Robust Portfolio Optimization with Options under VE Constraint using Monte Carlo

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Abstract—this paper proposes a robust portfolio optimization programming model with options. Under constraints of variance efficiency and shortfall preference structure, we derive optioned portfolios with the maximum expected return of robust counterpart. A numerical example using Monte Carlo illustrates some of the features and applications of this model.

Index Terms—Robust portfolio optimization; VE constraint; Monte Carlo

I. INTRODUCTION

The main problem an investor faces is to make an optimal portfolio. The classical portfolio model is generally only associated with stocks. The derivative instruments are no more considered as the hedging instruments, but now they are considered as the investment instrument. For example, options are the derivative instruments which can increase the liquidity and flexibility of return from the investment. And at the same time, they also can be regarded as an asset to be invested.

Numerous studies have investigated the integration of options in portfolio optimization models. Alexander, Coleman and Li (2006) [1] analyzed the derivative portfolio hedging problems based on value at risk (VaR) and conditional value at risk (CVaR). Papahristodoulou[2] proposed optioned portfolio model, and based on Black-Scholes (B-S) formula, they derived the values of all the Greek letters of the portfolio $\Delta, \Gamma, \Theta$ to hedge risk.

Their objective was to maximize the difference between the theoretical value and the market value of a portfolio with options. And they transformed the problem to a linear programming model. Their model is simpler, but it is intractable. Horasanli[3] extended the model proposed by Papahristodoulou to a multi-asset setting to deal with a portfolio of options and underlying assets. Gao[4] also extended the existing literature on options strategies. With the model and the method they mentioned, the investors can take the options strategies in terms of one’s subjective personality, and meanwhile, adjust the risks to suit the needs of the market change. Gerhard Scheuenstuhl, Rudi Zagst[5] examined the problem of managing portfolios consisting of both, stocks and options. However, the target function of their models associated with the stochastic properties of the portfolio return, which is intractable. Because we have to deal with the stochastic dynamics price model of the expected final portfolio value.

The mentioned above are related to the problem of parameter estimation. However, the framework requires the knowledge of some inputs, such both mean and covariance matrix of the asset returns, which practically are unknown and need to be estimated. The standard approach, ignoring estimation error, simply treats the estimates as the true parameters and plugs them into the optimal portfolio optimization model. But most frequently the uncertain parameters play a central role in the analysis of the decision making process. So the peculiarity of these parameters cannot be ignored without the risk of invalidating the possible implications of the analysis Wets [6].

During the last two decades, the idea of robust optimization has become an interesting area of research. Soyster [7] is the first who introduced the idea of robust optimization, but his idea turns to be very pessimistic which makes it unfavorable among practitioners. Ben-Tal and Nemirovski [8] developed new robust methodology where the optimal solution is more optimistic. Their idea uses interior point based algorithm to find the robust solution on a counterpart of the initial model. They also apply their robust method on some portfolio optimization problems and show that the final optimal solution remains feasible against the uncertainty on different input parameters. Steve Zymler proposed a novel robust optimization model for designing portfolios that include European-style options. This model trades off weak and strong guarantees on the worst-case portfolio return. The weak guarantee applies as long as the asset returns are realized within the prescribed uncertainty set, while the strong guarantee applies for all possible asset returns. Nemirovski[9] proposed robust portfolio selection under ellipsoidal uncertainty. There is rare literature about robust portfolio optimization with options as far as we know. Steve Zymler[10] proposed a novel robust optimization model for designing portfolios that include European-style options, extending robust portfolio optimization to accommodate options. But they only paid attention to portfolio return and ignored risk. Ai-fan Ling.etc [11] proposed robust portfolio selection models under so-called “marginal + joint” ellipsoidal uncertainty set and to test the performance of the
proposed models. In their paper one more robust portfolio selection model with option protection is proposed by combining options into the robust portfolio selection model. This paper considers the optioned robust portfolio return.

