(A_R\) can be obtained as:

\[
A_R^{-1} = \begin{bmatrix}
p_{22} + \lambda_R R_{0} - p_{21} \\
-p_{12}, p_{11} + \lambda_R R_{0},
\end{bmatrix}
\]

Where \[\{A_R\} = (p_{22} + \lambda_R R_0)(p_{11} + \lambda_R R_0) - p_{21} p_{12} \]

Further on, the spectral radius of \(A_R\) and \(A_{R}^{-1}\) can be presented by:

\[
\rho(A_R) = \max_i \{\lambda_i(A_R)\}, \\
\rho(A_{R}^{-1}) = \max_i \{\lambda_i(A_{R}^{-1})\}
\]

In order to solve this problem, the following derivation should be done:

\[
|\lambda I - A_R| = 0
\]
\[
=> \lambda^2 - \lambda(2\lambda_R R_0 + p_{22} + p_{11}) + [\lambda_R^2 R_0^2 + \lambda_R R_0 (p_{22} + p_{11}) + p_{22} p_{11} - p_{12} p_{21}] = 0.
\]

Because of \(\Delta_R = (p_{22} - p_{11})^2 + 4p_{12}^2 > 0\), the solutions \(\lambda\) can be got.

Here, let \(C = (p_{22} - p_{11})^2 + 4p_{12}^2\) be a constant factor. So \(\rho(A)\) and \(\rho(A^{-1})\) can be got as
\[
\rho(A) = \max_i \{\lambda_i(A)\} = \frac{2\lambda_R R_0 + p_{22} + p_{11} + \sqrt{C}}{2},
\]
and
\[
\rho(A^{-1}) = \max_i \{\lambda_i(A^{-1})\} = \frac{2\lambda_R R_0 + p_{22} + p_{11} + \sqrt{C}}{2|A_R|}.
\]

The condition number has the following form according to its definition:

\[
\text{cond}(A_R) = \frac{(2\lambda_R R_0 + p_{22} + p_{11} + \sqrt{C})^2}{4|A_R|}
\]
To make a minimum of \( \text{cond}(A_R) \), the way is to make the derivative of \( \text{cond}(A_R) \) on \( \lambda_R \) to zero, let
\[
\text{cond}'(A_R) = \frac{d(\text{cond}(A_R))}{d\lambda_R} \text{ be zero, so } \lambda_R \text{ can be got:}
\]
\[
\lambda_R = \frac{-\sqrt{C} - p_{22} - p_{11}}{2R_0}.
\]

While \( \lambda_R \) is got, it can be used to adjust the measurement noise \( R \), due to the analysis type (2) was turned into:
\[
R_{k} = \lambda_R R_{k-1}
\]
\[
K_k = P_{x/k-1} H_k^T (H_k P_{x/k-1} H_k^T + R_k)^{-1}
\]
\[
\hat{Z}_{k/k-1} = z_k - H_k \hat{x}_{k-1}
\]
\[
x_{k/k} = x_{k-1} + K_k \hat{Z}_{k/k-1}
\]
\[
P_{x/k} = (I - K_k H_k) P_{x/k-1}
\]
(5)

The other case, considering the case to adjust \( P \), which is similarly to the case \( R \), the following derivation should be done before \( \lambda_R \) was got:
\[
P_{x/k} = \Phi_{x/k-1} P_{x/k-1} \Phi_{x/k-1}^T + \Gamma_{k-1} Q_k \Gamma_{k-1}^T
\]
\[
R_k = \lambda_P R_{k-1}
\]
\[
K_k = P_{x/k-1} H_k^T (H_k P_{x/k-1} H_k^T + R_k)^{-1}
\]
\[
\hat{Z}_{k/k-1} = z_k - H_k \hat{x}_{k-1}
\]
\[
x_{k/k} = x_{k-1} + K_k \hat{Z}_{k/k-1}
\]
\[
P_{x/k} = (I - K_k H_k) P_{x/k-1}
\]
(6)

