Medical Image De-speckling Based on Improved Polar Coordinate

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Abstract—In recent years, medical image processing has been a hot issue for researchers at home and abroad. Considering the characteristics of medical image collection and speckle noise distribution, a de-speckling algorithm based on an improved polar coordinate is present in this paper. Firstly, a logarithmic transformation is applied to the noisy medical image in polar coordinates system and then full-domain sampling is adopted in any radial direction related to the point estimation. According to the distribution model of speckle noise and the space correlation between sample point and estimate point, the correlation between sample points and estimation points, weighting factors are constructed. Finally, experimental results show that the algorithm proposed in this paper not only can effectively remove the noise, but also can properly preserve the edge details of the medical image.

Index Terms—Medical Image; Image De-Noising; Polar Coordinate System; Simulation Research

I. INTRODUCTION

Noise reduction is one of the most important parts of image processing. Image de-noising has been an important problem that people devote to study. A better de-noising algorithm usually can preserve the image information and keep the contrast clear while removing noise. Traditional image de-noising methods, such as median filtering and Gaussian filtering etc., mainly filter the high-frequency component of the image. So the image details and texture regions of the reconstructed images are always blurring or even unrecognizable.

Because of the different imaging mechanism in medical image, the original images contain lots of different types of noise. The existence of the noise affects people’s visual feeling in medical image observation and disturbs people’s understanding of the image information. The extreme condition is so much noise that the image produces distortion, even making storage useless. It is clear that image de-noising is a necessary tool to correctly identify image information. So image de-noising plays an important role in improving the image quality. Image de-noising not only improves visual recognition accuracy, but also is a reliable guarantee for further image processing. The result is unsatisfactory if feature extraction, image alignment and fusion are performed in a noisy medical image. Especially for medical image processing, each step of processing must be as accurate and reliable as possible. Therefore, medical image de-noising is necessary.

Due to the limitation of imaging mechanism, this results in poor image quality in ultrasound image. As there are inherent speckle noises in the medical ultrasound images, it becomes more difficult to identify and analyze image details [1]. Therefore, image de-noising is necessary for lower image quality. It requires the de-noising algorithms not only can effectively remove the speckle noise, but also can protect the edge details of the medical image, which be used to provide an important basis for analyzing and diagnosing disease. The de-noising algorithms for medical ultrasound image are divided into two major categories: 1) de-noising methods based on transform domain [2]; 2) de-noising method based on spatial domain. The transform domain de-noising methods mainly include Wiener filtering [3], wavelet threshold [4] and Gaussian scale mixtures [5]. By analyzing the gray information of the image in spatial domain, the de-noising methods based on spatial domain revise the target image and de-noise the noisy images. Most spatial domain de-noising methods depend on the local information of the image to suppress the noise, which are called the de-noising method based on local spatial domain, including traditional median filter [6], Gaussian filter, and a relatively new method based on anisotropic model [7, 8] and bilateral filter [9] etc. Although these algorithms have high efficiency, when the local image affected by serious noise pollution, these methods have presented poor performance. In order to overcome the shortcomings of the de-noising method based on local spatial domain, [10] presented a method using global statistical information to suppress the noise. De-noising method based on global spatial-domain overcame the shortcoming of poor de-noising performance when the local image affected by serious noise pollution, but because of the global spatial domain methods required to sample the entire image and need a great deal of calculation to estimate the noisy image, it reduces the efficiency of the algorithm. In addition, so far the de-noising methods with better performance such as anisotropic, wavelet threshold etc. are performed on the premise that the ultrasound images were polluted by Gaussian white noise, which is not reasonable and may lead to inaccuracy of de-noising results. Meanwhile, these algorithms were failure to consider the physical characteristics of ultrasonic image acquisition and the distribution Character of noise.
Based on the analysis above, this paper proposes a method for suppressing the speckle noise in medical image based on improved polar coordinate system.

II. DEFINITION AND CLASSIFICATION OF THE NOISE IN MEDICAL IMAGE

For image noise, we can define it form two angles: one is from the perspective of the human senses, which consider the image noise is a factor of hindering human sense organ to recognize and understand the image information; the other is from the mathematical point of view, the image information is considered as a space function \( f \) and the image noise is a factor which makes the information expressed by this function degrade. That is to say, the image degrades to \( \hat{f} \) under the influence of noise. Image noise can be divided into different categories by different methods. Depending on how the image degrade from the mathematical point of view, the image noise can be divided into additive noise and multiplicative noise, which can be expressed as follows,

\[
\hat{f} = f + n \\
\hat{f} = f \cdot n
\]

(1) (2)

where, \( n \) is noise, represents additive noise and multiplicative noise in Eq. (1) and Eq. (2), respectively.