The rest of the paper is organized as follows. In section 2 we review robust portfolio optimization. In Section 3 we show how a portfolio that contains options can be modeled in a robust optimization framework. Section 4 gives an example based on Monte Carlo simulation to illustrate the application of the model and the method. Conclusions are also drawn.

II. ROBUST PORTFOLIO OPTIMIZATION

We consider the portfolio includes several European call options and puts options on different stocks. This portfolio makes extensive use of options to achieve the desired payoff profile. As we all know, the return of options depends on the return of the corresponding underlying stocks. And the inputs such as mean or variance are uncertain, which is lead to the returns of option are uncertain. However, if the uncertain sets of underlying inputs are determined, the ones of options are corresponding to. Mostly portfolio model integrated into options are only emphasized on portfolio return at the end of the investment horizon. Due to the resulting asymmetric portfolio return distribution mean–variance analysis will be not sufficient to identify optimal optioned portfolios. From the second half of the last century, options have been praised for their ability to give stock holders protection against adverse market fluctuations. A standard option contract is determined by the following parameters: the premium or price of the option, the underlying security price, the expiration date, and the strike price. A put (call) option gives the right to sell (buy) from the option writer the underlying security by the expiration, that is, at time T. We will pay attention to these options and volatility.

A. An Introduction to Option Pricing

It is necessary to introduce call option first. Suppose an investor is presented with an opportunity to enter into a position in a European call option written on a stock, with strike price K and expiration date T. The stock price process is assumed to follow a geometric Brownian motion with mean rate of return \( \mu > 0 \) and volatility \( \sigma > 0 \):

\[
dS_t = \mu S_t dt + \sigma S_t dW_t
\]

where \( \{W_t, t \geq 0\} \) is a standard Brownian motion with \( W_0 = 0 \). The basic model for call option of the B–S is:

\[
C_t = SN(d_1) - Ke^{-\rho T}N(d_2)
\]

\[
d_1 = \frac{\ln(S_t/K) + (\rho + \sigma^2/2)T}{\sigma\sqrt{T}}
\]

\[
d_2 = d_1 - \sigma\sqrt{T}
\]

where

\[
C \quad \text{call option price;}
\]

\[
S \quad \text{current stock price;}
\]

\[
K \quad \text{striking price;}
\]

\[
\rho \quad \text{riskless interest rate;}
\]

\[
T \quad \text{time until option expiration;}
\]

\[
\sigma \quad \text{standard deviation of return on the underlying security;}
\]

\[
N(d_1) \quad \text{cumulative normal distribution function evaluated at } d_1.
\]

The same as put option:

\[
P_t = Ke^{-\rho T}N(d_1) - SN(d_1)
\]

where \( P \) is put option price.

The meanings of the rest letters are similar to the formers.

Next, we will improve B-S formula using analytical method. It is well known that the basic assumption of B-S model is to assume the underlying price follows Geometric Brown motion:

\[
dS_t = \mu S_t dt + \sigma S_t dW_t
\]

Call option is an option is a security that gives its owner the right to trade in a fixed number of shares of a specified common stock at a fixed price at any time on or before a given date. The act of making this transaction is referred to as exercising the option. The fixed price is termed the strike price, and the given date, the expiration date. A call option gives the right to buy the shares; a put option gives the right to sell the shares.

For an European call option its value at the expired time T is

\[
C_T = (S_T - K)^+
\]

Because the future is uncertain, it is stochastic. And we need to know the current value of option. So it should to deduce from its expectation \( E(S_T - K)^+ \).

