Research on Model and Algorithm of Waveform Selection in Cognitive Radar

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Abstract—Cognitive radar can be aware of its environment, utilize intelligent signal processing, provide feedback from the receiver to the transmitter for adaptive illumination and preserve the information contents of radar returns. In this paper, based on the analysis of the parameters of radar measurements, range-Doppler resolution cell is built up, then stochastic dynamic programming model of waveform selection in cognitive radar is proposed, which is viewed as an important part of cognitive radar. Then optimal algorithm of waveform selection is proposed. The simulation results show the importance of adaptive waveform selection in cognitive radar, and the uncertainty of state estimation of using optimal selected waveform is lower than that of using fixed waveform.

Index Terms—cognitive radar, range-Doppler resolution cell, waveform selection model, dynamic programming

I. INTRODUCTION

The word radar is an acronym for radio detection and ranging. Radar is an electromagnetic system for the detection and location of reflecting objects such as aircraft, ships, spacecraft, vehicles, people, and the natural environment. It is widely used for surveillance, tracking, and imaging applications, for both civilian and military needs. All early radars use radio waves, but some modern radars today are based on optical waves and the use of lasers. Radar development was accelerated during World War II. Since that time development has continued such that present-day systems are very sophisticated and advanced.

Cognitive radar is a new framework of radar system proposed by Simon Haykin in 2006. Cognitive radar is an advanced form of radar system and it may adaptively and intelligently interrogate a propagation channel using all available knowledge including previous measurements, task priorities, and external databases. There are three basic ingredients in the composition of cognitive radar: Intelligent signal processing, which itself builds on learning through interactions of the radar with the surrounding environment; Feedback from the receiver to the transmitter, which is a facilitator of intelligence; Preservation of the information content of radar returns, which is realized by the Bayesian approach to radar signal processing[1]. Haykin suggests that such a cognitive radar system can be represented using a Bayesian formulation whereby many different channel hypotheses are given a probabilistic rating. As more information is collected, the parameters of the channel hypotheses and their relative likelihoods are updated. The goal of an illumination, therefore, is to efficiently reduce the uncertainty attributed to each channel hypothesis. Hard decisions are only made when confidence is sufficient or when necessity mandates an immediate action. Goodman have proposed and simulated a closed-loop active sensor by updating the probabilities on an ensemble of target hypotheses while adapting customized waveforms in response to prior measurement and compared the performance of two different waveform design techniques[2]. In [3], the author focuses on a cognitive tracking radar, the implementation of which comprises two distinct functional blocks, one in the receiver and the other in transmitter with a feedback link from the receiver to the transmitter. In [4], Informax principle aimed at maximizing the mutual information is used for designing the transmitted signal waveform. In [5], Arasaratnam have successfully solved the best approximation to the Bayesian filter in the sense of completely preserving second-order information, which is called Cubature Kalman filters. The problem of waveform selection can be thought of as a sensor scheduling problem, as each possible waveform provides a different means of measuring the environment, and related works have been examined in [6], [7]. In [8], adaptive waveform scheduling problem for new target detection as a stochastic dynamic programming problem is posed and Incremental Pruning method is used to solve this problem. In [9], it is shown that tracking errors are highly dependent on the waveforms used and in many situations tracking performance using a good heterogeneous waveform is improved by an order of magnitude when compared with a scheme using a homogeneous pulse with the same energy.
In this paper, according to the characteristic of cognitive radar, intelligent illuminator is viewed as an important part of cognitive radar, then stochastic dynamic programming model of waveform selection in cognitive radar is built up. In simulation, the importance of adaptive waveform in cognitive radar is shown. The simulation results also show that the uncertainty of state estimation of using optimal selected waveform is less than that of using fixed waveform.

II. THEORETICAL PRINCIPLE

Generally speaking, the most important parameters that a radar measures for a target are range, Doppler frequency, and two orthogonal space angles, we may envision a radar resolution cell that contains a certain four-dimensional hypervolume that defines resolution. A target for which measurements are to be made will fall in a resolution cell. Another target, conceptually, does not interfere with measurements on the first if it occupies another resolution cell different from the first. Thus, conceptually, as long as each target occupies a resolution cell and the cells are all disjoint, the radar can make measurements on each target free of interference from others.

