Multi-direction Fuzzy Morphology Algorithm for Image Edge Detection

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Abstract—A multi-direction fuzzy morphology algorithm, for image edge detection is proposed to deal with edge blur and inaccuracy of boundary localization. In the algorithm, two thresholds are selected to conduct image segmentation and image obtaining respectively, fuzzy enhancement for the image is adopted to resolve the loss of edge information and multi-directional structural elements are used to detect image edge. Based on it, experiments are carried out, the results show the proposed algorithm has strong anti-noise ability and superior to traditional edge detection algorithms.

Index Terms—multi-direction, fuzzy morphology, structural elements, image edge detection

I. INTRODUCTION

Edge is the basic characteristics of image, it depicts the outline of area shape and expresses most of the image information. Therefore, edge detection is an important preprocessing technology in image analysis, and also is the key to solve many complex problems.

Mathematical morphology is a new mathematic theory which is used in image processing and analysis. In mathematical morphology, image is regarded as a set, which coming of Minkowski’s morphological transform of addition and subtraction. Jin Zou [1] proposed color edge detection algorithm based on Morphology, Won Yeol Lee [2] completes edge detection by using morphological amoebasin noisy Images, In 1990’s, Sinha and Dougherty [3] introduce fuzzy mathematic to mathematical morphology, De Baets [4-5] applies fuzzy mathematical morphology in image analysis and achieves better effect than hard calculation method on certain occasion. However, image information is often complicated, and the processing precision may not be good.

This paper proposes a multi-directional algorithm for edge detection based on fuzzy mathematical morphology. Multi-directional structural elements and fuzzy features of image are integrated into mathematical morphology to detect image edge. Simulation experiments under various conditions are carried out, the results show that the proposed algorithm has better effect in edge detection, and superior to those traditional algorithms for edge detection.

II. MATHEMATICAL MORPHOLOGY

A. Basic Operations of Mathematical Morphology

Mathematical morphology is a powerful tool in image processing. It bases on set theory of mathematics and develops with image processing. Its basic operations are dilation and erosion, each elementary operation can be defined on the dimensions of Euclidean space [6]. In this paper, the morphological operations are confined to discrete two dimensions of Euclidean space.

Let A and B be two subsets of Z^2, A_a = {a + b | a ∈ A} , dilation operator is A ⊕ B , A ⊕ B = ∪_{b ∈ B} A_b ; erosion operator is A ⊙ B , A ⊙ B = ∩_{b ∈ B} A_{-b} .

Let f be the intensity function of the original image, g be structural elements, f : D ⊂ R^2 → R , g : E ⊂ R^2 → R , g is set to zero and to be symmetrical. Dilation and erosion are defined as:

\[ (f ⊕ B)(x) = \sup \{ f(x-t) + g(t) \} \] (1)

\[ (f ⊙ B)(x) = \inf \{ f(x+t) - g(t) \} \] (2)

Morphology opening and closing operators are compound operation of dilating and eroding, opening and closing operators are defined as:

\[ f ◦ B = (f ⊕ B) ⊙ B \] (3)

\[ f • B = (f ⊙ B) ⊕ B \] (4)

B. Morphological Edge Detection Operators

Morphological gradient operator is a nonlinear differential operator, and is from compound operation of dilation and erosion [6-7], the types are described as follows.

Dilating type: \[ g_1 = f ⊕ B - f \] (5)

Eroding type: \[ g_2 = f - f ⊙ B \] (6)

Dilating-eroding type: \[ g_3 = f ⊕ B - f ⊙ B \] (7)

White top-hat transformation: \[ g_4 = f - f ◦ B \] (8)

Black top-hat transformation: \[ g_5 = f • B - f \] (9)

Opening-closing type: \[ g_6 = f • B - f ◦ B \] (10)

Morphological opening operator can eliminate the burr incompatible with structural element and increase the brightness of image. Closing operator can break empty
shapes and smooth image. In order to eliminate noise and smooth image more efficient, the improved edge detectors are defined respectively as:

\[ g_7 = f \circ B \]

\[ G(g_7) = (g_7 \circ B) \oplus B - g_7 \circ B \]

III. MULTI-DIRECTION FUZZY MORPHOLOGY ALGORITHM

A. Definition of Fuzzy Morphological Operator

The definitions of fuzzy morphological operators are described as follows:

**Definition1:** The fuzzy dilation \( D_c(A,B) \) and fuzzy erosion \( E_r(A,B) \) of \( A \) by \( B \) are the gray-scale images, defined by:

\[ D_c(A,B)(y) = \sup_x C(B(x-y), A(x)) \]

\[ E_r(A,B)(y) = \inf_x I(B(x-y), A(x)) \]

Note that the reflection of a gray-scale image \( B \) denoted by \(-B\), and defined by \(-B(y) = B(-y)\), \( y \in \mathbb{R}^n \). Given two images \( B_1, B_2 \), we will say that \( B_1 \sqsubseteq B_2 \), when \( B_1(y) \leq B_2(y) \) for all \( y \in \mathbb{R}^n \).

**Definition2:** The fuzzy closing \( C_c(A,B) \) and fuzzy opening \( O_c(A,B) \) of \( A \) by \( B \) are the gray-scale images, defined by:

\[ C_c(A,B)(y) = E_r(D_c(A,B), B(y)) \]

\[ O_c(A,B)(y) = D_c(E_r(A,B), -B(y)) \]

In particular, it can be shown that fuzzy morphology is compatible with binary morphology and gray-scale morphology based threshold approach [8].