The noise can be divided into following categories according to the physical factors of the generation of the noise:

1. Electronic noise
   Noise generated the heating of the motion electron in the resistance device of the image acquisition circuits.

2. Photoelectron noise
   It caused by photoelectric and is serious especially in low-light situations.

3. Photosensitive particle noise
   It usually exists in film images and caused by the randomness of distribution and shape of emulsion Silver Halides grains changing into metallic silver particles.

4. Speckle noise
   In some coherent imaging system (such as medical imaging, synthetic aperture radar imaging and laser imaging), the noise produced in the images due to the interference effects of sound or light. It also has the relationship with the roughness of the target surface. Goodman analyzed the properties of speckle noise under condition of correlated irradiances. The difference between the laser speckle and ultrasonic speckle in interference and formulation is pointed out by Abbott and Thurstone.

How to define the parameters of characteristic of noise space is relied on whether these noises have relation to the image. Frequency characteristic of noise is content which the noise in Fourier domain. For example, when the Fourier spectrum of the noise is a constant, the noise often is called white noise. This term is derived from the physical characteristic, which will contain all spectrums in the visible spectrum with equal proportion. It is clear that the Fourier spectrum of the functions containing all the frequency is a constant associated with relevant knowledge.

Due to the abnormal of periodic noise in space, we often assume that the noise is independent of spatial coordinates and not associated with the image itself. These assumptions are invalid in some areas, but the situations of spatial independent and correlated noise is complex and let’s put this problem on ice for the moment. The main source of digital image noise is image acquisition (digital process) and transmission.

The working state of image sensor affected by various factors, such as environmental conditions of image acquisition and self-factors of sensors. For example, when we use CCD cameras to obtain images, light intensity and temperature of sensor is the main factor resulting in images. The images are polluted by noise mainly due to the interference of the transmission channel. For instance, the images transferred by wireless network may be polluted due to the interference of light or other atmospheric factors.

The ultrasound image is mainly depending on the principle of ultrasonic Pulse-Echo. When the ultrasonic wave spread in human body, it will produce reflection and refraction at the junction of human tissue or in the homogeneity of human tissue giving echo signal of different intensity. Collect these echo signals and convert into an electrical signal of different intensity by transfer circuit. Finally, we convert electrical signals into image signal of different intensity through the display circuits. In the imaging process of ultrasound images, speckle noise widely exists, which was mainly due to the mutual interference effect of ultrasound in imaging process, besides, has closely relationship with the roughness of tissue surface. It seems from the visual point of view, the noise presents in image like speckle spot. Speckle noise can be described by a generalized K distribution, and its probability density function as,

\[
p(x | \alpha, \nu, \eta) = \frac{2b}{\Gamma(\alpha)} \left( \frac{2\alpha}{b\eta} \right)^{\alpha} I_\nu \left( \frac{b\alpha}{\eta} \right) K_{\alpha-\nu} \left( b \frac{\alpha}{\eta} \right) (3)
\]

where, \( b = \sqrt{4\alpha + \nu^2} / \eta \), \( \eta \) is scaling factor, and \( \nu \) describe the coherent part of echo signal. For the speckle noise fully developed, the number of scattering particles is large and \( \alpha \to \infty \), and the generalized K distribution is equal to Rice distribution. If \( \nu = 0 \) further, the generalized K distribution equals to Rayleigh distribution.

III. PROPOSED SCHEME

The acquisition of ultrasound image is from the controlling the transducer array of ultrasonic transmitter to launch ultrasonic wave, which will touch different tissue and produce echo signals with different reflection and construct image of the scanning object according to the echo signals. It can be concluded from the physical characteristic of acquisition of ultrasonic data as: 1) Because the ultrasonic probe is sector scan, the appearance of speckle noise extends in a radial manner. 2) Speckle noise will become slim with the growth of distance from the scanning center. According to these two
points above, this paper propose to use polar coordinates \((r, w)\) instead of commonly used Cartesian coordinate system \((x, y)\) to study. Image model with speckle noise in polar coordinate system is described as follows,

\[
f(r, w) = f_0(r, w) * n_n(r, w) + n_n(r, w)
\]