B. Qualitative Analysis of Improved Kalman Filter Based on Perturbation Theorem

Quantitative analysis is a complex issue and can’t obtain good results in all cases, for example, if the measurement noise or noise covariance matrix is not obvious, it can’t get expected results. So qualitative analysis is actually appropriately for some cases. In this paper, kalman filter was used for indoor positioning, of which the measurement noise and noise covariance matrix are not vibrate badly. According to this reason, both quantitative analysis and qualitative analysis are done in this paper. Methods about how to change \( R \) and \( Z_k \) are mentioned in reference [2, 4]. In this paper, we made a compromise between them and method based on perturbation theorem. Consequently, the following formula can be obtained:
\[
\text{cond}(A_p) = \frac{(2R_0 + \lambda_R p_{22} + \lambda_R p_{11} + \sqrt{C})^2}{4|A_p|}
\]
To make a minimum of \( \text{cond}(A_p) \), let
\[
\text{cond}'(A_p) = \frac{d(\text{cond}(A_p))}{d\lambda_P} = 0,
\]
\[
\Rightarrow \lambda_P = \frac{2}{-\sqrt{C} - p_{22} - p_{11}}
\]
Since \( \lambda_P \) was derived, type (2) turned into:
\[
P_{x/k} = \Phi_{x/k-1} P_{x/k-1} \Phi_{x/k-1}^T + \Gamma_{k-1} Q_k \Gamma_{k-1}^T
\]
\[
P_{x/k} = \lambda_P P_{x/k-1}
\]
\[
K_k = P_{x/k-1} H_k^T (H_k P_{x/k-1} H_k^T + R_k)^{-1}
\]
\[
\hat{Z}_{k/k-1} = z_k - H_k \hat{x}_{k-1}
\]
\[
x_{k/k} = x_{k-1} + K_k \hat{Z}_{k/k-1}
\]
\[
P_{x/k} = (I - K_k H_k) P_{x/k-1}
\]
(7)
\[ r_i = \sqrt{M_i} (k+1) \] (8)

\[ d_i = D_i \cdot (k+1) + \varepsilon_i \quad i = 1, 2, ..., m. \]

In Eq. 7 and Eq. 8, \( b \) is the forgetting factor; \( f_i(r_i) \) is the activation function; \( \gamma_1 \), \( \gamma_2 \) and \( \gamma_3 \) were gained on experience. On one hand, if \( \text{cond}(A) \) is greater than a certain threshold, we think the sensitivity of \( A^{-1} \) to perturbation noise \( \sigma R \) is too high. And we choose to decrease innovations directly while adjusting \( R \). Otherwise, \( R \) was changed according to reference [4]. On the other hand, we changed \( Z_k \) by multiplying \( f_i(r_i) \) according to the truth that the activation function is inversely proportional to \( \text{cond}(A) \).

III. THEORY OF DIRECTION-CORRECTING ON KALMAN FILTER

Minimum Error Probability criterion and Bayesian criterion are usually used in multiple hypothesis testing problems in reference [8]. Suppose that we want to get the correct decision from the possible values of \( \{H_0, H_1, ..., H_{M-1}\} \), \( C_{ij} \) is the cost of the miscarriage of justice that \( H_j \) was judged as \( H_i \). The average total cost is

\[ C = \sum_{j=0}^{M-1} \sum_{i=0}^{M-1} C_{ij} p(H_j / z) p(z) dz. \]

The question of multiple hypothesis testing is to find an appropriate criterion to let \( C \) to a minimum.

Suppose that:

\[ \begin{align*}
H_0 : z_i &= -A + \omega, i = 0, 1, 2, ..., N - 1 \\
H_1 : z_i &= \omega, i = 0, 1, 2, ..., N - 1 \\
H_2 : z_i &= A + \omega, i = 0, 1, 2, ..., N - 1
\end{align*} \]

Where, \( A \) is the state, and \( \{w_i\} \) is a vector-valued, independent, gaussian random process, with zero mean.

It is actually a problem of minimum error probability criterion, which is equivalent to maximum likelihood criterion while they have equal prior probabilities. The probability can be represented as:

\[ p(z / H_j) = \frac{1}{2\pi \sigma^2} \exp\left[-\frac{1}{2\sigma^2} \sum_{i=0}^{N-1} (z_i - A_j)^2\right]. \] (9)

It’s a basic feature of objects to maintain their original state. Inertia factor \( g \) was introduced owing to this basic feature. In one direction, an object only has three states of motion: positive direction, negative direction and still. Here, \( g \) represents the probability when the current state and the previous state are the same. The probability when the current state and the previous state are not the same can be got as \( \frac{1 - g}{(N - 1)} \).