B. Selection of Membership Function

A \( M \times N \) gray scale image of \( L \) gray level can be represented in a fuzzy matrix via the concept of fuzzy sets, \( x = \bigcup_{m=1}^{M} \bigcup_{n=1}^{N} \mu_{mn}(1) \). Here, \( \mu_{mn} \) is the membership for image pixel \((m,n)\) which defines the membership as the ratio of the pixel gray scale \( I_{mn} \) relative to the largest gray level (L-1).

For every pixel, we obtain suitable membership function of fuzzy enhancement by an area around \( w \times w \) window:

\[ \mu_m = 1 - \min_{i,j\in[w]} x(i,j) / \max_{i,j\in[w]} x(i,j) \]

\[ \mu_m = G(l_{mn}) = abs(1- \min_{i,j\in[w]} x(i,j) / MeanMedW(i,j)) \]

Then the enhanced membership function is given as:

\[ \mu_m = \min(\mu_m, \mu_m) \]

Where, \( \min_{i,j\in[w]} x(i,j) \) and \( \max_{i,j\in[w]} x(i,j) \) are Minimum membership degree, maximum degree and windows gray scale average or median of center pixel \((i,j)\) in \( w \times w \) areas of window, respectively. It makes use of general characteristics of image edge and area pixel correlation of image and selects the algorithm parameters adaptively via different gray areas. However, the selection to the fuzzy characteristics of membership function can make the target edge not be enhanced which should be enhanced, it affects the edge detection results [9].

To cope with the deficiency of membership algorithm, two thresholds are selected to carry on the image segmentation and image obtaining separately, then, fuzzy enhancement for the image is completed and the maximum MAX and the minimum MIN are calculated. Let threshold \( TK = (Max + Min) / 2 \), the original image is segmented into target and background according to the threshold TK. Calculate the average grey scale of target and background MO and MB respectively and get a new threshold \( TK = (MO + MB) / 2 \).

Suppose \( TK = TK_1 \), then regard \( TK \) as the segmented threshold and perform the iterative operation.

In order to reduce information loss of gray area and improve the effect of fuzzy enhancement, a new membership function is defined in this paper:

\[ \mu_m = G(X_{mn}) = \begin{cases} \frac{x_{mn}}{x_T} & x_{mn} \leq x_T \\ \frac{x_{mn}}{x_{max}} & x_{mn} \geq x_T \end{cases} \]

Where \( X_{mn} \) is original data matrix, \( x_T \) is threshold.

The membership algorithm is used as the fuzzy enhancement operator via (17)-(19), and inverse transformed. Hence, we obtain following membership function:

\[ \mu_m = G(\mu_m) = \begin{cases} 0 & x_{mn} \leq x_T \\ \mu_m \cdot \mu_m \cdot x_{mn} & x_{mn} \geq x_T \end{cases} \]

C. Choice of Structural Elements

In the application of morphology, traditional structural elements are fixed-size and symmetric, a single structural element fails to detect image edge completely. Therefore, the structural elements of four different directions are built and structural element is designed into four templates of \( 3 \times 3 \):

\[
\begin{bmatrix}
0 & 0 & 0 \\
1 & 1 & 1 \\
0 & 0 & 0
\end{bmatrix}
\]

(a) \(0^\circ\)  
(b) \(45^\circ\)  
(c) \(90^\circ\)  
(d) \(135^\circ\)

IV. THE STEPS OF MULTI-DIRECTIONAL FUZZY MORPHOLOGY ALGORITHM

The steps of this algorithm for image edge detection are as follows:
(1) Let the values of image be in \([0, 1]\), transform it as a fuzzy set via (17)-(21) and get image A;
(2) Using structural element of one direction \(B_i\) (\(i = 1,2,..,n\)), perform fuzzy closing and get image C, then perform fuzzy opening and get image D via (11)-(12);
(3) Perform fuzzy eroding on image D and get image E, then perform fuzzy opening on E and get image F;
(4) Calculate \(G_i = F - D\) , get edges \(G_2, G_4, G_4\) by using other three directions structure elements;
(5) Finally, all results are weighted to obtain edge image \(g = \sum_{i=1}^{n} \omega_i G_i\), here \(g\) is output edge image, \(\omega_i\) is weight value and \(\omega_i = 1/n\).

V. EXPERIMENTAL RESULTS AND ANALYSIS

The original image has been experimented by using the proposed multi-direction fuzzy morphology (MDFM) algorithm and traditional algorithm such as Canny, Sobel, Prewitt operators and classical mathematical morphology, the results are shown in figure 1, figure 2 and figure 3. Based on the figures, analysis and comparison are made, the conclusion is drawn as follows:

(1) In the presence of no noise, traditional algorithms fail to separate the boundaries of the image. The proposed MDFM algorithm removes the noise of the image efficiently, without altering the outer edge of the regions that compose the image and maintains the edge features of the image.

(2) In the presence of Gaussian noise and salt & pepper noise, traditional algorithms fail to remove noise and fail to detect the outer edge of body. The proposed MDFM algorithm extracts adequately edge features and demonstrates strong anti-noise ability.
VI. CONCLUSION

In this paper, a multi-direction fuzzy morphology algorithm for image edge detection is proposed. The experimental results show that the proposed algorithm has strong anti-noise ability and better performances than corresponding traditional edge detection algorithms.

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