Blindman-Walking Optimization Method

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Abstract—Optimization methods are all implemented with the hypothesis of unknowing the mathematic express of objective function. Using the human analogy innovative method, the one-dimension blind-walking optimal method is proposed in this paper. The theory and the algorithm of this method includes halving, doubling, reversing probing step and verifying the applicability condition. Double-step is available to make current point moving to the extremum point. Half-step is available to accelerate convergence. The latter is deciding the following inequality whether right or not. The operation process, algorithmic flow chart and characteristic analysis of the method were given. Two optimization problems with unimodal or multimodal objective function were solved by the proposed method respectively. The simulation result shows that the proposed method is better than the ordinary method. The proposed method has the merit of rapid convergence, little calculation capacity, wide applicable range, etc. Taking the method as innovative kernel, the random research method, feasible direction method and complex shape method were improved. Taking the innovative content of this paper as innovative kernel, a monograph was published. The other innovations of the monograph are listed, such as applied algorithm of Karush-Kuhn-Tucker (KKT) qualifications on judging the restriction extremum point, the design step of computing software, the complementarity and derivation of Powell criterion, the method of keeping the complex shape not to deduce dimension and the analysis of gradual optimization characteristic, the reinforced wall of inner point punish function method, the analysis of problem with constrained monstrousity extremum point, the improvement of Newton method and the validation of optimization idea of blind walking repeatedly, the explanation of later-day optimization method, the conformity of seeking algorithm needing the objective function derivation, etc.

Index Terms—optimization method; blindman-walking optimization method; computing validation; teaching material; improved algorithm

I. INTRODUCTION

As a subject with more than 100 years history, Machine Optimization Design plays an important role in the college courses[1]. This subject can solve the optimization problem in the fields of mechanics, dynamics, complex engineering system, automobile etc. A numerous mathematic research on the optimization method has been done[2-3]. However, some complex optimization problem sometimes cannot be solved commendably because of the localization of algorithm at present.

As the kernel of multi-dimension optimization method, one-dimension method includes two kinds: (1) region elimination method, such as golden-section search method, Fibonacci sequence method etc. (2) interpolation or simulating function method, such as quadratic interpolation method (parabola method), Newton’s method, high-order interpolation method, etc. Under the hypothesis resupposition of unimodal objective function, the advance and retreat method decide the region bracketing the extremum point by doubling step search rule. After finding the extremum point existing in the scope of one step, the extremum point should be approached by the probing point if using halving step searching rule. This optimization method could be called one-dimension blindman-walking optimization method, because it likes a blindman walking. This proposed method provides the new material to the development of optimization subject. Taking it as innovative kernel, many methods can be improved to adapt more wide fields and get better optimization result.

II. ONE-DIMENTSION BLINDMAN-WALKING OPTIMIZATION METHOD

A. Basic idea

When blindman walking, he uses a pole to probe the road sometimes. Assuring the safety, he strides a long or short step. The idea of striding after probing gives birth to the mountain climbing method for multi-dimension problem and the advance and retreat method for deciding the region bracketing extremum point for one-dimension problem. The idea of adjusting the step gives birth to the one-dimension blindman-walking optimization method. Similar to the ordinary one-dimension method, it supposes that the objective function is unimodal. Its mathematical realization is the halving or doubling of step and the applicability condition checking of probed point. The latter is deciding the following inequality whether right or not.

\[ f\left(x^{(i+1)}\right) < f\left(x^{(i)}\right) \]  (1)

The probing point can be defined as the current optimal point (let \(k=k+1\)) no other than satisfying the applicability condition. The convergence ability is ensured by only halving step after assuring that the
extremum point is bracketed in the scope of one step. Its convergence domain is the scope of one step before halving step. The convergence condition can be expressed as the following inequality.

\[ |h| < \varepsilon \]  

The algorithm is composed of two parts. The first part is same as the advance and retreat method. The second part is double direction researching algorithm. During the first part, the step can be doubled to reduce the number of calculating objective function. During the second part, the halving step and change direction are done alternately.

B. Algorithm approach

According to the idea and mathematic realization of algorithm, the approach of this algorithm should be expressed as follows.

1) Taking the starting point \( x_1 \) as the current point, the initial step as \( h_0 = h \), the convergence precision value as \( \varepsilon < \|h/10\|\), the probing point as \( x_2 = x_1 + h \), \( y_1 = F(x_1) \) and \( y_2 = F(x_2) \) are calculated.