Due to the above analysis, the probability can be represented by a probability function:

\[ Q(H_j) = \begin{cases} g, & H_{kj} = H_{k-1} \\ 1 - g, & H_{kj} \neq H_{k-1} \end{cases}, \quad (N - 1), H_{kj} \neq H_{k-1}. \] (10)

So, type (10) was converted into:

\[ p(z / H_j) = \frac{Q(H_j)}{2\pi \sigma^2} \exp\left[-\frac{1}{2\sigma^2} \sum_{i=0}^{N-1} (z_i - A_j)^2\right]. \] (11)

According to the multiple hypotheses testing theory, the following criteria can be got. If it satisfies \( p(H_k / z) > p(H_i / z), i = 0, 1, 2, ..., M - 1 \) \( i \neq k \), the current state is determined to be \( H_k \). For \( p(z / H_i) p(H_j) = p(z / H_j) p(H_i) \) equals to make a minimum of \( p(z / H_j) p(H_i) \), Owing to type (11), the problem can be further transformed into form (12):

\[ \max \{\ln Q(H_j) \sum_{i=0}^{N-1} (z_i - \bar{z})^2 + N(\bar{z} - A_j)^2\}. \] (12)

Supposing that the three states are \( \{H_0, H_1, H_2\} \), the following form can be got by the criteria of form (12).

\[ \begin{align*}
H_0 : z &\leq \frac{[\ln Q(H_j) - \ln Q(H_0)]\sigma_j^2 - NA^2}{2A} \\
H_1 : z &\leq \frac{[\ln Q(H_j) - \ln Q(H_0)]\sigma_j^2 - NA^2}{2A} \\
H_2 : z &\geq \frac{[\ln Q(H_j) - \ln Q(H_0)]\sigma_j^2 + NA^2}{2A}
\end{align*} \] (13)

In actual, we consider two dimensions \( x, y \) and their states \( s_{-1x}, s_{0x}, s_{1x}, s_{-1y}, s_{0y}, s_{1y} \). So, form (14) can be got by multiple hypothesis testing theory.
\[
\begin{align*}
&\begin{cases}
  s_{-1x} : z_{wi} = -V_x + \kappa_i \\
  s_0x : z_{wi} = \kappa_i \\
  s_{1x} : z_{wi} = V_x + \kappa_i
\end{cases} & i = 0, 1, \ldots, N - 1 \quad \text{and} \\
&\begin{cases}
  s_{-1y} : z_{wi} = -V_y + \epsilon_i \\
  s_0y : z_{wi} = \epsilon_i \\
  s_{1y} : z_{wi} = V_y + \epsilon_i
\end{cases} & i = 0, 1, \ldots, N - 1.
\end{align*}
\]

And the following criteria can be got at the same while.

\[
\begin{align*}
&s_{-1x} : z \leq \frac{[\ln Q(H_1) - \ln Q(H_0)]\sigma_{V_x}^2 - NV_x^2}{2V_x} \\
&s_0x : \frac{[\ln Q(H_1) - \ln Q(H_0)]\sigma_{V_x}^2 - NV_x^2}{2V_x} \leq z \leq \frac{[\ln Q(H_2) - \ln Q(H_1)]\sigma_{V_x}^2 + NV_x^2}{2V_x} \\
&s_{1x} : z \geq \frac{[\ln Q(H_2) - \ln Q(H_1)]\sigma_{V_x}^2 + NV_x^2}{2V_x}
\end{align*}
\]

\[
\begin{align*}
&s_{-1y} : z \leq \frac{[\ln Q(H_1) - \ln Q(H_0)]\sigma_{V_y}^2 - NV_y^2}{2V_y} \\
&s_0y : \frac{[\ln Q(H_1) - \ln Q(H_0)]\sigma_{V_y}^2 - NV_y^2}{2V_y} \leq z \leq \frac{[\ln Q(H_2) - \ln Q(H_1)]\sigma_{V_y}^2 + NV_y^2}{2V_y} \\
&s_{1y} : z \geq \frac{[\ln Q(H_2) - \ln Q(H_1)]\sigma_{V_y}^2 + NV_y^2}{2V_y}
\end{align*}
\]

