Grey Differential Model of Groundwater Pollution

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Abstract—Contaminant transport in underground system usually occurs in varied flow fields and in anisotropic and heterogeneous media. Because the applicability of analytical solutions is extremely limited for such conditions, and the distribution and transport of pollutants in groundwater are controlled by physics and chemistry and biology courses, which include advection, mechanical disperse, molecule spreading, adsorption, solve and suck, biodegradation function, numerical techniques are essential for underground pollution modeling. Among the numerical techniques, the gray numerical method has become very popular and is recognized as a powerful numerical tool. The distribution and transport of pollutants mentioned by Liu et al. [6], Chen & Wagenet [7], Xu et al. [8] in groundwater are controlled by physical chemistry and biology functions, which include advection, diffusion, dispersion, sorption, decay and biodegradation. In the courses, there is not only the known information but also uncertain information. Therefore, it can be seen as one gray system. Considering the above mechanism synthetically, two-dimensional gray model about river water pollution is built in this paper. It has the significant practical value for the research of gray simulation of river water pollution.

Index Terms—grey model; truncation error; finite differential

I. INTRODUCTION

Gray systematic theory is proposed by Chinese professor Deng Jurong in 1982 [1]. In the theory, there is not only a large amount of known information called white system, but also much unknown and uncertain information called black system. The system including white system and black system is called gray system. Contaminant transport in natural river system usually occurs in varied flow fields and in anisotropic and heterogeneous media. Because the applicability of analytical solutions is extremely limited for such conditions, numerical techniques are essential for

underground pollution simulation. Mecarthy [2], Li et al. [3], Li & Wang[4], Basha & El-Habel [5] made much work about the uncertain issues. Among the numerical techniques, the gray numerical method has become very popular and is recognized as a powerful numerical tool. The distribution and transport of pollutants mentioned by Liu et al. [6], Chen & Wagenet [7], Xu et al. [8] in groundwater are controlled by physical chemistry and biology functions, which include advection, diffusion, dispersion, sorption, decay and biodegradation. In the courses, there is not only the known information but also uncertain information. Therefore, it can be seen as one gray system. Considering the above mechanism synthetically, two-dimensional gray model about river water pollution is built in this paper. It has the significant practical value for the research of gray simulation of river water pollution.

II. MECHANISM OF CONTAMINANT TRANSPORT

The contamination usually occurs at an isolated sites, and then infiltrates the soil and finally into groundwater. Following the groundwater flow, contaminants are transported and dispersed to the surrounding areas. Several processes are involved with this transport, including advection, diffusion, dispersion, sorption, decay and biodegradation.

A. Advection

Contaminant advection is defined as the movement of contaminants with the groundwater flow through the porous media. The gray advection flux is:

\[ \otimes F_x = \otimes v \cdot \otimes c \]  

Where \( \otimes F_x \) is the gray convection flux of pollutant in \( x \) direction (\( mg/(m^2 \cdot s) \)); \( \otimes v \) is the velocity in \( x \) direction(m/s); \( \otimes c \) is the gray density of pollutant (\( mg/m^3 \)).

B. Diffusion

Diffusion is the process of molecular and ionic movements. Because of the irregular moving of molecule, solutes in groundwater move from high density to low
density by diffusion. This course is called molecule diffusion. Even if in the static situation, materials can be transformed to make the interface between pollutant and groundwater gradually smudgy and gradually diffuse with time. The diffusion flux can be got through the first Fick law.

\[ \otimes M_1 = -\otimes D_m \frac{\partial \otimes c}{\partial x} \]  

Where \( \otimes M_1 \) is the gray molecule spreading flux, \((g/(m^2 \cdot s))\); \( \otimes D_m \) is Coefficient of molecule spreading \((m^2/s)\).

C. Dispersion

Dispersion is a mixing process during the computation of contaminant advection. The mean velocity and density represent the distribution value for simplicity. Generally, dispersion is recognized as an irreversible course. By analysis, the gray flux can also be described by the first Fick law, that is:

\[ \otimes M_2 = -\otimes D_o \frac{\partial \otimes c}{\partial x} \]  

Where \( \otimes M_2 \) is dispersion flux in x direction \((g/(m^2 \cdot s))\), \( \otimes D_o \) is Coefficient of dispersion \((m^2/s)\).