2) Comparing \( y_1 \) and \( y_2 \).
   (a) If \( y_1 \geq y_2 \), then let \( h = h/2 \).
   (b) If \( y_1 < y_2 \), then exchange \( x_1 \) and \( x_2 \), let \( h = -h \).

3) Taking the probing point as \( x_2 = x_1 + h \), calculating \( y_2 = F(x_2) \).

4) Comparing \( y_1 \) and \( y_2 \).
   (a) If \( y_1 \geq y_2 \), then let \( y_1 = y_2, x_1 = x_2, h = h/2 \).
   (b) If \( y_1 < y_2 \), then let \( h = -h \), turning to (6).

5) Taking the probing point as \( x_2 = x_1 + h \), calculating \( y_2 = F(x_2) \), turning to (4).

6) Taking the probing point as \( x_2 = x_1 + h \), calculating \( y_2 = F(x_2) \).

7) Comparing \( y_1 \) and \( y_2 \).
   (a) If \( y_1 \geq y_2 \), then let \( y_1 = y_2, x_1 = x_2, h = h/2 \).
   (b) If \( y_1 < y_2 \), then the next estimation. If the previous probing is done after the change of searching direction, then let \( h = h/2 \), otherwise let \( h = -h \).

8) Taking the probing point as \( x_2 = x_1 + h \), calculating \( y_2 = F(x_2) \). If \( h > \varepsilon \), then turning to (7).

9) Taking the optimum as the better of current point \( x_1 \) and probing point \( x_2 \).

In the above, the detail of doubling step, changing direction, etc. could be changed.

C. Program flow chart

It is shown in fig.1, where Flag is the sign of whether doubling step or not, FFlag is the sign of whether changing searching direction or not, \( k \) is the sequence number of current point, \( x_i \) is the current point, \( x_{k+1} \) is the probing point, \( h \) is the probing step, \( \varepsilon \) is the convergence precision value of step, \( F(x_i) \) is the objective function value of current point, \( F(x_{k+1}) \) is the objective function value of probing point.

III. VALIDATION EXAMPLE

A. Unimodal function

\[ F(x) = x^2 \]  

If taking the starting point \( x_0 = -10.0 \), the initial probing step \( h_0 \) as 1.0, then the extremum point is founded after 8 times of probing. The series of points are -10.9, -7.3, 5.1, 3.3, -1.0. Considering the general instance, if taking the starting point \( x_0 \) as -10.1, the convergence precision value \( \varepsilon \) as 0.02, then the searching process is tabulated below.

<table>
<thead>
<tr>
<th>Serial number of current point ( k )</th>
<th>Probing step ( h )</th>
<th>Probing point ( x_k )</th>
<th>Probing point ( x_{k+1} )</th>
<th>Function value ( F(x_k) )</th>
<th>Function value ( F(x_{k+1}) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1.0</td>
<td>-10.1</td>
<td>-9.1</td>
<td>102.01</td>
<td>82.81</td>
</tr>
<tr>
<td>1</td>
<td>2.0</td>
<td>-9.1</td>
<td>-7.1</td>
<td>82.81</td>
<td>50.41</td>
</tr>
<tr>
<td>2</td>
<td>4.0</td>
<td>-7.1</td>
<td>-3.1</td>
<td>50.41</td>
<td>9.61</td>
</tr>
<tr>
<td>3</td>
<td>8.0</td>
<td>-3.1</td>
<td>4.9</td>
<td>9.61</td>
<td>24.01</td>
</tr>
<tr>
<td>4</td>
<td>16.0</td>
<td>-3.1</td>
<td>0.9</td>
<td>9.61</td>
<td>0.81</td>
</tr>
<tr>
<td>4</td>
<td>2.0</td>
<td>0.9</td>
<td>2.9</td>
<td>0.81</td>
<td>8.41</td>
</tr>
<tr>
<td>5</td>
<td>-1.0</td>
<td>0.9</td>
<td>-0.1</td>
<td>0.81</td>
<td>0.01</td>
</tr>
<tr>
<td>5</td>
<td>-0.5</td>
<td>-0.1</td>
<td>0.6</td>
<td>0.01</td>
<td>0.36</td>
</tr>
<tr>
<td>5</td>
<td>-0.25</td>
<td>-0.1</td>
<td>0.35</td>
<td>0.01</td>
<td>0.1225</td>
</tr>
<tr>
<td>5</td>
<td>0.25</td>
<td>-0.1</td>
<td>0.15</td>
<td>0.01</td>
<td>0.0225</td>
</tr>
<tr>
<td>5</td>
<td>0.125</td>
<td>-0.1</td>
<td>0.025</td>
<td>0.01</td>
<td>6.25e-4</td>
</tr>
<tr>
<td>6</td>
<td>0.0625</td>
<td>0.025</td>
<td>0.0875</td>
<td>6.25e-4</td>
<td>7.6563e-3</td>
</tr>
<tr>
<td>6</td>
<td>0.03125</td>
<td>0.025</td>
<td>0.0562</td>
<td>6.25e-4</td>
<td>3.1641e-5</td>
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<tr>
<td>6</td>
<td>-0.03125</td>
<td>0.025</td>
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<td>3.9062e-5</td>
<td></td>
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<td>-</td>
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<td></td>
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<tr>
<td>6</td>
<td>0.015625</td>
<td>0.00625</td>
<td>-</td>
<td>4</td>
<td></td>
</tr>
</tbody>
</table>