Here, \( N \) is the length of selected window; \( z_{ij} \) is the velocity derived from the Kalman filter; and \( z = \frac{1}{N} \sum_{j=k}^{k+N} z_{ij} \) is the mean value of \( z_{ij} \) within the window. \( Q(v_{-1}) \), \( Q(v_0) \) and \( Q(v_1) \) can be obtained by form (10). We assume that \( Q(v_{-1}) = Q(v_0) = Q(v_1) \), Eq. 15 was converted into:

\[
\begin{align*}
  v_{-1x} & : z \leq -V_x / 2 \\
  v_{0x} & : -V_x / 2 \leq z \leq V_x / 2 \quad \text{and} \\
  v_{1x} & : z \geq V_x / 2
\end{align*}
\]

The motion direction of the target can be detected by Eq. 16 and the output of the improved Kalman filter can be corrected simultaneously. Considering the case of x-axis, suppose the state predicted is \( v_{-1} \), if \( x - x_{pre} > 0 \), we corrected \( x \) to \( x = x_{pre} + \mu [x - x_{pre}] \). Suppose the state predicted is \( v_0 \), if \( x - x_{pre} < 0 \), we corrected \( x \) to \( x = x_{pre} - \mu [x - x_{pre}] \). Otherwise, if \( x - x_{pre} > 0 \), we corrected \( x \) to \( x = x_{pre} + \mu [x - x_{pre}] \). Suppose the state predicted is \( v_1 \), if \( x - x_{pre} < 0 \), we corrected \( x \) to \( x = x_{pre} - \mu [x - x_{pre}] \). The case of y-axis is similarly to this one. And the state predicted can be used as the input of \( Q(H_i) \) for the next time if \( Q(H_i) \) was considered. Direction-correcting can be done according to the result of the predicted state.

IV. RESULTS AND ANALYSIS

In this part, we made a comparison of raw data, data filtered by Kalman filter and the improved Kalman filter using Matlab toolbox EKF/UKF which was introduced in reference [9]. We also made a comparison between quantitative analysis and qualitative analysis. The experimental test bed is located in the 4th floor of Beijing University of Posts and Telecommunications. Fig. 1 shows the top view of it.

The raw data was obtained by the method of KNN and SVN. And the sampling interval of the test is 0.5s. Initial parameters of the filter were got on experience and the initial state values and initial covariance matrix were obtained by two-point method. They are:

\[
\begin{align*}
  v_{-1x} & : z \leq -V_x / 2 \\
  v_{0x} & : -V_x / 2 \leq z \leq V_x / 2 \\
  v_{1x} & : z \geq V_x / 2
\end{align*}
\]

The raw data was obtained by the method of KNN and SVN. And the sampling interval of the test is 0.5s. Initial parameters of the filter were got on experience and the initial state values and initial covariance matrix were obtained by two-point method. They are:
\[
A = [1, 0, 0.5, 0; 0, 1, 0.5; 0, 0, 0.5; 0, 0, 1; 0, 0, 0, 0.1] \\
Q = [0.2, 0, 0, 0; 0, 0, 0, 1; 0, 0, 0, 0.00005; 0, 0, 0, 0.00005, 1] \\
H = [1, 0, 0; 0, 1, 0; 0, 0, 1], R = [2, 0; 0, 2] \\
x[0] = [58.2700, -2.5125, 0.5043, -0.1671]^T \\
P[0] = [2.0000, 2.0000, 0; 2.0000, 8.2500, 0; 0.20000, 0.20000, 0; 0.20000, 8.25000] \\
\]

The factors of \( b, \lambda_1, \lambda_2 \) and \( \lambda_3 \) are 0.99, 1.02, 3 and 3 in the testing.