(4)

where, \(f(r, w)\) is noisy image, \(f_0(r, w)\) is noise-free image, \(n_n(r, w)\) and \(n_n(r, w)\) are multiplicative noise and additive noise, respectively. \(r\) is radial coordinate, \(w\) is the angular coordinate. Under general circumstances, the effects of additive noise are much smaller than the multiplicative noise, so we rewrite Eq. (4) as \([12-14]\),

\[
f(r, w) \equiv f_0(r, w) * n_n(r, w)
\]

(5)

According to the research of literature \([14]\), the noise \(n_n(r, w)\) in Eq. (2) obey the generalized gamma distribution, the PDF as,

\[
p_n(n) = \frac{\gamma^\alpha n^{\gamma-1}}{\alpha^\Gamma(\gamma)} \exp\left\{-\left(\frac{n}{\alpha}\right)^\gamma\right\}, \ n \geq 0, \ \alpha, \ \gamma, \ \nu > 0
\]

(6)

where, \(\Gamma(\nu)\) denotes the gamma function. The ultrasound speckle noise can be removed using the generalized gamma distribution because it encompasses lots of other commonly used denoising distribution model and can dynamically adapt to different noise and achieve better denoising effect. For example, if \(\gamma = 2\), the generalized gamma distribution becomes the Nakagami distribution. Alternatively, if \(\nu = 1\), the generalized gamma becomes the Weibull distribution. Specifically, if \(\gamma = 2, \ \nu = 1\), the generalized gamma distribution becomes the Rayleigh distribution.

De-noising method based on partial spatial-domain is sensitive to SNR of local regions of the image. When the local region is affected by serious noise pollution, the de-noising effect is often poor. De-noising algorithm based on spatial-domain overcome the defects of local spatial-domain algorithm which is sensitive to local serious noise pollution, but its efficiency is too low. Using Monte Carlo method to do research, it can obtain trade-off between efficiency against the de-speckling effect. The main steps of the likelihood weighted Monte Carlo method presented in this paper as follows: 1) decouple the speckle noise in polar coordinate system and use logarithmic transformation to separate the multiplicative random noise; 2) generate the sample sequence based on the two-dimensional Gaussian distribution of the center of estimated point; 3) likelihood weighted Monte Carlo estimation, analyze the space and gray correlation between sample points and estimated points, and construct the weighted factors between sample points and estimated points combining the characteristics of distribution model of speckle noise. Finally, carry out the weighted likelihood Monte Carlo estimation.

A. Decoupling the Speckle Noise in Polar Coordinate System

Because ultrasonic probe scan with sector scanning and speckle noise model satisfies the two characters described in section one, the polar coordinate is more suitable to describe speckle noise model. Meanwhile, the noise-free image and multiplicative noise in Eq. (6) can be separated using a logarithmic calculation expressed as follows.

\[
\ln\{f(r, w)\} \equiv \ln\{f_0(r, w)\} + \ln\{n(n(r, w))\}
\]

(7)

For the ease of representation, we simplify Eq. (7) as,

\[
\hat{f}(r, w) \equiv \hat{f}_0(r, w) + \hat{n}(r, w)
\]

(8)

where, \(f_0(r, w)\), \(f_0(r, w)\) and \(n(n(r, w))\) denote \(\{f(r, w)\}\), \(\ln\{f_0(r, w)\}\) and \(\ln\{n(n(r, w))\}\), respectively. According to the noise model in Eq. (4), we can estimate the noise-free image using \(\hat{f}(r, w)\) and \(\hat{n}(r, w)\).

B. Sample Sequence Generation

Among all algorithms in the global spatial-domain, the estimation of any point must consider the other pixels in the whole image, which increase the complexity of the algorithm. In order to optimize the global spatial complexity of the algorithm, in this paper we will take the following approach to estimate the pixels. In the polar coordinate space, for a point \(f_0(r, w)\) in the original image, we extract the sample sequence \(\{f_0(r, w)\}, \ln\{f_0(r, w)\}\) and \(\ln\{n(n(r, w))\}\) from the noisy image \(\hat{f}(r, w)\) to estimate the sample point \(f_0(r, w)\), where \(y(r, w)\), \(k\) is the number of sample points in the sequence. The sample sequence obeys a two-dimensional Gaussian distribution centered at \((r, w)\)

(9)
optimum value of \( \sigma \) depends on the spatial resolution of the captured image, therefore, for different ultrasound images collected with ultrasonic probe, the optimal value is not the same.