The financial market is perfect, that is the current value is equal to the discount of future value.

\[
C_0 = e^{-\rho T} E(S_T - K)^+
\]

Now, to calculate the expectation based on the hypothesis of lognormal distribution.

\[
E(S_T - K)^+ = E \left( S_0 e^{\sigma \sqrt{T} Z} \left( e^{\frac{\rho T}{2}} - K \right) \right)
\]

where \( Z \sim N(0,1) \) whose density function is

\[
f(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}
\]

Let \( S_0 e^{\sigma \sqrt{T} Z} \left( e^{\frac{\rho T}{2}} - K \right) = 0 \) then

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The basic notion follows [12]. Consider a portfolio consisting of quantities \(X = (x_1, x_2, \ldots, x_n)\) of stocks 1, 2, \ldots, \(n\) with the return vector \(R = (r_1, r_2, \ldots, r_p)\). We assume that for each stock there are \(m\) put and \(m\) call options that mature in one year. The \(m\) strike prices of the put and call options for one particular stock are located at equidistant points between 70\% and 130\% of the stock’s current price. \(R^p\) and \(R^c\) are denoted the corresponding calls and puts returns in the portfolio with stock price \(S^k\), \(k = 1, 2, \ldots, m\) means the \(k\)-th strike price based on the \(i\)-th stock, call price \(C^k_i\), and put price \(P^k_i\), whose exercise prices are \(K^k\) and \(\gamma_i\) denote the (decision) variables on the numbers of the corresponding calls and puts option. \(S^0\) denotes the initial price of stock, which then can be expressed as \(S^0\) at the end of the period. Using the payoff functions of call and put options, we can explicitly express the returns of options as:

\[
\begin{align*}
R^c_{i,k} &= \frac{1}{C_{i,k}} \max \left\{ 0, S^k_T - K_{i,k} \right\} = \max \left\{ 0, a_{i,k} + b_{i,k} r_{i} \right\} \\
\text{with } a_{i,k} &= \frac{K_{i,k}}{C_{i,k}} \\
\text{and } b_{i,k} &= \frac{S_0}{C_{i,k}} \\
R^p_{i,k} &= 1 - \max \left\{ 0, K_{i,k} - S^k_T \right\} = \max \left\{ 0, a_{i,k} + b_{i,k} r_{i} \right\} \\
\text{with } a_{i,k} &= \frac{S_0}{P_{i,k}} \text{ and } b_{i,k} = \frac{K_{i,k}}{P_{i,k}}
\end{align*}
\]

Similarly, the return of a put option is

\[
R^p = \frac{1}{P^p_{i,k}} \max \left\{ 0, K^p_{i,k} - S^p_T \right\} = \max \left\{ 0, a^p_{i,k} + b^p_{i,k} r_{i} \right\}
\]

where \(P^p_{i,k}\) will be calculated from Black–Scholes formula.

Within this investment framework, the value a portfolio at the expired time the investor wishes to maximize can thus be formulated as:

\[
\max F = \sum_{i=1}^{n} \left[ x_i r_i + \sum_{k=1}^{m} \beta^i_k R^c_{i,k} + \gamma^i_k R^p_{i,k} \right]
\]

Constrains concluded in this paper will be developed based on [13], whose model also contained in optioned portfolio. The risk-return preferences of the investor are specified as mean–variance efficiency with additional shortfall constraints expressing the downside risk preferences.

\[
(I - L) w^p_{ij} = C^p_{ij} r_{ij} \text{ and } Q(V \geq B(\alpha)) \geq 1 - \alpha
\]

where the meanings of the parameters are explained as: \(w^p_{ij}\) is the share vector of stocks, call options and put options. Set \(L = C^{-1} r\) and \(I\) being the matrix with 1 in the diagonal and 0 else. Let \(C\) be the covariance matrix of the (discrete) returns, \(r = (r_1, r_2, \ldots, r_p)\) the vector of expected returns and \(e\) the \(p\)-dimensional vector filled with 1 in each component of the instruments.

