A Game Theoretic Resource Allocation Model Based on Extended Second Price Sealed Auction in Grid Computing

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Abstract—In resource-limited environment, grid users compete for limited resources, and how to guarantee tasks’ victorious probabilities is one of the most primary issues that a resource scheduling model cares. In order to guarantee higher task’s victorious probabilities in grid resources scheduling situations, a novel model, namely ESPSA (Extended Second Price Sealed Auction), is proposed. The ESPSA model introduces an analyst entity, and designs analyst’s prediction algorithm based on Hidden Markov Model (HMM). In ESPSA model, grid resources are sold through second price sealed auction. Moreover, to achieve high victorious probabilities, the user brokers who are qualified to participate in the auctions will predict other players’ bids and then carry out the most beneficial bids. The ESPSA model is simulated based on GridSim toolkit. Simulation results show that the ESPSA model assures a higher victorious probability and superior to other traditional algorithms. Moreover, we analyze the existence of Nash equilibrium based on simulation results, thus, any participant who changes its strategy unilaterally could not make the results better.

Index Terms—grid resource scheduling, game theory, extended second price sealed auction

I. INTRODUCTION

The concept of grid technology originates from electricity grid. Idle resources in grids are connected through internet and grid users can access the resources no matter where they are. Grid system is heterogeneous, multi-zone managed, large-scale and distributed. Furthermore, grid system is concerned with a dynamic collection of diverse resources and services across multiple domains, and resources apply and demand is dynamic either. As resources join and exit dynamically, the loads of grid resources change continuously. All these characteristics show that grid system needs a dynamic and high-efficiency resources scheduling algorithm, which can manage dynamic resources flexibly.

The majority of grid systems fall in the category of resource-rich ones, where resources compete for limited tasks. However, sometimes, numerous tasks are submitted at the same time, leading to the undersupplying of resources, which makes tasks compete for limited resources. Whether tasks compete for resources or resources compete for tasks, the resources providers will benefit from assigning resources and the thing that users need to do is paying for the resources. Thus, it is reasonable to model the resource scheduling by economic models. However, current literature on economic grid models only limits to the basic auction models where the bidders bid according to a single factor. Some algorithms integrate simple prediction methods into the bidding algorithms without assuring the victorious probability. Thus, with the increase of the complexity of grid systems, how to improve those simple bidding strategies, prediction algorithms and victorious probability is becoming urgent.

This paper is an extension in terms of both theoretic and simulation results of our conference paper [1]. The major contributions of this paper are as follows.

• To solve the task’s low victorious probability problems using game theory in grid resource scheduling situations, an ESPSA (Extended Second Price Sealed Auction, ESPSA) model is proposed in this paper. An analyst entity is introduced in the model to fulfill the trend that roles in grids are increasing dramatically.

• The ESPSA model extends second price sealed auction mechanism. In the extended auction, the total price is sealed while keep the resource demand quantity unsealed.

• This paper introduces a prediction algorithm based on Hidden Markov Model, improves basic bidding algorithms and increases participants’ victorious probability.

• GridSim toolkit is utilized to simulate the algorithms and the existence of Nash equilibrium is analyzed, and then proved theoretically.

The rest of paper is organized as follows: section II is the related work; section III describes the ESPSA model;
section IV elaborates the simulation and analyzes the simulation results and the computational load of the ESPSA, also the existence of Nash Equilibrium is proved theoretically in this part. Section V draws the conclusion and discusses about the future work.

II. RELATED WORK

At present, resources management and dynamic scheduling based on game theory is becoming a focus of researches in the settings of P2P, grid and cloud computing. A lot of work has been done by researchers, and it will be overviewed in this section from the perspective of grid and cloud computing.

Game theoretic methods have been adopted in settings of cloud and grid computing. In the first place, from the grid computing perspective, Lei Yao and Guanzhong Dai (2008) [2] focus on resource-rich environment. They propose a method, namely GVP, which could guarantee resources’ victorious probabilities. In their method, resources win the auction by predicting other resources’ bids based on the mean value of historical bids. However, the prediction algorithm is too simple to get accurate predictions. Maheswaran and Basar(2003) [3] discuss a divisible auction and adopts a decentralized strategy, but users only make decisions according to resources’ historical load information without consideration of the changing trend of resources’ loads. Kwok and Song (2005) [4] propose a hierarchical grid model, in which resources are selfish. The selfishness of resources makes them only consider their own needs when executing tasks. This kind of action delays tasks’ execution even though the tasks could accurately predict their opponents’ bids. This model couldn’t guarantee tasks’ victorious probabilities. Bredin and Kotz (2003) [5] propose a resources allocation algorithm based on mobile brokers’ competing for resources. In this model, user brokers make an inaccurate prediction for resources, and they neglect the prediction of other user brokers’ behavior. Li Zhijie and Cheng Chuntian (2006) [6] propose a resource allocation model that uses sequential game to predict resource load for time optimization. In this model, user brokers offer their bids in sequence, causing first-mover advantage, which makes the model less popular in practical applications. Abramson and Buyya (2002) [7] propose grid resource scheduling model which only considers the relationship between users and resources without considering the interactions between users. Thus, this model is too idealized to be used in practical circumstances.