C. Weighted Likelihood Monte Carlo Estimation

For the sample sequence \( \{ \hat{f}(\psi_1), \hat{f}(\psi_2), \ldots, \hat{f}(\psi_n) \} \) in Sec.2.2, we adopt Monte Carlo method to estimate point \( \hat{f}_0(r*, w*) \). The traditional Monte Carlo estimation methods are described below,

\[
\hat{f}_0(\psi) = \frac{1}{k} \sum_{i=1}^k \hat{f}(\psi_i) 
\]  

(10)

Since the correlation between each point of sampling sequence and \( \hat{f}_0(r*, w*) \) is not the same, so the direct use of Eq. (10) to calculate will lead to high variance estimation, and cause inaccurate estimation. If the weight between sample points and estimated points can be estimated according to their impact or relevance, it will reduce the variance estimation. Therefore, we propose the weighted likelihood Monte Carlo estimation against \( \hat{f}_0(r*, w*) \) as follows.

Firstly, let us define the spatial correlation coefficient as,

\[
\rho = \frac{1}{\sqrt{\Gamma(\alpha)}} \exp \left\{ \frac{\tau^2 \sigma^2 + r^2 - 2\tau \sigma \cos(\alpha) - \ln \alpha}{\sqrt{\Gamma(\alpha)}} \right\} 
\]  

(11)

where, \( \rho \) denotes the space correlation between sample points and estimated points. When the sample points infinitely close to the estimated point, then \( \rho \rightarrow 1 \). While the sample points are away from the estimated point, so \( \rho \rightarrow 0 \). \( \tau \in \mathbb{R}^+ \) is the convergence control parameter which is used to control the velocity of convergence.

According to the analysis of [14], under the logarthmic compression, the speckle noise model in Eq. (5) can be expressed as,

\[
p_n = \frac{1}{\Gamma(\alpha)} \exp \left\{ \gamma(n - \ln \alpha) \right\} 
\]  

(12)

From the Eq. (12) we can see that after logarithmic compression noise distribution is asymmetrical and has a double exponential distribution. Compared with the single Gaussian distribution, using Eq. (12) to estimate the noise distribution may be better [14]. Considering the noisy ultrasound image model in polar coordinates system, the spatial correlation between sample points and estimated points and the speckle noise model in Eq. (9) together, we define weighting factors between sample points \( \psi_i \) and the estimated points \( \psi_e \) as follows,

\[
g(\psi_i | \psi_e) = \frac{1}{\Gamma(\alpha)} \exp \left\{ \frac{\gamma \exp \left\{ \frac{\gamma [\hat{f}(\psi_i) - \hat{f}(\psi_e) - \ln \alpha]}{\sqrt{\Gamma(\alpha)}} \right\} - \exp \left\{ \gamma [\hat{f}(\psi_i) - \hat{f}(\psi_e) - \ln \alpha] \right\} }{\Gamma(\alpha)} \right\} 
\]  

(13)

The meaning of the weighting factor is that if the relationship between \( \hat{f}(\psi_i) \) and \( \hat{f}(\psi_e) \) can be used to approximately estimate the distribution of Eq. (12), then the sample points \( \psi_i \) are more close to speckle noise. Therefore, in the estimation process, the bigger the correlation between such sample point and speckle noise, the higher the weight. Finally, we get the weighted Monte Carlo likelihood estimation of \( n \)

\[
\hat{f}_0(\psi) = \frac{\sum_{i=1}^k g(\psi_i | \psi_e) \hat{f}(\psi_i)}{\sum_{i=1}^k g(\psi_i | \psi_e)} 
\]  

(14)

We apply natural exponential for \( \hat{f}_0(\psi) \) to restore the image, then will get the de-noised image \( f_0(\psi) = \exp \{ \hat{f}_0(\psi) \} \).

IV. NUMERICAL RESULTS

A. Evaluation Index

In order to evaluate the performance of this de-noising algorithm objectively, it is necessary to determine the suppression index of speckle noise. This paper will measure the de-noising quality from the following three aspects [15]. In the following formula, \( i \) and \( ir \) denote original image and de-noised image, correspondingly. The size of the image is \( M \times N \).