The steps of calculating the parameters are followings:

1. Covariance matrix of the (discrete) returns \(C\) is estimated from history data.

2. Expected returns vector \(r = (r_1, r_2, \ldots, r_p)\) is also estimated from history data.

3. \(a = e^C r, b = r^C e, c = e^C e, d = bc - a^2\)

\[
r_a = \left( \frac{b - ar_i}{d} \right)_{i=1,2,\ldots,p}, r_c = \left( \frac{cr - a}{d} \right)_{j=1,2,\ldots,p}
\]

4. \((I - L) w^p_{ij} = C^{-1} r_{ij}\)

The following portfolio optimization problem corresponds to this model.
\[
\max \sum_{i=1}^{n} \left\{ x_i r_i + \sum_{k=1}^{m} \left( \beta_{ik} R_k^c + \gamma_{ik} R_k^p \right) \right\} \\
\text{s.t.} \quad \begin{cases} (I - L) w_{ijp} = C^{-1} r_j \\ Q(V \geq B(\alpha)) \geq 1 - \alpha \end{cases}
\]

C. Parameter Uncertainty

Most of the parameter such as the expected returns and covariance are estimated from noisy data. Hence, these estimates are no accurate. As a result, if the model amplifies any estimation errors, the portfolio holding will extremely perform badly in out-of-sample tests. So it needs to solve this problem. And the robust optimization is a good choice. Generally speaking, robust optimization aims to find solutions to a given optimization problems with uncertain parameters which could achieve good objective values for all or most of realizations of the uncertain parameters. We will assume that the estimate is reasonably accurate such that there is no uncertainty about it. This assumption is justified since the estimation error in expectation by far outweighs the uncertainty about it. This assumption is justified since the covariance is reasonably accurate such that there is no uncertain parameters. We will assume that the estimate objective values for all or most of realizations of the uncertain parameters which could achieve good aims to find solutions to a given optimization problem. And the robust optimization will extremely perform badly in out-of-sample tests. So it amplifies any estimation errors, the portfolios yielding estimates are no accurate. As a result, if the model covariance are estimated from noisy data. Hence, these quantity to an uncertain LP is given as

\[
\max \left[ \min \left( c^T x \right) \right] \\
\text{Subject to} \quad Ax \leq b, \forall (A, b, c) \in U
\]

There are two forms for transfer the robust into a set, linear or Ellipsoidal.

1. Linear interval

In the robust optimization framework, the true value \( a_i \) is not certain which is given by the following equation

\[ a_i = \bar{a}_i + \eta_i, \forall i \]

where \( \bar{a}_i \) is an estimate for \( a_i \), and \( \eta_i \) is the maximum distance that \( a_i \) deviated from \( \bar{a}_i \) and \( \eta_i \) is a random variable which is bounded by and symmetrically distributed within the interval \([-1, 1]\). That is, the true value \( a_i \) is symmetrically distributed with respect to \( \bar{a}_i \) on the interval \([\bar{a}_i - \eta_i, \bar{a}_i + \eta_i]\).

2. Ellipsoidal uncertainty sets are given by

\[
\left\{ \alpha : \sum_{i} \left( \frac{a_i - \bar{a}_i}{\alpha_i} \right)^2 \leq \Omega^2 \right\}
\]

where \( \Omega \) is a user defined parameter and adjusts the trade-off between robustness and optimality.

Next, the problem is how to transfer the uncertainty set to a series equations or in-equations.

Let \( |P| \) be the number of parameters. For Soyster’s and Ben-Tal and Nemirovski’s model [16],

\[
\sum_{i} a_i - \bar{a}_i = |P|
\]

or

\[
\sum_{i} |\eta_i| = |P|
\]

Bertsimas and Sim (2004) relaxed this condition by defining a new parameter \( \Gamma \) the budget of uncertainty as the number of uncertain parameters that take their worst case value \( a_i - \bar{a}_i \). Therefore \[ |\eta_i| \leq \Gamma, \text{such that } \Gamma \in \mathbb{R} \], then the optimal problem can be rewritten as

\[
\max \left( \sum \bar{a}_i w_i + \min \sum \eta_i w_i \right) \\
\text{s.t. } \sum w_i = 1 \\
\sum |\eta_i| \leq \Gamma \\
w_i \geq 0, -1 \leq \eta_i \leq 1, \forall i
\]