During the second phase, the convergence ability is validated by halving step per one or two times of probe. Using the golden-section search method, the function value of midpoint of searching region is 0.062 after 15 times of calculating objective function. It is obvious that the one-dimension blindman-walking method has the searching effect close to that of golden-section search method.
**B. Multimodal function**

\[ F(x) = \sin(x) \]  

(4)

There are several local extremum points for multimodal function. In order to find the macrocosm extremum point, all local extremum point should be found out. It mains that different starting point and probing step should be chosen time after time. Taking the step convergence precision value \( \varepsilon \) as \( 1.0 \times 10^{-6} \), the searching process is tabulated below.

<table>
<thead>
<tr>
<th>( h_0 )</th>
<th>( x_0=-1.5708 )</th>
<th>( x_0=1.0 )</th>
<th>( x_0=0 )</th>
<th>( x_0=0.5 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.00</td>
<td>0/12/-1.57</td>
<td>14/34/-1.57</td>
<td>16/34/-1.57</td>
<td>16/38/-1.57</td>
</tr>
<tr>
<td>0.1</td>
<td>1/16/-1.57</td>
<td>12/31/-1.57</td>
<td>12/37/-1.57</td>
<td>15/33/-1.57</td>
</tr>
<tr>
<td>0.5</td>
<td>0/18/-1.57</td>
<td>10/29/-1.57</td>
<td>11/33/-1.57</td>
<td>12/33/-1.57</td>
</tr>
<tr>
<td>1</td>
<td>0/19/-1.57</td>
<td>6/31/-1.57</td>
<td>7/35/-1.57</td>
<td>12/30/-1.57</td>
</tr>
<tr>
<td>3</td>
<td>0/21/-1.57</td>
<td>13/27/-1.57</td>
<td>8/33/-1.57</td>
<td>11/31/4.7124</td>
</tr>
<tr>
<td>10</td>
<td>0/22/-1.57</td>
<td>8/29/-1.57</td>
<td>7/41/29.85</td>
<td>13/32/10.996</td>
</tr>
</tbody>
</table>

Where, the column 1 represents the initial step \( h_0 \), the row 1 represents the starting point \( x_0 \), the tabulated data represents the updating number of current point, the calculating number of objective function and the final optimum.

With different starting point and different initial step, the optimum with objective function value -1.0 is gotten without exception. Taking the initial step(6.2832) little less than \( \pi \), the starting point (1.571) little great than 0.5\( \pi \), the optimum with objective function value -1.0 is gotten after 27 times of updating current point and 84 times of calculating objective function. The above computing result validates the adaptability of algorithm. However, it needs 20 times of calculating objective function during deciding the region including optimum, 56 times of calculating objective function during optimizing and 55 times of region elimination with golden-section search method. The optimum with objective function value 0.78 is gotten in the end. The found optimum is different largely from the extremum point. It is obvious that the new optimization method is more adaptable than the ordinary method.

**IV. ALGORITHM CHARACTERISTIC**

According to the computing results and optimization idea, the algorithm has the followed characteristics.

(1) Wider applicable fields. The proposed method can be applicable to the consecutive design variable optimization problem, so much as hidden expression objective function optimization problem. The applicability condition is the foundation of updating the current point. For the multimodal objective function, the local extremum points can be found out by choosing different starting point and probing step. It can be used as the subterminum of any one-dimension method not needing objective function derivative. It can also be used as the kernel algorithm of multi-dimension optimization methods(shown in part 5).

(2) Faster convergence speed. At the beginning of halving step, the probing step is two-part of the zone decided by advance and retreat method. One halving step needs one or two times of calculating objective function. For the golden-section search method, the zone shor ted by 38.2% per time of calculating objective function. Therefore, the convergence speed of the proposed method is faster than that of golden-section search method. The convergence domain is bracketed by one step before halving step.