D. Radioactive Decay

Substances containing radioelement in the groundwater will have disintegration with time so as to reduce density. The rule of Radioactive decay can be described as follows:

\[ \otimes c = \otimes c_0 \exp(-\lambda f) \]  

So \[ \frac{\partial (\otimes c)}{\partial t} = -\lambda_o (\otimes c) \]  

E. Adsorption and Desorption

Absorption and desorption is an appearance which occurs in the cross section between solid and liquid phase. Solute in liquid phase may be absorbed by solid phase, while, solute in solid phase can enter liquid phase by the effect of solute and ion exchange. Here, we use Henery absorption constant temperature formula to calculate:

\[ \otimes s = (\otimes k)(\otimes c) \]  

Where \( \otimes s \) is gray concentration of solid when equilibrium of absorption arrives (ug/g), \( \otimes k \) is experience constant, which is relevant to factors such as water temperature and nature of pollutant and so on; \( \otimes c \) is gray concentration of pollutants in groundwater environment when absorption equilibrium arrives.

F. Biodegradation

Organic contaminants in groundwater system degrade into inorganic matters and synthesize new cell under the effects of microbial population gradually, which makes the concentration of organic contaminants decreased. Degradation velocity of microbes and pollutants is relevant to biochemical action. The velocity has a vital and common sense to anticipate and control of organic contaminants.

Biodegradation velocity of organic contaminants can be described as follows:

\[ \frac{d(\otimes C)}{dt} = -\frac{v_m (\otimes C)}{K_c + (\otimes C)} \]  

Where \( X \) is concentration of microbes at present time (mg/l), \( K_c \) is constant. Concentration of organic contaminants in groundwater is generally low, that’s to say, \( \otimes C \) is much smaller than \( K_c \), the formula above can be transformed to:

\[ \frac{d(\otimes C)}{dt} = -\frac{v_m X}{K_c} \]  

Because microbes are short of food under the environment of low contaminant concentration, increment speed of organic contaminants is rather little, which means that concentration of microbes is turning balanced population. \( \frac{v_m}{K_c} = k_1 \), here, \( k_1 \) can be treated as a constant, so

\[ \frac{d(\otimes C)}{dt} = -(\otimes k_1)(\otimes C) \]  

G. Retardation

Sorption results in retardation which is the phenomenon that the contaminant solute does not move as fast as the general groundwater flow. Transportation of organic chemicals in geological medium is influenced by groundwater flow, adsorption and desorption, ion exchange, chemical precipitation/solution, mechanical filtration and many other physico-chemical reactions. The transportation way is nearly the same with the way of groundwater flow. Transportation velocity of contaminants has the relationship with the velocity of groundwater flow as follows:

\[ \frac{d(\otimes v^\prime)}{dt} = \otimes v/\otimes R_d \]  

Where \( \otimes R_d \) is the gray retardation coefficient of pollutants in porous medium

III. ESTABLISHMENT OF FINITE DIFFERENTIAL EQUATION OF GROUNDWATER POLLUTION

Assuming pollutant particles and river environment particles have the same hydromechanics characteristics. On the basis of discussing parameters and variables, the finite differential equation of air pollution is educed according to the principles of mass conservation and energy conservation. The process is as follows.
As shown in Fig. 1, pollutant inputting of each volume element in X direction at unit time is

\[ u_x \cdot c + \left( -D_{ss} \cdot \frac{\partial c}{\partial x} \right) \Delta y \Delta z = \Delta \mathcal{Y} \Delta z \]

Pollutant outputting of each volume element in X direction at unit time is

\[ u_x \cdot c + \left( -D_{ss} \cdot \frac{\partial c}{\partial x} \right) \Delta y \Delta z = \Delta \mathcal{Y} \Delta z \]

So the pollutant change quantity in X direction at unit time is

\[ \Delta \mathcal{Y} \Delta z \]

Similarly, the pollutant change quantities in Y,Z direction at unit time respectively are

\[ \Delta \mathcal{X} \Delta z \]