1) Mean Squared Error (MSE)

The smaller the MSE value, the better the de-speckling. It can be computed as,

\[
MSE(i, ir) = \frac{1}{MN} \sum_{m=1}^M \sum_{n=1}^N (i(m, n) - ir(m, n))^2
\]  

(15)

2) PSNR

Peak Signal to noise Ratio (PSNR) is used to evaluate the de-speckling effect for the grayscale image. The larger the PSNR value indicates the better performance of the algorithm, the definition as follows,

\[
PSNR = 20 \times \log \left( \frac{255}{\sqrt{MSE(i, ir)}} \right)
\]  

(16)

3) Edge Keeping Index (EKI)

Edge keeping index (EKI), is used to describe the image edge retention. A good de-noising algorithm should suppress speckle noise as much as possible while preserving image edges feature information. EKI is defined as,

\[
EKI = \frac{\sum_{j=1}^N (\Delta y - \Delta \hat{y})^2 (\Delta w - \Delta \hat{w})}{\sqrt{\sum_{j=1}^N (\Delta y - \Delta \hat{y})^2} \sqrt{\sum_{j=1}^N (\Delta w - \Delta \hat{w})^2}}
\]  

(17)

where, \( \Delta t, \Delta tr \) is the high-pass filtering result of \( \Delta \hat{y}, \Delta \hat{w} \) via a standard 3\times3 Laplacian operator.

B. Evaluation of Noise Removal Results

To verify the effectiveness of our method, we select ultrasound image of the human renal in our experiment.
All the experiments are run under MATLAB 2009 on PCs with Intel Pentium Dual Core at 1.60GHZ and 2GB memory. Comparing our method with current better ultrasonic image de-speckling algorithms including bilateral filtering [9], NL-means [16], PK model ($k = 40$), SRAD [8] and algorithms in [17]. Visual effect, operating efficiency, MSE, PSNR and EKI are used to evaluate the performance of all the de-noising methods in this paper. The experimental results show in Figs.1-2 and Table 1. Figure 2 is the edge detection results of Figure 1, which use canny operator with threshold [0.05 0.3]. It is noted that all TIME of these methods in Table 1 are averaged over ten independent trials.

The visual comparison is illustrated in Fig.1. We can observe that our algorithm can effectively suppress speckle noise in ultrasound images and provides better visual quality. Meanwhile, the edge details are better preserved in our algorithm which can be further illustrated in Fig.2. Our algorithm removes most speckle noise scattered in ultrasonic kidney images and better preserved details of kidney, such as center part of the medulla, the edge of outer part of renal cortex. The data in Table 1 show the index of different methods. It can be seen that our method has some improvement in MSE and PSNR compared with other de-speckling methods. The speed of our algorithm is just slower than PM ($k = 40$) algorithm, but the de-noising performance markedly superior to PM.

![Comparison of various algorithms](image)

TABLE I. PERFORMANCE COMPARISON

<table>
<thead>
<tr>
<th>Algorithm</th>
<th>MSE</th>
<th>PSNR</th>
<th>EKI</th>
<th>TIME/S</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bilateral filtering</td>
<td>0.0224</td>
<td>34.1334</td>
<td>0.6570</td>
<td>1.5957</td>
</tr>
<tr>
<td>NL-means</td>
<td>0.0276</td>
<td>33.2269</td>
<td>0.4632</td>
<td>1.8672</td>
</tr>
<tr>
<td>PM($k=40$)</td>
<td>0.0250</td>
<td>33.6647</td>
<td>0.5696</td>
<td>0.2447</td>
</tr>
<tr>
<td>SRAD</td>
<td>0.0212</td>
<td>34.3679</td>
<td>0.6432</td>
<td>0.6288</td>
</tr>
<tr>
<td>[17]</td>
<td>0.0264</td>
<td>33.3609</td>
<td>0.4830</td>
<td>0.9749</td>
</tr>
<tr>
<td>Our method</td>
<td>0.0204</td>
<td>34.5339</td>
<td>0.7047</td>
<td>0.4852</td>
</tr>
</tbody>
</table>

V. CONCLUSION

This paper analyzes the physical properties of image acquisition and the distribution of speckle noise, and introduces the polar coordinate system to represent speckle noise model more reasonably. Apply logarithmic transformation to noise model under polar coordinate system and estimate the noise distribution model using the generalized gamma distribution in the logarithmic domain. Meanwhile, integrate Monte Carlo method into the global spatial-domain and obtain sample sequence based on two-dimensional Gaussian distribution of the centered on the estimated point. We can obtain the weight factors between sample points and estimated points considering the speckle noise distribution model and the space correlation between sample points and estimated points. Finally, suppress the speckle noise and preserve the image details.

REFERENCES