For a single radar pulse, we may give a general sort of definition by considering the resolution cell to bounded in range by the compressed pulse’s duration (the time-equivalent extent of range), in Doppler by the reciprocal of the transmitted pulse’s duration, and in the two angles by the antenna pattern’s two orthogonal-plane beamwidths.

If a radar seeks to simultaneously make measurements on targets resolved in Doppler frequency, it can provide a bank of matched filters operating in parallel. Each target will excite the filter matched to its Doppler frequency, and its response can be used for measurements. Targets resolved in the range(time) coordinate can be separated with range gates followed by measurements. Thus a radar can, in principle, perform simultaneous measurements on targets unresolved in angle, provided the targets are resolved in range, or Doppler frequency, or both. On the other hand, it is difficult to simultaneously measure targets in angle coordinates (regardless of resolution in range or Doppler frequency). Such measurements require either a bank of main beams (which are possible to generate but not often implemented) or the time-sharing (switching) of one main beam among the various targets.

Angle resolution can be considered independently from range and Doppler resolution. While this result is not strictly true, we will give some conditions often satisfied in radars, such that when true, the resolution properties of the radar in angle are independent of the resolution properties in range and Doppler frequency[10].

III. MODEL FOR WAVEFORM SELECTION

We define range–Doppler resolution cell for waveform selection model.

Range resolution, denoted as \( \Delta R \), is a radar metric that describes its ability to detect targets in close proximity to each other as distinct objects. Radar systems are normally designed to operate between a minimum range \( R_{min} \), and maximum range \( R_{max} \). The distance between \( R_{min} \) and \( R_{max} \) is divided into \( M \) range bins, each of width \( \Delta R \)

\[
M = \frac{R_{max} - R_{min}}{\Delta R} \quad (1)
\]

Targets separated by at least \( \Delta R \) will be completely resolved in range.

Radas use Doppler frequency to extract target radial velocity (range rate), as well as to distinguish moving and stationary targets or objects such as clutter. The Doppler phenomenon describes the shift in the center frequency of an incident waveform.

In general, a waveform can be tailored to achieve either good Doppler or good range resolution, but not both simultaneously. So we need to consider the problem of adaptive waveform scheduling. The basic scheme for adaptive waveform scheduling is to define a cost function that describes the cost of observing a target in a particular location for each individual pulse and select the waveform that optimizes this function on a pulse by pulse basis.

We make no assumptions about the number of targets that may be present. We divide the area covered by a particular radar beam into a grid in range-Doppler space, with the cells in range indexed by \( \tau = 1, \ldots, N \) and those in Doppler indexed by \( \nu = 1, \ldots, M \). There may be \( 0 \) target, \( 1 \) target or \( NM \) targets. So the number of possible scenes or hypotheses about the radar scene is \( 2^{NM} \). Let the space of hypotheses be denoted by \( \chi \). The state of our model is \( X_{t} = x \) where \( x \in \chi \). Let \( Y_{t} \) be the measurement variable. Let \( u_{t} \) be the control variable that indicates which waveform is chosen at time \( t \) to generate measurement \( Y_{t+1} \), where \( u_{t} \in U \). The probability of receiving a particular measurement \( X_{t} = x \) will depend on both the true, underlying scene and on the choice of waveform used to generate the measurement.