It also can be rewritten as

\[
\max \left( \sum \bar{a}_i w_i - \max \sum \eta_i w_i \right) \\
\text{s.t. } \sum w_i = 1 \\
\sum |\eta_i| \leq \Gamma \\
w_i \geq 0, 0 \leq \eta_i \leq 1, \forall i
\]

However, this problem is not well-defined. Because it is difficult to obtain a different optimal solution for each return realization, there are multiple ways to specify the linear set. A nature choice is to construct an ellipsoidal uncertainty set

\[
\Theta_r = \left\{ \gamma : (r - \mu)^T \Sigma^{-1} (r - \mu) \leq \delta^2 \right\}
\]

According to El Ghaoui et al [17], when \( r \) has finite second-order moments, then, we can choose

\[
\delta = \sqrt{\frac{p}{1-p}} \text{ for } p \in (0,1) \quad \text{ and } \delta = +\infty
\]

for \( p = 1 \), it means the following probabilistic guarantee for any portfolio \( w \):
\[ P \left\{ w^T \hat{r} \geq \min_{r \in \Theta} w^T r \right\} \geq p \]

The optimal problem reduces to a convex second-order cone program [18]:

\[ \max_w \left\{ w^T \mu - \delta \| \Sigma^{1/2} w \|_{2} \right\} \text{subject to } \| \Sigma^{1/2} w \|_{2} = 1, l \leq w \leq u \]

According to the Central Limit Theorem, it is concluded that the sample mean \( \mu \) is approximately normally distributed. That is, it follows:

\[ \hat{\mu} \sim N \left( \mu, \frac{\sigma^2}{n} \right) \]

Similarly, the ellipsoidal uncertainty set for the mean \( \mu \) can be expressed as:

\[ \Theta_{\kappa} = \left\{ \mu: \left( \mu - \mu \right)^T \left( \Sigma^{1/2} \right)^{-1} \left( \mu - \mu \right) \leq \kappa^2 \right\} \]

where \( \kappa = \sqrt{q/1-q} \) for some \( q \in [0,1) \)

The problem reduces to:

\[ \max_w \left\{ w^T \mu - \kappa \| \Sigma^{1/2} w \|_{2} \right\} \text{subject to } \| \Sigma^{1/2} w \|_{2} = 1, l \leq w \leq u \]

See [19] the problem is finally reduced to:

\[ \max_w \left\{ w^T \mu - \kappa \| \Sigma^{1/2} w \|_{2} \right\} \text{subject to } \| \Sigma^{1/2} w \|_{2} = 1, l \leq w \leq u \]

where \( \Omega = \frac{\Sigma}{n} - \frac{1}{n^2} \left( \Sigma - 1 \right) \left( \Sigma - 1^T \frac{\Sigma}{n} \right) \)

In this paper, firstly, we consider the uncertain set for return mean. We define \( r^* \) as the estimation of real value \( r \) , the uncertainty set \( \Theta \) as:

\[ I = \left\{ r: r^* - s \leq r \leq r^* + s \right\} \]

According to [19], the robust counterpart:

\[ \min \sum r_i x_i \geq r_p \]

can be transferred to the following form:

\[ \sum r_i x_i + \sum s_i m_i \geq r_p \]

The robust counterpart of objective function is:

\[ \max_{x, \beta, \gamma} \sum_{i=1}^{n} \left( \beta_i d_i \gamma_i + \gamma_i^P \right) \]

let \( x_{ad} \) is the share of stock and options.

The goal is to determine the solution of above problem under the constraints.

III. MONTE CARLO SIMULATION AND EMPIRICAL EXAMPLE

A. Monte Carlo Method and the Simulation Process

Comparing with other numerical methods, Monte Carlo simulation has two major advantages: first, more flexible, easy to implement and improvement; secondly, the simulation of estimation error and convergence speed in solving the problem has strong independence of dimension. European option because of its execution time is fixed, not to be executed in advance, therefore it only need to calculate the earnings of the option of each sample path at expiration date, which is available by Matlab programming. [21-23] discuss the application of the simulation methods in various area. Monte Carlo method can overcome the obstacle and we use it further to improve the accuracy of simulated price with the enhancement of reduction variate technique for more complex options whose payoff function is dependent on the underlying asset path and sum of asset is more than one.