If pollutant takes place attenuation reactions and contains sources and sinks in the volume element, corresponding pollutant change quantity will be \( (S - \delta k \cdot \Delta c) \Delta x \Delta y \Delta z \). Thereupon, in the unit time pollutant change quantity of the volume element is

\[ \frac{\partial (u_x \cdot c)}{\partial t} + \left( -D_{ss} \cdot \frac{\partial c}{\partial x} \right) \Delta y \Delta z = \Delta \mathcal{Y} \Delta z \]

In a homogeneous flow field, \( u_x \) and \( E_y \) are the non-dynamic volumes, which can be taken as constants. Order the volume element \( \Delta x \Delta y \Delta z = 1 \), then the following equation can be obtained

\[ \Delta c = E_y \cdot \frac{\partial^2 c}{\partial x^2} + E_x \cdot \frac{\partial^2 c}{\partial y^2} + E_z \cdot \frac{\partial^2 c}{\partial z^2} - u_x \cdot \frac{\partial c}{\partial x} - u_y \cdot \frac{\partial c}{\partial y} - u_z \cdot \frac{\partial c}{\partial z} + s \cdot k \cdot c \]

Where, \( c \) —— the concentrations of pollutants, \( mg/m^3 \);

\[ E_y \] —— the grey diffusion coefficients in the landscape orientation, \( m^2/s \);

\[ u_x \] —— the grey wind speed in the predominant direction (vertical), \( m/s \);

\( s \) —— the sources and sinks of air pollutants, \( mg/m^3 \cdot s \);

\( k \) —— the attenuation coefficient of air pollutants, \( s^{-1} \);

\( t \) —— the migration time of air pollutants, \( s \);

According to the above analysis, the governing gray equation of groundwater pollution is:

\[ \frac{\partial (\mathcal{C} c)}{\partial t} = \frac{\partial^2 (\mathcal{C} c)}{\partial x^2} + \frac{\partial^2 (\mathcal{C} c)}{\partial y^2} + \frac{\partial^2 (\mathcal{C} c)}{\partial z^2} \]

The initial and boundary condition:

\[ \mathcal{C} c(x, 0) = 0 \quad x > 0 \]

\[ \mathcal{C} c(0, t) = c_0 \quad t \geq 0 \]

\[ \mathcal{C} c(\infty, 0) = 0 \quad t = 0 \]

Using finite difference equation to replace differential equation as followings:

\[ \frac{\partial (\mathcal{C} c)}{\partial t} = \left( \frac{\mathcal{C} c}{\Delta t} \right)^{i+1} - \left( \frac{\mathcal{C} c}{\Delta t} \right)^{i} \]

\[ \frac{\partial^2 (\mathcal{C} c)}{\partial x^2} = \left( \frac{\mathcal{C} c}{\Delta x} \right)^{i+1} - 2 \left( \frac{\mathcal{C} c}{\Delta x} \right)^{i} + \left( \frac{\mathcal{C} c}{\Delta x} \right)^{i-1} \]

\[ \frac{\partial^2 (\mathcal{C} c)}{\partial y^2} = \left( \frac{\mathcal{C} c}{\Delta y} \right)^{i+1} - 2 \left( \frac{\mathcal{C} c}{\Delta y} \right)^{i} + \left( \frac{\mathcal{C} c}{\Delta y} \right)^{i-1} \]

Because the finite difference approach uses limited developments of derivatives, it is only an approximation of partial differential equations leading to truncation errors. Truncation errors affect the accuracy of numerical simulations. A Taylor series expansion of \( c \) about any grid point is used to determine the form of truncation errors [7], [9], [10]. If terms of third and higher orders are neglected, then:

\[ \left( \frac{\mathcal{C} c}{\Delta t} \right)^{i+1} \approx \left( \frac{\mathcal{C} c}{\Delta t} \right)^{i} + \frac{\partial^2 (\mathcal{C} c)}{\partial t^2} \Delta t + \frac{\partial^3 (\mathcal{C} c)}{\partial t^3} \Delta t^2 + \ldots \]

\[ \left( \frac{\mathcal{C} c}{\Delta x} \right)^{i+1} \approx \left( \frac{\mathcal{C} c}{\Delta x} \right)^{i} + \frac{\partial^2 (\mathcal{C} c)}{\partial x^2} \Delta x + \frac{\partial^3 (\mathcal{C} c)}{\partial x^3} \Delta x^2 + \ldots \]