We define \( d_{x} \) is state transition probability where

\[
a_{x} = P(x_{t+1} = x' \mid x_{t} = x) \quad (2)
\]

We define \( b_{x} \) is the measurement probability where

\[
b_{x} = P(x_{t+1} = x' \mid x_{t} = x) \quad (2)
\]
Assume the transmitted baseband signal is \( s(t) \), and the received baseband signal is \( r(t) \). The matched filter is the one with an impulse response \( h(t) = s^*(t) \), so an output process of our matched filter is

\[
x(t) = \int h^*(\lambda - t)r(\lambda)d\lambda
\]

In the radar case, the return signal is expected to be Doppler shifted, then the matched filter returns a signal with an expected frequency shift \( \nu \) and an impulse response

\[
h(t) = s^*(-t)e^{j2\pi\nu t}
\]

The output is given by

\[
x(t) = \int h^*(\lambda - t)e^{-j2\pi\nu(t-\tau)}r(\lambda)d\lambda
\]

where \( \nu \) is an expected frequency shift.

The baseband received signal will be modeled as a return from a Swerling target:

\[
r(t) = A\delta(t - \tau)e^{j2\pi\nu t}I + n(t)
\]

where \( s(t, \nu, A) = s(t - \tau)e^{j2\pi\nu t} \) is a delayed \( \tau \) and Doppler-shifted \( \nu \) replica of the emitted baseband complex envelope signal \( s(t) \); \( I \) is a target indicator. \( A \) approaches a complex Gaussian random variable with zero mean and variance \( 2\sigma_A^2 \). We assume \( n(t) \) is complex white Gaussian noise independent of \( A \), with zero mean and variance \( 2N_0 \).

At time \( t \) the magnitude square of the output of a filter matched to a zero delay and a zero Doppler shift is

\[
|x(t)|^2 = \int_0^T |r(\lambda)s^*(\lambda - t)|^2d\lambda
\]

When there is no target

\[
r(t) = v(t)
\]

\[
x(t_0) = \int_0^\tau n(\lambda)s^*(\lambda - t_0)d\lambda
\]

The random variable \( x(t_0) \) is complex Gaussian, with zero mean and variance given by

\[
\sigma_x^2 = E\{x(t_0)x^*(t_0)\} = 2N_0
\]

\( \xi \) is the energy of the transmitted pulse.

When target is present

\[
r(t) = As(t - \tau)e^{j2\pi\nu t}I + n(t)
\]

\[
x(t_0) = \int_0^\tau [As(\lambda - \tau)e^{j2\pi\nu t} + n(\lambda)]s^*(\lambda - t_0)d\lambda
\]

This random variable is still zero mean, with variance given by

\[
\sigma_x^2 = E\{x(t_0)x^*(t_0)\} = \sigma_0^2(1 + \frac{2\sigma_A^2\xi^2}{\sigma_0^2}A(t_0 - \tau, v_0 - v))
\]

\( A(\tau, v) \) is ambiguity function, given by

\[
A(\tau, v) = \frac{1}{(\int s(\lambda)2\pi\nu d\lambda)^2} \int |s(\lambda)s^*(\lambda - \tau)|e^{j2\pi\nu t}d\lambda
\]

Recall that the magnitude square of a complex Gaussian random variable \( x \sim N(0,\sigma_x^2) \) is exponentially distributed, with density given by

\[
y = x^2 \sim \frac{1}{2\sigma_x^2} e^{-\frac{y}{2\sigma_x^2}}
\]

We consequently have that the probability of false alarm \( P_f \) is given by

\[
P_f = \int_0^{\infty} \frac{1}{2\sigma_x^2} e^{-\frac{x}{2\sigma_x^2}} dx = e^{-\frac{D}{2\sigma_x^2}}
\]

And the probability of detection \( P_d \) by

\[
P_d = \int_0^{\infty} \frac{1}{2\sigma_x^2} e^{-\frac{x}{2\sigma_x^2}} \frac{2\sigma_A^2}{(1 + \frac{2\sigma_A^2\xi^2}{\sigma_0^2}A(t_0 - \tau, v_0 - v))} dx = e^{-\frac{D}{2\sigma_A^2}}
\]

In the case when a target is present in cell \((\tau, v)\), assuming its actual location in the cell has a uniform distribution

\[
P_d = \frac{1}{|A|} \int_{(\tau, v) \in A} e^{-\frac{2\sigma_A^2(1 + \frac{2\sigma_A^2\xi^2}{\sigma_0^2}A(t_0 - \tau, v_0 - v))} d\tau_a d\nu_a
\]

where \( A \) is the resolution cell centred on \((\tau, v)\) with volume \( |A| \).

