The Translation Invariant Wavelet-based Contourlet Transform for Image Denoising

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Abstract—A new method of image denoising using wavelet-based contourlet transform (WBCT) is proposed. Due to the lack of translation invariance of WBCT, image denoising by means of WBCT would lead to Gibbs-like phenomena. In the paper, cycle spinning-based technique is applied to develop translation invariant WBCT denoising scheme. Many simulation experiments with images contaminated by additive white Gaussian noise demonstrate that the performance of the proposed approach substantially surpasses that of previously wavelets methods using the cycle spinning both visually and in terms of the PSNR values, especially for the images that include mostly fine textures and contours.

Index Terms—wavelet-based contourlet transform (WBCT), cycle spinning, image denoising, translation invariance

I. INTRODUCTION

During the acquisition and transmission of image, there always exists noise. We must remove these noise from image to improve the image quality. Traditional denoising methods are classified into two groups: the spatial method such as wiener filter which eliminates the noise in image at the cost of blurring the edge and texture; the other one is using image transform method, for example, the popular wavelet threshold denoise technology.

Over the past decade, wavelet transforms have been paid a lot of attention to many image processing areas such as compression, noise removal, feature extraction, edge enhancement and detection. In denoising, the wavelet with a single orthogonal wavelet function has played an important role because of its ability to capture the energy of an image with few energy transform values[1,2,3]. But wavelet has limited ability in capturing the directional information of the natural images. Aimed at improving the representation sparsity of an image over the Wavelet, some new transforms have been introduced for image denoising. Do and Vetterli developed the contourlet transform (CT)[4,5] based on anisotropy scaling law and directional which can sparsely represent natural images. It is implemented by Laplacian pyramid and a double filter banks structure that can simultaneously hold multiresolution, localisation, nearly critical sampling, flexible directionality and anisotropy.

As we turn to use the transform tool, more advanced denoising methods [6,7,8,9,10,11,12] were proposed. But, due to the redundancy of the Laplacian pyramid, the contourlet transform has a redundancy factor of 4/3 and hence, some approaches have been attempted to introduce non-redundant image transforms. Eslami and Radha proposed a new non-redundant image transform [13], the Wavelet-Based Contourlet Transform (WBCT), with a construction similar to the contourlet transform. The proposed WBCT achieves both radial and angular decomposition to an arbitrary extent and obeys the anisotropy scaling law. Compared to the aforementioned DFB-based non-redundant transforms, the WBCT can easily be realized by applying DFB on the wavelet coefficients of an image. But, the DFB stage of WBCT involves downsampling and therefore, it is shift variant. Translation invariance is a required feature in denoising, which can significantly improve the performance [14,15].

In this paper, we propose a new approach for image denoising based on the Wavelet-Based Contourlet Transform (WBCT). To compensate for the lack of translation invariance property of the WBCT, we apply the principle of cycle spinning to the WBCT which can improve the denoising performance of WBCT. Experimental results show that the significant improvements have feasibility and superiority, and demonstrate that the proposed scheme can achieve better PSNR values when compared with the wavelet transform using cycle spinning (WTCS), and visually, the method is capable of better retaining edges and textures in the denoised images.

This paper is organized as follows: In Section II , Wavelet-based Contourlet Transform (WBCT) and basic denoising and cycle spinning are discussed. Details of the proposed denoising method is given in Section III. Experimental results are discussed in Section IV. Concluding remarks are given in Section V.

II. THEORY

A. The Wavelet-based Contourlet Transform

The contourlet transform based on a multiscale and multidirectional filter bank developed by Do and Vetterli,
is one of the new geometrical image transforms, which can capture nearly arbitrarily directional information of the natural images. This transform consists of two major stages: the subband decomposition and the directional transform. At the first stage, Laplacian pyramid (LP) is employed, while directional filter banks (DFB) are used for the second stage. The main feature of this transform is the potential to efficiently deal with 2-D singularities such as edges, unlike wavelet which manage point singularities exclusively. We can attribute the success to the two main properties of contourlet transform: (1) the directionality property, i.e. having basis functions at many directions, whereas wavelet has only 3 directions; (2) the anisotropy property, meaning that the basis functions appear at various aspect ratios (depending on the scale), as opposed to equal aspect ratio of wavelet. The main advantage of the contourlet transform over other geometrical-driven representations, e.g. curvelet, is its relatively simple and efficient and wavelet-like implementation using iterative filter banks. Due to its structural resemblance with the wavelet transform, many image processing tasks applied on wavelet can be seamlessly adapted to contourlet. But, the contourlet transform is a redundant image transform based on LP. The Wavelet-Based Contourlet Transform (WBCT) developed by Eslami and Radha, is a new non-redundant image transform with a construction similar to the contourlet. It also consists of two filter bank stages: the first stage provides subband decomposition using wavelet transform rather than the Laplacian pyramid; The second stage of the WBCT is a directional filter bank (DFB), which provides angular decomposition. The graph of the detail decomposition process of CT and WBCT is shown in Fig.1. In the first stage of WBCT, the image is decomposed into the low frequency subband corresponding to the LL and the traditional three highpass bands corresponding to the LH, HL, and HH. In the second stage, perform DFB with the same number of directions on each high frequency subband in a given level, and start from the desired maximum number of directions on the finest level, and decrease the number of directions at every other dyadic scale when proceeded through the coarser levels.