The second-order temporal derivative of \( c \) is written in terms of spatial derivatives using the differentiated form of \[ \text{[11]} \]. The transport parameters are assumed to be constant within each combination of time and space increments in the finite difference calculations. Thus to second order accuracy:
\[
\frac{\partial^2 (\sigma c)}{\partial t^2} = \left[(\partial u)^2 - 2(\partial k)(\partial D_{xy})\right] \frac{\partial^2 (\sigma c)}{\partial x^2} - 2(\partial k)\partial (\sigma c) + \frac{\partial^2 (\sigma c)}{\partial y^2} + 2(\partial u)\partial (\partial k)\partial (\sigma c) + (\partial k)^2 \frac{\partial^2 (\sigma c)}{\partial y^2}.
\]

Equation (8) may then be written as:

\[
\frac{\partial (\sigma c)}{\partial t} \left[\frac{\partial (\sigma c)}{\partial x} + (\partial u)\partial (\partial k)(\partial (\sigma c))\right] = \frac{\partial^2 (\sigma c)}{\partial x^2} - (\partial u)\partial (\partial k)\partial (\sigma c) + (\partial k)^2 \frac{\partial^2 (\sigma c)}{\partial y^2}.
\]

namely:

\[
(\partial D_{xy})^2 = (\partial D_{xy}) - \frac{\Delta t}{2} [(\partial u)^2 - 2(\partial k)(\partial u)].
\]

\[
(\partial D_{xy}) = (\partial D_{xy}) + \Delta t(\partial k)(\partial D_{xy})
\]

\[
(\partial u) = (\partial u) + \Delta t(\partial k)(\partial u).
\]

\[
(\partial k) = (\partial k) + \frac{\Delta t}{2} (\partial k)^2.
\]

Equation (9) can be simplified as

\[
\frac{\partial (\sigma c)}{\partial t} = (\partial D_{xy})^2 \frac{\partial^2 (\sigma c)}{\partial x^2} + (\partial D_{xy}) \frac{\partial^2 (\sigma c)}{\partial y^2} - (\partial u) \frac{\partial (\sigma c)}{\partial x^2} + a(x, y, t).
\]

To remove the induced truncation errors from the finite difference model, the model can be rewritten as

\[
(D_{xy})^2 (\sigma c) = D_{xy}^2 (\sigma c) + (\partial D_{xy})^2 (\sigma c) + \frac{\Delta t}{2} (\partial u)^2 (\sigma c) + \frac{\Delta t}{2} (\partial k)^2 (\sigma c).
\]

\[
(\partial D_{xy}) = (\partial D_{xy}) - \frac{\Delta t}{2} [(\partial u)^2 - 2(\partial k)(\partial u)].
\]

\[
(\partial D_{xy}) = (\partial D_{xy}) + \Delta t(\partial k)(\partial D_{xy})
\]

\[
(\partial u) = (\partial u) + \Delta t(\partial k)(\partial u).
\]

\[
(\partial k) = (\partial k) + \frac{\Delta t}{2} (\partial k)^2.
\]

Where

\[
\begin{align*}
\frac{(\partial E_x)^2}{(\Delta x)^2} &= \sigma A \\
\frac{(\partial E_y)^2}{(\Delta y)^2} &= \sigma B \\
\frac{(\partial u)^2}{2\Delta x} &= \sigma M
\end{align*}
\]

\[
[2(\partial A) + 2(\partial B) + (\partial k) \cdot \Delta t + 1] (\sigma c)_{i,j,k}^{l+1} = [(\partial A) + (\partial M)] (\sigma c)_{i,j,k}^{l+1} - [(\partial A) - (\partial M)] (\sigma c)_{i,j,k}^{l+1} - (\partial B)(\sigma c)_{i,j,k}^{l+1} - (\partial B)(\sigma c)_{i,j,k}^{l+1} = (\sigma c)_{i,j,k}^{l+1}
\]