A target for which measurements are to be made will fall in a resolution cell. Another target, conceptually, does not interfere with measurements on the first if it occupies another resolution cell different from the first. Thus, conceptually, as long as each target occupies a resolution cell and the cells are all disjoint, the radar can make measurements on each target free of interference from others.

Define \( \pi = \{u_0, u_1, \ldots, u_T\} \) where \( T+1 \) is the maximum number of dwells that can be used to detect
and confirm targets for a given beam. Then \( \pi \) is a sequence of waveforms that could be used for that decision environment. We can obtain different \( \pi \) according to different environment in cognitive radar. Let

\[
V'_{t}(X_{t}) = E[\sum_{t=0}^{T} \gamma^{t} R(X_{t}, u_{t})] \tag{20}
\]

where \( R(X_{t}, u_{t}) \) is the reward earned when the scene \( X_{t} \) is observed using waveform \( u_{t} \) and \( \gamma \) is discount factor. Then the aim of our problem is to find the sequence \( \pi^{*} \) that satisfies

\[
V^{*}(X_{t}) = \max_{\pi} E[\sum_{t=0}^{T} \gamma^{t} R(X_{t}, u_{t})] \tag{21}
\]

However, knowledge of the actual state is not available. Using the method of [11], we can obtain that the optimal control policy \( \pi^{*} \) that is the solution of (21) is also the solution of

\[
V^{*}(p(0)) = \max_{\pi} E[\sum_{t=0}^{T} \gamma^{t} R(p_{t}, u_{t})] \tag{22}
\]

where \( p_{t} \) is the conditional density of the state given the measurements and the controls and \( p_{0} \) is the a priori probability density of the scene. \( p \) is a sufficient statistic for the true state \( X_{t} \). So we need to solve the following problem

\[
\max_{\pi} E[\sum_{t=0}^{T} \gamma^{t} R(p_{t}, u_{t})] \tag{23}
\]

This is the waveform selection model in cognitive radar.

The refreshment formula of \( p_{t} \) is given by

\[
p_{t+1} = \frac{B A p_{t}}{1^{T} L A p_{t}} \tag{24}
\]

where \( B \) is the diagonal matrix with the vector \((b_{ik}(u_{k}))\) the non-zero elements and \( 1 \) is a column vector of ones. \( A \) is state transition matrix.

IV. ALGORITHM OF WAVEFORM SELECTION

If we wanted to solve this problem using classical dynamic programming, we could have to find the value function \( V'_{t}(p_{t}) \) using

\[
V'_{t}(p_{t}) = \max_{u_{t} \in U_{t}} (R(p_{t}, u_{t}) + \gamma E[V'_{t+1}(p_{t+1})] \tag{25}
\]

for each value of \( p_{t} \). With backward dynamic programming, we step forward in time.

Solving a finite horizon problem, in principle, is straightforward. We simply have to start at the last time period, compute the value function for each possible state \( p_{t} \in P \), and then step back another time period. This way at time period \( t \) we have already computed \( V'_{t+1}(p_{t+1}) \). The critical element that attracts so much attention is the requirement that we compute the value function \( V'_{t}(p_{t}) \) for all states \( p_{t} \in P \).

Our optimal algorithm is given below.

Step1. Initialization:

Initialize the terminal contribution \( V'_{T}(p_{T}) \).

Set \( t = T - 1 \).

Step2. Calculate:

\[
V'_{t}(p_{t}) = \max_{u_{t}} (R(p_{t}, u_{t}) + \gamma \sum_{p' \in P} P(p' | p_{t}, u_{t}) V'_{t+1}(p')) \tag{26}
\]

for all \( p_{t} \in P \)

Step3. If \( t > 0 \), decrement \( t \) and return to step 1. Else, stop.

Generally speaking, reward function can be different forms according to different problems. It represents the value that we stand in certain place and take some action. In the problem of adaptive waveform selection, two forms of reward function are usually used. They are linear reward function and entropy reward function.