Now, we illustrate the key steps in Monte Carlo. It is seen that to draw samples of the terminal stock price \( S(T) \) it suffices to have a mechanism for drawing samples from the standard normal distribution. For now we simply assume the ability to produce a sequence \( Z_1, Z_2, \cdots \) of independent standard normal random variables. Given a mechanism for generating the \( Z_i \), we can estimate \( E\left[ e^{-rT} (S_T - K)^+ \right] \) using the following algorithm:

For \( i = 1, 2, \cdots, n \)

1. generate \( Z_i \)
   \[ S_i(T) = S_0 \exp \left( r \frac{T}{2} - \sigma \sqrt{T} Z_i \right) \]

2. set \( C_i = e^{-rT} (S_i - K)^+ \)
   \[ C_n = \frac{1}{n} \sum_{i=1}^{n} C_i \]

For any \( n \geq 1 \), the estimator \( \hat{C}_n \) is unbiased, in the sense that its expectation is the target quantity:

\[ E\left( \hat{C}_n \right) = C = E\left[ e^{-rT} (S_T - K)^+ \right] \]

The estimator is strongly consistent meaning that as \( n \to \infty \).

In this paper, we suppose \( z = z(t) \) is a random process, the change in a very small time interval \( \Delta t \) is expressed as \( \Delta z \) . If \( \Delta z \) satisfies that \( \Delta z = \epsilon \Delta t \) where \( \epsilon \sim N(0, 1) \) . For different time interval \( \Delta t \) , \( \Delta z \) are independent, then call \( z = z(t) \) follows Wiener process. Suppose the stock price follows \( ds = \mu sd \Delta t + \sigma s \Delta z \), where \( \Delta z \) is the Standard Brown motion. In the practical
application, more accurate simulation not starts from S, but log-price ln S. Monte Carlo simulation steps:
(1) To generate sample paths for underlying asset, given the initial value
\[ S_{t+1} = S_t + \mu S_t\Delta t + \sigma S_t\sqrt{\Delta t} \epsilon_i \]
(2) To calculate option price of each sample path.
(3) To average option price for each sample path.

IV. Empirical Example

In order to illustrate the features and applications of this model, we make a numerical example. For simple, we only consider two stocks. And there is a call and a put option based on each stock. Suppose that the investment horizon is \( T = 1 \) year, including 20 trading days in each month, so there is 240 trading days in total. To divide \( T \) by daily, that is \( \Delta t = \frac{1}{240} \) year. The price of each stock is supposed to follow log-normal distribution, then the price of stock \( i, (i = 1, 2) \) in \( t + 1 \) day is:
\[ S_{t+1}^i = S_t^i + \mu S_t^i\Delta t + \sigma S_t^i\sqrt{\Delta t} \epsilon_i, \quad \epsilon_i \sim N(0,1) \]

Generate a path \( S_{01}^i, S_{11}^i, \cdots, S_{240}^i \) for stock \( i \) by Monte Carlo method.
Each stock will only correspond to a European call option and a European put option, asset specific parameters are as follows:
\( \mu_i = 11\%, \sigma_i = 26.86\%; \mu_2 = 8.05\%, \sigma_2 = 16.3\% \)
Other parameters are as shown in the following. The kind of option, underlying, market price, option price, time and strike price respectively are:

For call option \( C_i \) whose underlying is \( S_i \) with initial value \( S_i(0) = 15.59 \), the option premium \( C_i(0) = 2.17 \), the strike price is \( K_{i1} = 14.5 \), the expired time is 6 month.

For call option \( P_1 \) whose underlying is \( S_1 \) with initial value \( S_1(0) = 15.59 \), the option premium \( P_1(0) = 1.87 \), the strike price is \( K_{12} = 16.5 \), the expired time is 12 month.