Adopting the same picking-number with the grey number, the two following equations can be obtained [3].

\[
\begin{align*}
(\partial A) + 2(\partial B) + (\partial k) \cdot \Delta t + 1] (\sigma c)_{i,j,k}^{l+1} = [(\partial A) + (\partial M)] (\sigma c)_{i,j,k}^{l+1} - [(\partial A) - (\partial M)] (\sigma c)_{i,j,k}^{l+1} - (\partial B)(\sigma c)_{i,j,k}^{l+1} - (\partial B)(\sigma c)_{i,j,k}^{l+1} = (\sigma c)_{i,j,k}^{l+1}
\end{align*}
\]

The equation has the special structure, which can be solved by the special method [3,11].

\[
\begin{align*}
&[a_{11} C_{11} + a_{12} C_{21} + a_{13} C_{31} + \cdots + a_{1n} C_{n1}] = f_{11} \\
&a_{21} C_{11} + a_{22} C_{22} + a_{23} C_{32} + \cdots + a_{2n} C_{n2} = f_{22} \\
&\vdots \\
&a_{11} C_{11} + a_{12} C_{22} + a_{13} C_{33} + \cdots + a_{1n} C_{n3} = f_{1n} \\
&[b_{11} C_{11} + b_{12} C_{22} + b_{13} C_{33} + \cdots + b_{1n} C_{n3}] = f_{1b} \\
&b_{21} C_{11} + b_{22} C_{22} + b_{23} C_{33} + \cdots + b_{2n} C_{n3} = f_{2b} \\
&\vdots \\
&b_{11} C_{11} + b_{12} C_{22} + b_{13} C_{33} + \cdots + b_{1n} C_{n3} = f_{1b} \\
&[c_{11} C_{11} + c_{12} C_{22} + c_{13} C_{33} + \cdots + c_{1n} C_{n3}] = f_{1c} \\
&c_{21} C_{11} + c_{22} C_{22} + c_{23} C_{33} + \cdots + c_{2n} C_{n3} = f_{2c} \\
&\vdots \\
&c_{11} C_{11} + c_{12} C_{22} + c_{13} C_{33} + \cdots + c_{1n} C_{n3} = f_{1c}
\end{align*}
\]

Where \( l = m \times n \)

\[
\begin{align*}
a_{11}^{i,j,k} &= 1 + \Delta t \cdot k_{11} + 2 \Delta t \cdot E_{i,j,k} + \Delta t \cdot E_{i,j,k} \\
a_{12}^{i,j,k} &= -\Delta t \cdot E_{i,j,k} + \Delta t \cdot E_{i,j,k} \\
a_{13}^{i,j,k} &= -\Delta t \cdot E_{i,j,k} + \Delta t \cdot E_{i,j,k} \\
a_{1n}^{i,j,k} &= -\Delta t \cdot E_{i,j,k} + \Delta t \cdot E_{i,j,k} \\
b_{11}^{i,j,k} &= 1 + \Delta t \cdot k_{11} + 2 \Delta t \cdot E_{i,j,k} + \Delta t \cdot E_{i,j,k} \\
b_{12}^{i,j,k} &= -\Delta t \cdot E_{i,j,k} + \Delta t \cdot E_{i,j,k} \\
b_{13}^{i,j,k} &= -\Delta t \cdot E_{i,j,k} + \Delta t \cdot E_{i,j,k} \\
b_{1n}^{i,j,k} &= -\Delta t \cdot E_{i,j,k} + \Delta t \cdot E_{i,j,k}
\end{align*}
\]

The two equations can be solved by turn as follows, the gray concentration of groundwater

\[
\begin{align*}
C_{i} &= C_{i}^{0} + \Delta t \cdot S_{i} + \Delta t \cdot E_{i} \\
C_{i} &= C_{i}^{0} + \Delta t \cdot S_{i} + \Delta t \cdot E_{i} \\
C_{i} &= C_{i}^{0} + \Delta t \cdot S_{i} + \Delta t \cdot E_{i}
\end{align*}
\]