Linear reward function is usually used in the circumstance that \( R(p, u) \) is required to be a piecewise linear function. The form of this function is simple and easy to calculate. However, it can not reflect the whole value. The form of linear reward function is

\[
R_{l}(p, u) = p'p - 1 \tag{27}
\]

Entropy reward function is usually used in the circumstance that \( R(p, u) \) is not required to be a piecewise linear function. It comes from information theory. It can reflect the whole value accurately. But it is more complex than linear reward function. The form of entropy reward function is

\[
R_{e}(p, u) = \sum_{k \in \mathbb{K}} p_{k}(k) \log(p_{k}(k)) \tag{28}
\]

We can choose different form of reward function according to different problems.

V. SIMULATION

In order to show the importance of adaptive waveform selection in cognitive radar, curves of measurement probability versus SNR are simulated.
Figure 3 is curve of measurement probability versus SNR with three different waveforms. From this figure we can see that measurement probability is becoming large with the increase of SNR. Under the same SNR, different waveform corresponds different measurement probability. So measurement can be improved through appropriately scheduling waveform in cognitive radar. Generally speaking, the wider the pulse duration is, the larger the measurement probability is. However, wide pulse duration means large energy of the transmitted pulse. We should make a balance between pulse duration and energy of the transmitted pulse and appropriately schedule waveform in order to obtain large measurement probability.

Figure 4 is measurement probability versus SNR with different targets. From this figure we can see that under the same SNR, measurement probability is different to different targets. So according to different targets we should select different waveforms. In actuality, path of target is so complex. We should change waveform according to different environment.

We consider a simple scenario. The state space is $4 \times 4$. We consider 5 different waveforms where for each waveform $u$, and each hypotheses for the target $x$, the distribution of $x'$ is given in table 1. The discount factor $\gamma = 0.9$.

<table>
<thead>
<tr>
<th>TABLE I. MEASUREMENT PROBABILITIES FOR THE EXAMPLE SCENARIO</th>
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<tr>
<td>$x'=1$</td>
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<tr>
<td>$x=1$</td>
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<tr>
<td>$x=2$</td>
</tr>
<tr>
<td>$x=3$</td>
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<tr>
<td>$x=4$</td>
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</tbody>
</table>

The matrix $A$ is given by

$$A = \begin{bmatrix}
0.96 & 0.02 & 0.01 & 0.04 \\
0.01 & 0.93 & 0.03 & 0.04 \\
0.02 & 0.03 & 0.95 & 0.02 \\
0.01 & 0.02 & 0.01 & 0.9 \\
\end{bmatrix} \tag{29}$$

The matrix $A$ is given by

$$A = \begin{bmatrix}
0.96 & 0.02 & 0.01 & 0.04 \\
0.01 & 0.93 & 0.03 & 0.04 \\
0.02 & 0.03 & 0.95 & 0.02 \\
0.01 & 0.02 & 0.01 & 0.9 \\
\end{bmatrix} \tag{29}$$

Figure 5 is curve of uncertainty of state estimation using formula (27) and figure 6 is curve of uncertainty of state estimation using formula (28). In fact, the optimal adaptive waveform selection can be viewed as minimizing the uncertainty in the state estimation or target tracking errors. We can see the tracking errors are becoming lower with the increase of time. The tracking errors using optimal algorithm are lower than fixed waveform. Moreover, the advantages of optimal algorithm do not depend on the form of reward function we use.

VI. CONCLUSIONS
In this paper, the characteristic of cognitive radar is introduced. After all, cognitive radar is an ideal form of radar, and there are many problems to solve. Intelligent transmitter may be an important and crucial part of cognitive radar. The importance of waveform selection is shown in our simulations. The proposed waveform selection model can be used in the adaptively scheduling of waveform in cognitive radar. We also give an optimal algorithm for waveform selection. However, actual situation may be hundreds of states and hundreds of waveforms. So we need to research highly efficient algorithm to solve the problem.

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