By means of this way, the anisotropy scaling law is achieved. Moreover, the wavelet filters are not perfect in splitting the frequency space to the lowpass and highpass components, the scheme using fully DFB decomposition on each band can compensate for the drawbacks of the wavelet filters. Fig.2(a) shows an example of the WBCT using 3 wavelet levels and applying 8,4,2 number of directions to each level from the finest level to the coarse level. Fig.2(b) shows the example of WBCT representation on Peppers image with size of 512*512. For clear visualizing, the image is only decomposed into three levels, which are then transformed into 16,4,4 directional subbands, respectively. Here, small coefficients are shown in black while large coefficients are shown in white. We see that WBCT can produce the significant coefficients in both location and direction of image contours, and most of the coefficients in the LH subbands are in the horizontal directional subbands (the lower half of the subbands) while those in the HL subbands are in the vertical directional subbands (the upper half of the subbands).

Because wavelet transform has more advantages than LP decomposition in representing sparsely image, so WBCT can capture image structure features more efficiently. It has been shown to be a better alternative choice than the contourlet transform for image denoising.

B. Basic Denoising

Image denoising by wavelet thresholding was introduced by Donoho and Johnstone in [14]. Let \( x \) be a noiseless image and \( y \) be the image corrupted image with independent Gaussian noise \( n \), the noise-corrupted image

![Diagram of CT and WBCT decomposition](image-url)
The plot of WBCT using 3 wavelet levels and 8 directions at the finest level.

Figure 2. A flow graph of WBCT scheme

is defined as follows:

\[ y = x + n \]  

(1)

Perform the orthogonal wavelet transform on the noise-corrupted image \( y \) and then zero out the detail coefficients that fall below a certain threshold, which are likely to contain mostly noise. An inverse wavelet transform is applied to the thresholded image to yield an estimate for the noise-free image \( x \). Threshold \( T \) can be calculated as below:

\[
T = \sigma \sqrt{2 \log N}
\]

(2)

\[
\sigma = \frac{\text{median}(d_i)}{0.6745}
\]

(3)

where \( N \) is length of the corresponding data, \( d_i \) represents all of the coefficients in the finest level. The wavelet threshold denoise includes hard threshold denoise and soft threshold denoise. Hard-threshold consists of replacing each wavelet coefficient \( d_i \) by the value

\[
d_i' = \begin{cases} 
  d_i, & \text{if } d_i > T \\
  0, & \text{else}
\end{cases}
\]

(4)

Each wavelet coefficient \( d_i \) of Soft-threshold is replaced by the value \( d_i' \), where

\[
d_i' = \begin{cases} 
  \text{sgn}(d_i)(|d_i| - T), & \text{if } d_i \geq T \\
  0, & \text{else}
\end{cases}
\]

(5)

This method was remarkable for its simplicity, it performs well under a number of applications of image denoising. But since the denoise principle of threshold is to set finescale wavelet coefficients to zero, and the feature also lies in the high frequency bands, it brings about visual artifacts.

Since the wavelet-based contourlet transform with a construction similar to the wavelet transform, has the compaction property that there is only a small number of large coefficients, and all the rest coefficients are very small. We can apply threshold-based image denoising to the domain of wavelet-based contourlet transform, however, it can not escape to eliminate visual artifacts or blur the feature. What should we do to resolve it?

C. Cycle Spinning

The wavelet-based contourlet transform and wavelet transform are not time invariant. Consequently, if the noisy image is shifted in time, denoised, and then shifted back, the result will, in general, be different from the estimate obtained from denoising without shifting. One way of improving upon basic wavelet transform denoising is through cycle spinning, where the image to be denoised is translated by various time shifts. For a shift variant wavelet transform \( P \), if the following procedure is applied to the noisy image \( y \):

\[
x^\wedge_{k_1,k_2} = \frac{1}{k_1k_2} \sum_{i=0,j=0}^{k_1,k_2} X_{i,j}^\wedge (P^{-1}(T[P(X_{i,j}(y))]))
\]

(6)

\[
x = \frac{1}{k_1k_2} \sum_{i=0,j=0}^{k_1,k_2} x^\wedge_{i,j}
\]

(7)

Where \( X_{i,j} \) and \( T \) respectively represent the 2-D circulant shift and its threshold operators. \( k_1k_2 \) are the maximum number of shifts on the row and col direction, \( P^{-1} \) expresses the inverse transform. The cycle spinning estimate \( x^\wedge \) is obtained by simply linearly averaging the \( k_1k_2 \) estimates (As in (7)). The errors in the estimates are not completely dependent. Consequently averaging these estimates yields a reduction in noise. Similar to wavelet, if an image of size \( N \times N \) is decomposed using WBCT, the maximum number of decomposition levels in the scale stage will be \( k(N=2^k) \), and therefore, the maximum number of shifts are \( k \times k \) in the row and column directions. We applied this procedure to WBCT and
achieved superior performance in our denoising experiments as briefly demonstrated in Section IV.