For call option \( C_2 \) whose underlying is \( S_2 \) with initial value \( S_2(0) = 13.71 \), the option premium \( C_2(0) = 1.32 \), the strike price is \( K_{21} = 13 \), the expired time is 3 month.

For call option \( P_2 \) whose underlying is \( S_2 \) with initial value \( S_2(0) = 13.71 \), the option premium \( P_2(0) = 1.48 \), the strike price is \( K_{22} = 15 \), the expired time is 9 month.

If the option \( j \) based on stock \( i \) is exercised on the \( l \) (\( l \leq 240 \)) day, the option value is
\[ \max(0, S_{l|j} T^j r_j^l \cdots r_j^l - K_j) \], and in the rest of investment horizon, that is, in the following \( 240 - l \) days, the value is treated as risk free asset, so the total value of the option in the investment horizon is:
\[ \max(0, S_{01} T^j r_1^0 \cdots r_1^0 - K_j) e^{\sigma^2 / 2} + \sigma^2 / 2 - \frac{1}{2} \nu^2 \]
where \( \sigma \) is the risk free interest rate.

The European call option price before expiration day, for example, on the \( v - \Delta t \) day is:
\[ \max(0, S_{v-\Delta t} T^j r_{v-\Delta t} \cdots r_{v-\Delta t} - K_j) e^{-\nu^2 / 2} - \frac{1}{2} \nu^2 \]
where \( C_{v-j} \) is the option current price (option premium) based on stock \( i \).

If \( v > l \), then on the \( v - \Delta t \) day, the call option price is
\[ e^{-\nu^2 / 2} \max(0, S_{v-\Delta t} T^j r_{v-\Delta t} \cdots r_{v-\Delta t} - K_j) \]
Suppose that the portfolio assets real returns are
\( \mu = (\mu_1, \mu_2, r_1, r_2, r_p, r_p, r_p, r_p) \), and the means of samples return are
\( \mu' = (\mu_1', \mu_2', r_1', r_2', r_p', r_p', r_p', r_p') \) with investment share
\( x = (x_1, x_2, w_1, w_2, w_1, w_1) \). We construct the model:
\[ \max \mu_x x_1 + \mu_2 x_2 + r_1 w_1 + r_2 w_2 + r_p w_3 + r_p w_4 \geq 0.001 \]
\[ (I - L) w_{p} = C^{-1} \mu_2 \]
\[ \mu_2 = \frac{b - a \mu}{d}, \mu' = \frac{c \mu - a}{d}, L = C^{-1} \mu_2' \]
\[ x_1 + x_2 + w_1 + w_2 + w_1 + w_1 = 1 \]
\[ \mu' x_1 + \mu_2 x_2 + r_1' w_1 + r_2' w_2 + r_p' w_3 + r_p' w_4 - \sum s_i m_i \geq r_p \]
\[ m_i \geq 0 \]
where \( C \) is the covariance matrix between the assets, the minimum return for an investor is 2%.

By solving the above model, we obtain the optimal portfolio is \( (0.4, 0.04, 0.1, 0.3, 0.16) \), the objective is 0.00682456. If it is set \( x_1 = 0 \), that is there is no robust of return mean, the result is 0.0070175439. It is easy to understand that under robust, investment is more conservative. Because the advantage of combining option in portfolio is option could hedging with risk. In order to test it, we change the variance from small to large, for example, suppose \( \sigma_1 = 30\%; \sigma_2 = 25\% \), we find that the objective is 0.0052984, if there is without options, the objective is 0.000215. That is, options in portfolio could hedge risks.

V. Conclusion

This paper extends the general portfolio model in two aspects. The first is to combined option in the portfolio could hedge the risk, and the options can also considered as an asset in the portfolio, extending the general model. And we use Monte Carlo method to simulate the
option prices. The second point is to propose the model of maximizing the return under constrains of variance efficiency and shortfall preference structure in the robust counterpart, taking account of uncertain inputs. It extends the general portfolio model, putting forward some feasible suggestions to investors.

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