(3) Less computing quantity. The step calculation only includes halving and doubling. The main computing quantity is calculating and comparing objective function value between the current point and probing point.

(4) Simplified and scientific optimization idea. The optimization idea is directed by the common sense of blindman walking. So the idea is in focus and prone to be understood.

(5) Higher extending worthiness. The proposed method could be inserted into other optimization method in order to improve optimization effect. For example, if the idea is inserted into the random search method, the searching efficiency can be improved, the searching direction can be utilized sufficiently and the false optimum can not be found. Another example, if the idea is inserted into the complex shape method. Taking the reflection point as the optimum point along the direction from the rejected point to centroid, the optimal efficiency should be improved greatly. Another example, if the idea is inserted into the feasible direction method, the point of intersection between searching direction and constraint border can be found out efficiently.

(6) Lower requirement for objective function. Only the objective function value is needed, so the continuity, differentiability and expressibility of function are not needed.

**V. IMPROVEMENT OF OTHER OPTIMIZATION METHOD BASED ON THE IDEA**

One-dimension optimization method is the kernel of multi-dimension method. So introducing the blindman-walking idea into multi-dimension method, the algorithm can be ameliorated and the optimization effect can be improved. For the random search method, complex shape method and feasible direction method, the melioration is obvious[4].

For the random direction method, blindman-walking idea can realize double direction optimization. The optimum point can be found out in the given directions efficiently[3].

Feasible direction method is a kind of optimization method for solving large optimization problem. If the probing point is out of feasible domain, the intersection of searching direction and constraint border should be selected. In general, theoretic method based on Taylor series or numerical method setting constraint tolerance is used. Those methods all have bigger error and lower efficiency. The blindman-walking idea may get the nearest point from the constraint border efficiently.

The reference frame is founded with taking \( x_0 \) as origin and taking the probing direction as positive direction. Near the constraint border, the constraint function is degression. So introducing the blindman-
walking idea can get the point on constraint border according to the constraint function value.

Complex shape method is an important direct solving method for constraint optimization problem. Firstly, an initial complex shape with k feasible points is formed. Secondly, the false point \((x_F)\) with the worst objective function value is found. Thirdly, the reflection point \((x_R)\) on the direction from the false point to the centroid \((x_c)\) of remained points is found. Lastly, \((x_R)\) is substituted with \((x_F)\). As a result, the complex shape is updated. Continue the complex shape changed time after time untill it approaches to the extremum point. The kernel of this method is searching the reflection point. Introducing the blindman-walking idea, the optimum point along the direction from \((x_F)\) to \((x_c)\) can be found easily. Taking this point as reflection point, the optimization process can be realized efficiently. The reference frame is founded with taking \(x_0\) as origin and taking this point as reflection point, the time untill it approaches to the extremum point. The improved algorithm is shown in fig.2. Though the improved algorithm loses the gradual optimization characteristic of ordinary complex shape method, it can make complete this defect by other algorithm improvement[6].

![Figure 2. Flow sketch of improvement on searching reflect point](image)

VI. THE CORRESPONDING MONOGRAPH INNOVATION

Taking the innovative content of this paper as innovative kernel, a monograph is published[6]. The monograph improves each optimization method nearly and provides many flow sketches with readability. The other innovations of the monograph are followed mostly.

A. Applied algorithm of Karush-Kuhn-Tucker (KKT) qualifications on judging the restriction extremum point

On judging the constrained point is whether the extremum point or not, Karush-Kuhn-Tucker qualification plays an important place in optimal algorithm. The algorithm and program flow chart of three situations are given. The redundancy of working restrictions should be eliminated before computing the KKT qualifications. For the number of working restriction greater than the number of dimension, all basic grads groups should be checked. If one group Lagrangian multipliers are all un-negative, the studied point meets the KKT conditions. For the number of working restriction less than the number of dimension, the Lagrangian multiplier get from some equations and then use in testing other equations. The computing steps of algorithm are shown by three examples.