III. ALGORITHM DESCRIPTION

Since the WBCT can decompose image in multi-scale and multi-direction, and using the idea of cycle spinning, we know that the edge can be kept best when the anisotropic filter’s long axes is in accord with the edge, and with the angle between the edge and the anisotropic filter’s long axes becomes larger the edge becomes blurrier. The DFB decomposition can be seen as the DFB filter’s long axes is placed at different orientations and which resulting in the many directional subbands. Every directional subbands has a decomposition direction and the edge in this direction has a largest gray scale comparing with which in any other direction.

On the base of the idea that the coefficients of image WBCT lying in the edges is larger than the noise WBCT coefficients, we can compare the WBCT coefficients with a threshold and kill the smaller coefficients to remove the noise from image.

The whole denoising algorithm consists of 6 steps as follows:
1) Shift the image in each direction, and the maximum numbers of shifts will be $k(N=2^k)$;
2) Perform WBCT on the shifted image;
3) Apply hard thresholding to the resulting coefficients;
4) Perform inverse WBCT on the thresholded coefficients;
5) Implement inverse shift the processed image, and get the noise-free image $x_{i,j}$;
6) Average over the all results using (7) to get the denoised image $\hat{x}$.

IV. EXPERIMENTAL RESULTS

To evaluate the proposed scheme, we applied four approaches to four images with size of $512 \times 512$: Barbara, Lena, Cameraman and Peppers. The four approaches are: the wavelet transform (WT), the translation invariant wavelet transform (WTCS) and the wavelet-based contourlet transform (WBCT), in addition to the proposed method based on WBCT using cycle spinning (WBCTCS). We used biorthogonal Daubechies 9/7 wavelet transform. For the first stage of WBCT, we also used the same biorthogonal filters and applied 4 levels and 8 directions decomposition at the finest level. The images are corrupted by a zero-mean Gaussian noise with a standard deviation of $\sigma$, ranging from 20 to 100. Since hard-thresholding usually yields better results than soft-thresholding, we used hard-thresholding with a fixed...
threshold value equal to $T=3\sigma$ [Ref.14], here. Table I shows the PSNR value achieved by the four methods.

<table>
<thead>
<tr>
<th>Method</th>
<th>Lena</th>
<th>Barbara</th>
<th>Cameraman</th>
<th>Peppers</th>
</tr>
</thead>
<tbody>
<tr>
<td>WT</td>
<td>28.4322</td>
<td>27.4251</td>
<td>29.7387</td>
<td>30.7387</td>
</tr>
<tr>
<td>WBCT</td>
<td>28.7156</td>
<td>27.9530</td>
<td>29.8135</td>
<td>30.9357</td>
</tr>
<tr>
<td>WTCS</td>
<td>29.1364</td>
<td>28.3121</td>
<td>30.9251</td>
<td>31.4459</td>
</tr>
<tr>
<td>WBCTCS</td>
<td>30.1378</td>
<td>29.1024</td>
<td>31.7154</td>
<td>31.9269</td>
</tr>
</tbody>
</table>

Table I. PSNR VALUES FOR THE FOUR IMAGES APPLIED TO THE VARIOUS METHODS ($\sigma=20$)

It is clear that the use of the new method improves the quality of denoising image. For the Cameraman and Peppers image, which are more smooth images, and hence they are "wavelet-friendly" images, the WTCS performs almost the same as the WBCTCS at a range of the input noise power. However, in case of the images containing mostly textures and contours such as the Barbara and Lena images, the WBCTCS yields significant improvements up to 0.7 dB over the WTCS. The PSNR vs. standard deviation curves for the images are provided in Fig.3. Our method is seen to clearly
outperform the other three over the entire range of noise levels, it achieved better results at all cases. To visually compare the estimated images, we show the denoised images of Barbara image when $\sigma=20$ in Fig.4. We can see that there are many visual artifacts in the experimental result of the WBCT due to the Gibbs-like phenomena; however, we could reduce these artifacts to a large extent and achieve the superior PSNR values by using our proposed method. To compare visibility of the artifacts of the various denoising methods, Fig.5 and Fig.6 show two small different part of the Barbara image. As can be seen, Our method provides fewer artifacts as well as better preservation of edges and other details. Significantly more levels of detail and texture are retrieved by the proposed scheme, most of the visual artifacts due to the Gibbs-like phenomena in the WBCT denoising are reduced by using cycle spinning. From the comparison of the outcomes of both sets of experiments, we may conclude that our method contributes significantly to the performance advantage.

V. CONCLUSION

We propose an efficient method of image denoising. We utilize the cycle spinning algorithm in developing a translation invariant WBCT-based denoising. Our experimental results clearly demonstrate that the proposed scheme can provide smoothness and better edge preservation, especially for those images possessing detailed textures. We can eliminate most of the visual artifacts resulting from the wavelet transform denoising function that uses hard thresholding. The experimental evaluation showed that the proposed methods have far better performance than the translation invariant wavelet denoising.

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Figure 6. Comparison of denoising results of Barbara image (cropped to 200×200 for visibility of the artifacts).


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