IV. Application

A. Grey numerical model for one-dimensional water quality

The mathematic model of solute transport equation can be got as following:

\[
\begin{align*}
\frac{\partial (\sigma c)}{\partial t} = (\partial E) \left[\frac{\partial^2 (\sigma c)}{\partial x^2} - (\partial u) \frac{\partial (\sigma c)}{\partial x} - (\partial k) \frac{\partial (\sigma c)}{\partial x}\right]
\end{align*}
\]

Where \( \sigma c \) — grey concentration of pollutant in section, mg/l; \( \sigma E \) — the grey diffusion coefficients in
the landscape orientation, $m^2/s$; $\otimes u$—grey velocity in section, $m/s$; $t$—time, $s$; $x$—distance, $m$

To any differential equation, its solution only can be solved in special initial condition and boundary condition. In this paper, the constraint condition can be described as followings:

\[
\otimes c(x, 0) = 0 \quad x > 0
\]
\[
\otimes c(0, t) = c_0 \quad t \geq 0
\]
\[
\otimes c(L, t) = 0 \quad t = 0
\]

Using finite difference equation to replace differential equation as followings

\[
\frac{\partial (\otimes c)}{\partial t} = \frac{(\otimes c)^{i+1}_j - (\otimes c)^{i}_j}{\Delta t}
\]
\[
\frac{\partial^2 (\otimes c)}{\partial x^2} = \frac{(\otimes c)^{i+1}_j - 2(\otimes c)^i_j + (\otimes c)^{i-1}_j}{(\Delta x)^2}
\]
\[
\frac{\partial (\otimes c)}{\partial x} = \frac{(\otimes c)^{i+1}_j - (\otimes c)^i_j}{\Delta x}
\]

The transport parameters are assumed to be constant within each combination of time and space increments in the finite difference calculations. Thus to second order accuracy:

\[
\frac{(\otimes E) \cdot \Delta t}{(\Delta x)^2} = \otimes A \cdot \frac{\partial (\otimes u)}{\partial x} = \otimes B
\]
\[
[(\otimes B) - (\otimes A)(\otimes c)^{i+1}_j] + [1 + 2(\otimes A) + (\otimes A)(\otimes c)^{i-1}_j] - (\otimes A)(\otimes c)^i_j = (\otimes c)^{i+1}_j
\]

Adopting the same picking-number with gray number, so the two following equations can be obtained.

\[
([B_2] - [A_1])[c^i_j]^{n+1} + (1 + 2[A_1]) + [A_2][\Delta t][c^i_j]^{n+1} - [A_1][c^i_j]^{n+1} = [c^i_j]^n
\]
\[
([B_1] - [A_1])[c^i_j]^{n+1} + (1 + 2[A_1]) + [A_2][\Delta t][c^i_j]^{n+1} - [A_1][c^i_j]^{n+1} = [c^i_j]^n
\]

The equation has an special structure as followings:
The equation (17) is a five-diagonal matrix. Solving the equation, \( C_{h+1}^i \) (i=1,2,...,m) can be easily got, then put it into the below equation, \( C_{w+1}^i \) can be also obtained.

Considering one-dimensional groundwater pollution, and the groundwater flow and hydro dispersion both are one dimension. When \( t=0 \), the seepage region has no pollutant, at start of \( t=0 \), water containing contaminant (concentration is \( C_0 \)) is injected continuously from the port. In the course of calculation, only the relation of the coefficient of dispersion \([D_a, D_b]\) and \([u_a, u_b]\) with the contaminant density \([C_a, C_b]\) is considered. According to the gray numerical model, the figure 1 can be got. The values of parameters are as follows:

- \( u = 0.4 \text{cm/h} \)
- \( u_a = 0.38 \text{cm/h} \)
- \( u_b = 0.42 \text{cm/h} \)
- \( D = 0.4 \text{cm}^2/\text{h} \)
- \( D_a = 0.38 \text{cm}^2/\text{h} \)
- \( D_b = 0.420 \text{cm}^2/\text{h} \)
- \( k_a = 0.095 \text{cm/h} \)
- \( k_b = 0.105 \text{cm/h} \)

Seen from the figure 1, the curve of contaminant transport by gray numerical model is one “gray strip”. That is to say the value of gray numerical model changes in some ranges, but not one certain value. The analytical solution is rightly within the ranges, which reveals that the method is reliable. At present, the data of hydraulic and water quality in groundwater system is not enough, which provides wide space for the gray mathematic application.