For the followed multi-dimension optimization problem,
\[
\min f(x), \quad x \in D \subset \mathbb{R}^N
\]
\[s.t. \quad g_u(x) \geq 0 \quad (u = 1, 2, \ldots, m)\]
\[h_v(x) = 0 \quad (v = 1, 2, \ldots, l \leq N)\]

The Lagrange function concomitantly with objective function is following.
\[
L(x, \lambda) = f(x) - \sum_{u=1}^{m} \lambda_u g_u(x) - \sum_{v=1}^{l} \mu_v h_v(x) \quad (5)
\]

If \(x^*\) is a constraint extremum point, it satisfies the KKT qualification as following.
\[
\nabla f(x^*) = \sum_{u=1}^{m} \lambda_u^* \nabla g_u(x^*) - \sum_{v=1}^{l} \mu_v^* \nabla h_v(x^*) = 0 \quad (6)
\]
\[
\lambda_u^* \geq 0, \quad h_v(x^*) = 0, \quad \mu_v^* \neq 0 \quad (7)
\]

There into, \(\lambda_u^*\) and \(\mu_v^*\) are the Lagrange multipliers. The \(\lambda_u^*\) matches along with the inequality constraints. The \(\mu_v^*\) matches along with the equality constraints. Due to the equality constraint has no direction, \(\mu_v^*\) has no sign limit. Formula (7) means the complimentary slackness condition. The KKT qualification indicates that the grad of objective function is the summation of the acting inequality constraint function grads non-negative linear combination and the equality constraint function grads linear combination.

In fact, the number of acting constraint function is indeterminate at the studied constraint point. In general, there are some superfluous constraints. If an inequality constraint does not influence the size of feasible domain, it is a superfluous constraint. At a constraint point, the superfluous constraint has another definition. Its grads direction locates in the angle-off cone of other constraints grads. The superfluous constraint does not influence the size of feasible domain, its grads direction has another definition. Its grads direction locates in the angle-off cone of other constraints grads. The superfluous constraint does not influence the size of feasible domain and the size of superfluous constraint does not influence the calculation result and adds calculation loads in vain. So it should be eliminated. For the KKT qualifications, if the grads direction terms corresponding to the superfluous constraints were eliminated, the judgment result has no changes. For the equality constraint, it the following inequality is satisfied,
\[
[\nabla h_v(x)][-\nabla f(x)] > 0 \quad (8)
\]

The positive direction of its grads should be examined. Otherwise, its grads negative direction should be examined. If the direction of \(\nabla h(x)\) coincides with the direction of \(\nabla f(x)\), that is to say their quantities are in proportion to each other, it should be treated according to the position of curvature center and the size of curvature radius.
The process of eliminating the superfluous constraints is shown in figure 3.

![Figure 3. Flow sketch of eliminating the superfluous constraints](image)

After eliminating the superfluous constraints, there is positive or negative direction of equality constraints in the grads group. If the equality constraint grad is negative, it should be treated as inequality constraint after adding a opposite sign. If there are J numbers of acting constraint function after eliminating, the followed three instances will be met when judging the constraint point whether extremum point or not using KKT qualifications.

1) \( J = N \). A system of linear equation with \( N \) unknowns should be solved. In general, the exclusive Lagrange multipliers can be gotten. Due to the direction of equality constraints grads has been treated, if all the multipliers are non-negative, the KKT qualification was satisfied, or else not satisfied.

2) \( J > N \). If only the direction of \( \nabla f \) locates in the cone of random \( N \) grads, which means the basic grads group, the KKT qualification is satisfied. If arranging the edges of cone in turn, only the combination sequence including a given edge should be checked, where have \( N - 2 \) groups. However, this combination sequence is difficult to be gotten in general. So all basic grads groups should be checked, where has

\[
C_J^N = \frac{J \times (J - 1) \times \cdots \times (J - N + 1)}{N \times (N - 1) \times \cdots \times 1}
\]

3) \( J < N \). The number of equation is bigger than the number of undecided coefficient. Selecting \( J \) equations from the equation group, if its corresponding multiplier are not all non-positive, the studied constraint point is not extremum point. Then the multipliers should be validated by substituting into other equations. If the remanent \( N-J \) equations are all satisfied, the studied point satisfied the KKT qualification.

For the third instance, a definite error should be allowed when validating the equation whether satisfied or not because of the character of numerical calculation. For example, if the difference between two sides of equal sign less than the 1 percent of max term in equation, the equation should be considered being satisfied.

For the constraint point with only one constraint acting, satisfying the KKT qualification means the superposition between the grad directions of this constraint function and objective function.

For the constraint optimization problem with non-convex feasible domain or constrained monstrosity extremum points, the getting optimum point has a little difference with the extremum point generally because of the limitation of optimization method. Therefore, the selecting of acting constraint and the validating of equation need a definite error much more.