From the left part of Fig. 2, we can find that data filtered by improved quantitative Kalman filter is closer to the true value than the raw data and data filtered by classic Kalman filter. And its result is smooth. We can also make a conclusion that the error of the data filtered by improved Kalman filter is least compared to the true value from Fig. 2’s right part. But the improvement is not obvious compared to qualitative case.

According to Table 1, we know that the average error, variance, 3 meters accuracy and 5 meters accuracy of the data filtered by improved quantitative Kalman filter are 1.49m, 3.72 m^2*m, 86% and 98%. From the comparisons we know that the improved Kalman filter we proposed has some improvement on improving positioning accuracy and enhancing positioning stability but not good enough compared to qualitative case.

Qualitative analysis had a better result through our experiment. From the left part of Fig. 3, we can find that data filtered by improved Kalman filter is obviously closer to the true value than the raw data and data filtered by classic Kalman filter. And its result is smoother. We can also got that the error of the data filtered by improved Kalman filter is least compared to the true value from Fig. 3’s right part.

According to Table 2, we know that the average error, variance, 3 meters accuracy and 5 meters accuracy of the raw data are 1.87m, 5.45 m^2*m, 79% and 96%; The average error, variance, 3 meters accuracy and 5 meters accuracy of the data filtered by classic Kalman filter are 1.66m, 3.84 m^2*m, 83% and 98%; And the average error, variance, 3 meters accuracy and 5 meters accuracy of the data filtered by improved Kalman filter are 1.33m, 2.86 m^2*m, 91% and 100%. From the comparisons we know that the improved Kalman filter we proposed has great performance on improving positioning accuracy and enhancing positioning stability. Compared the quantitative case and qualitative case of the improved

\[
\begin{align*}
\text{TABLE I. COMPARISONS OF DATA NON-FILTERED, FILTERED BY KF AND FILTERED BY IKF OF QUANTITATIVE CASE} \\
\hline
\text{class} & \text{mean[m]} & \text{Variance [m^2*m]} & \text{3 meters accuracy} & \text{5 meters accuracy} \\
\hline
\text{Non-filtered} & 1.87 & 5.45 & 0.79 & 0.96 \\
\text{Filtered by KF} & 1.66 & 3.84 & 0.83 & 0.98 \\
\text{Filtered by IKF} & 1.49 & 2.81 & 0.86 & 1 \\
\hline
\end{align*}
\]

\[
\begin{align*}
\text{TABLE II. COMPARISONS OF DATA NON-FILTERED, FILTERED BY KF AND FILTERED BY IKF OF QUANTITATIVE CASE} \\
\hline
\text{class} & \text{mean[m]} & \text{Variance [m^2*m]} & \text{3 meters accuracy} & \text{5 meters accuracy} \\
\hline
\text{Non-filtered} & 1.87 & 5.45 & 0.79 & 0.96 \\
\text{Filtered by KF} & 1.66 & 3.84 & 0.83 & 0.98 \\
\text{Filtered by IKF} & 1.49 & 2.81 & 0.86 & 1 \\
\hline
\end{align*}
\]
kalman filter we can conclude that the qualitative case had better result in indoor positioning filtering.

V. SUMMARY AND FUTURE WORK

Owing to the complexity of the indoor environment, it’s hard to get good results by positioning algorithm based on ranging. The non-ranging algorithm based on fingerprint matching was a good choice. However, it brings out new problems, such as the result is unstable and not continuous. For these shortcomings, we did the following research: 1) proposed an improved Kalman filter algorithm by two kinds of anti-perturbation method which were derived according to the perturbation theorem of inverse matrix; 2) did the direction-correcting on the result of the improved algorithm by using multiple hypothesis testing theory which can detect the current direction of a target. The improved kalman filter had been proved to be efficient. The proposed kalman filter had limiting conditions that it would be best to use in a steady-state, for example, it’s not suitable for the convergence procedure.

Future directions for research on filtering techniques are as follows: To do an extended discussion on the noise perturbation method, for example, apply the algorithm to a suitable case to verify the effectiveness; considering $\lambda_0$ as a matrix not a constant; to extend the direction-correcting method to an accurate way; aiming at the application limits, looking for some new methods to improve the new algorithm.

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