The parameters are listed in table 1. The results are shown in Fig. 1 and Fig. 2.

![Graph](image_url)

**B. Two-dimensional groundwater pollution**

By the two dimensional gray equation of groundwater pollution with instantaneous source, the location of source is at \( X_0=0, Y_0=0 \), the region of seepage is infinite and the groundwater flow is one dimensional, \( Q = nu \).
Fig. 1 and Fig. 2 shows that the curve of contaminant transport by gray numerical model is one "gray strip". That is to say the value of gray numerical model changes in some ranges, but not one certain value. The analytical solution is rightly within the ranges, which reveals that the method is reliable. At present, the data of hydraulic and water quality in groundwater system is not enough, which provides wide space for the gray mathematic application.

TABLE I PARAMETER VALUE USED IN TWO-DIMENSIONAL ADVECTION-DISPERSION EQUATION

<table>
<thead>
<tr>
<th>Caption</th>
<th>Unit</th>
<th>Analytical Solution</th>
<th>Gray Upper Value</th>
<th>Gray Lower Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$u_x$</td>
<td>mm/s</td>
<td>2</td>
<td>19</td>
<td>21</td>
</tr>
<tr>
<td>$c_0$</td>
<td>mg/l</td>
<td>1000</td>
<td>950</td>
<td>1050</td>
</tr>
<tr>
<td>$k_1$</td>
<td>$h^{-1}$</td>
<td>2</td>
<td>1.9</td>
<td>2.1</td>
</tr>
<tr>
<td>$\Delta x$</td>
<td>mm</td>
<td>40</td>
<td>38</td>
<td>42</td>
</tr>
<tr>
<td>$\Delta y$</td>
<td>mm</td>
<td>40</td>
<td>38</td>
<td>42</td>
</tr>
<tr>
<td>$\Delta x$</td>
<td>s</td>
<td>10</td>
<td>9.5</td>
<td>10.5</td>
</tr>
<tr>
<td>$\Delta y$</td>
<td>s</td>
<td>10</td>
<td>9.5</td>
<td>10.5</td>
</tr>
<tr>
<td>$D_{xx}$</td>
<td>mm$^2$/s</td>
<td>50</td>
<td>47.5</td>
<td>52.5</td>
</tr>
<tr>
<td>$D_{yy}$</td>
<td>mm$^2$/s</td>
<td>30</td>
<td>28.5</td>
<td>31.5</td>
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<tr>
<td>$Q$</td>
<td>mm$^2$/s</td>
<td>10</td>
<td>9.5</td>
<td>10.5</td>
</tr>
<tr>
<td>$c^*$</td>
<td>0.2</td>
<td>0.19</td>
<td>0.21</td>
<td></td>
</tr>
</tbody>
</table>

V. CONCLUSIONS

According to the analysis of gray numerical model of groundwater pollution, some conclusions can be gotten as following:

1) It is reasonable and reliable that simulation and prediction of groundwater quality with uncertain information is made by gray mathematic, which provides one new method to simulate and predict the groundwater quality.

2) Compared with analytical solution, some uncertain parameters in gray model such as dispersion coefficient and seepage velocity can be given gray ranges. The result is one gray strip which has great advantages for the application of model and decision-making.

3) The numerical model in this paper also has the common applicability to the surface water pollution and advection-diffusion equation.

VI. ACKNOWLEDGMENTS

This study was funded by the National Basic Research Program of China (2010CB951101), the National Natural Science Foundation of China (Grant No. 40830639, 50879016, 50979022 and 50679018), the program for Changjiang Scholars and Innovative Research Team in University (IRT0717) and the Special Fund of State Key Laboratory of Hydrology-Water Resources and Hydraulic Engineering (1069-50985512, 1069-50986312) and State Key Laboratory of Hydraulics and Mountain River Engineering(SKLH-OFF-0807)

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