B. The design step of computing software

All optimization algorithms should be computed. The computing software is very important to the subject. The step of programming was proposed as following. Firstly, designing the programming structure correctly is important. Secondly, writing the program by hand carefully is the kernel of step. Thirdly, debugging the program by comparison of the calculating result is the practice step. Lastly, composing the corresponding document is important to user. The design step aims at avoiding the delay of accomplishment because of difficult to find the logic error. It is worth to emphasize that writing programming by hand is very important to the program jackaroo.

C. Complementarity and derivation of Powell criterion

Powell method is one of the most useful optimization methods. Based on the no linearity degradation principle of basic direction group, the deriving process of updating direction group criterion is proposed. The complementarity is given. The first original condition guarantees the forecast optimum located the area of current series optimum. The second original condition guarantees the forecast downtrend of objective function. The complementarity condition guarantees the linearity of basic direction group with a bigger value of matrix quantity. The new condition can avoid the linearity degradation of basic direction group. Based on the principle of visualizing teaching, the flow chart of improved Powell method is designed. It has a good readability.

The Powell criterion is deduced for avoiding the direction conjugacy and linearity degradation of seeking direction group according to the idea of quadratic interpolation. The \( x_0^{(k)}, x_N^{(k)} \) and \( x_r^{(k)} \) are defied as the starting, ending and mapping point of the current seeking direction group respectively. Their corresponding objective function value are \( f_0, f_2 \) and \( f_3 \) respectively.

The \( x_r^{(k)} \) and \( x_0^{(k)} \) are symmetrical on the both sides of \( x_N^{(k)} \).

\[
x_r^{(k)} = 2x_N^{(k)} - x_0^{(k)} \quad (9)
\]

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The drop of objective function value along the basic seeking direction \( s_{i}^{(k)} \) can be defined as follow.

\[
\Delta_{i} = f\left(x_{i}^{(k)}\right) - f\left(x_{i-1}^{(k)}\right)
\]

(10)

Where, \( x_{i-1}^{(k)} \) and \( x_{i}^{(k)} \) are the corresponding initial and optimum point respectively. The worst direction has the biggest drop of objective function value, \( \Delta_{m} = \max_{i \in \mathcal{N}} \Delta_{i} \). It is impossible to get a bigger objective function value drop along the worst direction again.

If only the \( x_{0}^{(k)} \), \( x_{N}^{(k)} \) and \( x_{r}^{(k)} \) on the seeking direction have the character of “high-low-high” or “high-low-low”, it is necessary to seek along the new direction.

\[
f_{3} < f_{0} \quad (11)
\]

The quadratic function is constructed with the three points and their objective function value. Due to the symmetry of mapping point, the position of the quadratic points and their objective function value. Due to the each seeking direction unitization, this condition can be expressed as follow (in figure 4).

1. If \( f_{2} < f_{0} < f_{1} \), it locates between \( x_{0}^{(k)} \) and \( x_{N}^{(k)} \).
2. If \( f_{2} < f_{3} < f_{0} \), it locates between \( x_{N}^{(k)} \) and \( x_{r}^{(k)} \).
3. If \( f_{3} < f_{2} < f_{0} \), it locates the right side of \( x_{r}^{(k)} \).

\[f(x) = a_{0} + a_{1}x + a_{2}x^{2} \quad (12)\]

According to the reference [6], its extremum value is following.

\[
f_{p} = f_{2} + \frac{\left[-f_{1} + f_{3}\right]^{2}}{2 \times 4 \left[-f_{1} + 2f_{2} - f_{3}\right]} \quad (13)
\]

The drop of objective function value from \( x_{N}^{(k)} \) to the above extremum point is following.

\[
\Delta_{N+1} = -\frac{\left[-f_{1} + f_{3}\right]^{2}}{2 \times 4 \left[-f_{1} + 2f_{2} - f_{3}\right]} \quad (14)
\]

The above objective function value drop is a determinant for judging the new direction whether useful or not.

If \( \Delta_{N+1} \) is too little, the new direction is not necessary to be used. Therefore, it is supposed that the new direction can be used if \( \Delta_{N+1} \) is bigger than a coefficient times of sum of other drop, \( f_{0} - f_{2} - \Delta_{m} \).

The coefficient can be defined as follow.

\[
f_{0} - f_{2} - \Delta_{m} \quad (15)
\]

Therefore,

\[
\Delta_{N+1} > \frac{f_{0} - f_{2} - \Delta_{m}}{4\Delta_{m}} \quad (16)
\]

The above expression can output the following expression.

\[(f_{0} - 2f_{2} + f_{3})(f_{0} - f_{2} - \Delta_{m})^{2} < 0.5\Delta_{m} (f_{0} - f_{3})^{2} \quad (17)\]

The linearity degradation of basic seeking direction group means that at least one direction can be expressed as the linear combination of other directions. At that time, the determinant value of basic seeking direction group equal to zero or nearly. So, the corresponding determinant condition of updating basic seeking direction group is the enough big value of the supposed new basic seeking direction group, \( S_{N+1}^{(k+1)} \). Due to the each seeking direction unitization, this condition can be expressed as follow.

\[
|S_{N+1}^{(k+1)}| \geq \varepsilon \quad (18)
\]

Above all, Powell criterion includes expression (11), (17) and (18).

D. The method of keeping the complex shape not to deduce dimension and the analysis of gradual optimization characteristic

According to the matrix theory, whether deducing dimension or not can be judged by the matrix rank of searching direction group.

The optimal point is not near to the extremum point before changing the research direction. The sequence optimum point approaches to the extremum. The above two cases can be called gradual optimization. The optimization method with characteristic of gradual optimization can get good result. The classical complex
shape method has the characteristic of gradual optimization. The other optimization methods with this character are successive linear programming, random search method, punish function method, etc. The complex shape method based on the blind-walking idea has no characteristic of gradual optimization. So its optimization result is not perfect for the problem with un-convex feasible domain and constrained monstrosity extremum points.

If the dimension deduction happens, the probing point should be adjusted. The improvements of complex shape method are following. The bad point should close to the good point in order to avoid the updating complex failure. Some points should be moved in order to avoid the dimension deduction of complex. The computing result shows that the improved algorithm can solve the complicated optimization problem with un-convex feasible domain or constrained monstrosity extremum points.

E. The reinforced wall of inner point punish function method

Inner point method can be called enclosure method. During the seeking, the probing point maybe out of the feasible domain. If thickening the enclosure, the phenomenon can be avoided.

The punish function is expressed as subsection function by heightening subsection. In order to maintain the strengthening wall function, the wall height can increase along with the minish of punish factor or keep a big value. The punish function can be expressed as follow (shown in figure 5).

\[
G_u(x) = \begin{cases} 
\frac{r^{(i)}}{g_u(x)} & g_u(x) \geq 10^{-6} \\
0.5(r^{(i)} - 1)10^{12}g_u(x) + 0.5(r^{(i)} + 1)10^6 & g_u(x) < 10^{-6} \\
\frac{r^{(i)}}{g_u(x)} & g_u(x) \geq 10^{-6} \\
0.5(r^{(i)} - 1)10^{12}g_u(x) + 0.5(r^{(i)} + 1)10^6 & -10^{-6} \leq g_u(x) < 10^{-6} \\
10^6 + [g_u(x) + 10^{-6}]^2 & g_u(x) \leq -10^{-6}
\end{cases}
\]  

(19)

![Figure 5. The sketch map of reinforcing wall](image)

The computation validates the conclusion that the reinforced wall algorithm improvement can avoid the computational failure of outing the feasible domain. The new algorithm improvement has the characteristic of simple idea, strong maneuverability. Through the computation of two examples, it is proved that the punish method with the algorithm improvement do not need the starting point being in the feasible domain. This provides a feasible way to some engineering optimal problem.

F. The analysis of problem with constrained monstrosity extremum point

Near the constrained monstrosity extremum point, the objective function isoline nearly parallels with constrain border. The computation results of random direction method, classical complex shape method, inner and outer penalty function method are contrasted. The former two methods are easy to give false optimal point because of little feasible and applicable region when the current point is on the border. With the characteristic of gradual optimization, the penalty function method can approach the extremum point. For the problem with multi-extremum point, all the local optimal point could be seek out if only starting at proper point. The reinforced wall inner penalty function method does not need the feasibility of initial point. So its optimal result is close to the outer method. Therefore the penalty function method is the best choice for the problem with constrained monstrosity extremum point.

G. The improvement of Newton method and the validation of optimization idea of blind walking repeatedly

The existence of maximum point, oddity point and saddle point often leads to computation failure. The optimization idea of blind walking repeatedly is based on the reality that the optimum towards the local minimum related to the initial point. After getting several optimal results with different initial point, the best result is taken as the final optimal result. The algorithm improvement of multi-dimension Newton method is improved. The improvement is important for all optimization method whose searching direction is constructed by grads.

Because the iterative formula has no seeking idea of following the descend direction, the objective function value of new iterative point may large than the previous point. Indeed, the series iterative point may jump over the extremum point times and times. In order to strengthen the convergence stability, the Newton direction, \[-\nabla^2 f(x^{(i)})^{-1}\nabla f(x^{(i)})\], is taken as the seeking direction. Then the iterative formula is following.

\[
x^{(i+1)} = x^{(i)} - \alpha^{(i)} [\nabla^2 f(x^{(i)})]^{-1}\nabla f(x^{(i)})
\]

(20)

Where, the \(\alpha^{(i)}\) is the best step of one dimension optimization method. This method can be defined as Newton direction method.

In order to strengthen the convergence stability, the damping factor (0<\(\alpha<1\)) can be introduced also. Then the damped Newton method is gotten. Its iterative formula is following.

\[
x^{(i+1)} = x^{(i)} - \alpha [\nabla^2 f(x^{(i)})]^{-1}\nabla f(x^{(i)})
\]

(21)
At the objective function Hessian matrix singularity point, due to the failure of getting matrix inverse, the computing will be stopped. So after the computation of Hessian matrix, the following step should be done.

\[
\text{if } (|H| < c) x^{(k+1)} = x^{(k)} + \Delta x \quad (22)
\]

At the maximum point, due to the zero grads vector, the iterative point can not be changed. So after the current point satisfied the convergence condition, the following step should be done.

\[
\text{if } (H \text{ is not positive definite}) x^{(k+1)} = x^{(k)} + \Delta x \quad (23)
\]

At the saddle point, due to the zero grads vector and positive definite Hessian matrix, the false optimum point may be put out. So after the current point satisfied the convergence condition, it is also needed to seek along 3-5 proportional spacing directions along positive and negative direction. No other than there is no descendant direction among those directions, the current iterative point can be output as optimum point.

A computational example with saddle point, maximum point and oddity point is studied by multidimension Newton method, damped Newton method and Newton direction method (shown in fig.6 and fig.7). The importance of the idea of blind walking repeatedly is testified. The parallel algorithm is the kernel idea of modernistic optimization method. This idea is also same to blind walking repeatedly. So the optimization problem with seriate feasible domain does not need to be solved by modernistic optimization method.

H. The explanation of later-day optimization method

Its kernel idea is seeking order in huddle and seeking necessity in contingency. A lot of optimization problem has no consecutive feasible domain, such as the traveling shopping problem and the knapsack problem etc. So the quality of probing point can not be measured by a uniform criterion. Those problems can not be solved by the classical optimization method. In recent years, some later-day methods are developed recurring to the development of other subject, such as genetic algorithms, ant colony algorithm, simulated annealing algorithm, artificial neutral network algorithm, expert system algorithm, generalized knowledge algorithm etc. Those methods study macrocosm optimization problem based on probability and randomization theory, just like many blindmen seek the macrocosm extremum point according to the same rule. In the monograph, the above six later-day optimization methods are analyzed based on this idea.

I. The conformity of seeking algorithm needing the objective function derivation

The monograph simplified those all algorithm. There are many complementarity and perfection of current method. Except for the above, there are many innovative contents, such as the best step with the hypothesis of two-order in steepest ascent method, the conclusion of the simplified function curve is not necessarily the tangent of the original function curve, the improvement of the general simplify gradient methods, the deduction of imitating Newton condition, etc.

The heredity and variation phenomenon is very complex. The variation style includes gene recombination, gene exchange and chromosome aberration. The genetic algorithms simulate the gene recombination.

The living of ant colony is very complex. There are similar global position system and radio system among individual ants. The ant colony algorithm simulates a tiny fraction of ant colony living.

When the solid metal annealed, the change of inner metal crystal grain is very complex. The simulated annealing algorithm simulates the simplest phenomenon of crystal grain change.

The structure and function of biologic neural network is not possible to be explained by modern science and technology. The human being only can understand there are relations between stimulus and response. Based on the simple knowledge, the artificial neutral network algorithm establishes an approximative corresponding relation between the design variables and objective function value.
VII. CONCLUSION

One dimension blindman-walking optimization method and its idea enriched the theoretic content of optimization design. It provided a new choice for solving some complex optimization problem in any fields. So some inextricable mechanics or dynamics optimization problem could be solved by it. Taking the new method and its visualized characteristic as the innovative kernel, many conventional methods could be improved. As a result, the subject gets a great development.

The improved algorithm takes one blindman to probe the extremum point. So he is easy to stop at the local extremum point. If select the best optimum point among the result of many initial point, the macrocosm extremum point is easy to be found.

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