Game theoretic solutions also attracted a lot researchers’ attention in the research area of cloud computing [8, 9, 10, 11, 12, 13, 14, 15]. Chonho, et al. (2010) [12] study an evolutionary game theoretic mechanism for adaptive and stable application deployment in cloud computing settings. Their algorithm, called Nuage, allows applications to adapt their locations and resource allocation to the environmental conditions in a cloud. Wei,G., et al. (2010) [13] consider a QoS constrained resource allocation problem, in which service demanders try to solve sophisticated parallel computing problem. Their solution consists of two steps. First, each participant solves its optimal problem independently, without consideration of the multiplexing of resource assignments. Then, an evolutionary mechanism which takes both optimization and fairness into account is designed. Rajkumar Buyya and Manzur Murshed (2002) [14] present a game theoretic method to schedule dependent computational cloud computing services and an evolutionary mechanism is designed. Fei and Frederic (2010) [15] introduce a new Bayesian Nash Equilibrium Allocation algorithm to solve resource management problem in cloud computing. Through experiments, they show that cloud users can receive Nash equilibrium allocation solutions by gambling stage by stage and the resource will converge to the optimal price.

The impact of selfish behavior entities has been paid a lot of attention in self-organized Mobile Ad hoc Networks (MANNEs) and Peer-to-Peer (P2P) networks [16, 17, 18, 19, 20, 23, 24, 25]. In such networks, each node specified as player is under the authority of a self-interested user.

All the literatures [2-25] listed above have made remarkable contributions to economic resources scheduling and allocation algorithm in the settings of grid and cloud computing, but some of them have a low accuracy prediction method for agents to predict tasks’ bids. In response to these issues, a model, namely ESPSA, based on extended second price sealed auction and Hidden Markov Chain prediction method is proposed. In ESPSA model, a mass of tasks compete for limited resources, and tasks decide their best bidding price according to their resource demand quantities, their budgets and predictions of their opponents’ bids. In this way, the ESPSA model will guarantee the tasks a higher victory probability when competing for grid and cloud resources than other methods do in the above literatures.

III. ESPSA RESOURCE ALLOCATION MODEL AND ALGORITHM

The ESPSA model [1] is consisted of extended second price auction algorithm, resource broker’s algorithm, user broker’s algorithm with HMM. Specifically, the extended second price auction sealed the total price of each trade and unsealed the resource demand quantity. By invoking the extended auction, the users’ and resources’ brokers represent as bidders and auctioneers respectively.

In the common second price sealed auction [22], the bidders only submit prices that they wish to pay to the auctioneer without specifying a total price or a unit price. The ESPSA model extends the common second price sealed auction by specifying that the bidders should seal their total price and unseal their resource demand quantity. Then, after each auction, the auctioneer announces the winners and amount of resource they demand. This improvement is reasonable for that the resource broker can attract more customers (resources) by announcing the amount of resources processed in each auction. It is also applicable to seal customers’ total price for privacy reason, because in some reality, the customers are not willing to let others know the total price they paid.
As Figure 1 shows, when a user wants to execute some tasks, it sends the task list to a user broker \( UB_i \). After receiving the task list, in order to be qualified to take part in an auction, \( UB_i \) sends an application message with its expected victorious probability and amount of resource that it demands to resource broker \( RB_i \). If resource broker \( RB_i \) agrees \( UB_i \) to take part in the auction, \( UB_i \) sends prediction request to an Analyst. On receiving the predictions, \( UB_i \) calculates its bid according to the prediction results. After that, it will send its bid to \( RB_i \). Finally, \( RB_i \) sends winners to \( UB_i \) and resource demand quantity of \( UB_i \) to information service entity.

One of the improvements of our model is the analyst entity that we proposed. An analyst gets information from the information service entity, and provides prediction services to user brokers. The analyst entity’s prediction algorithm is based on HMM. Obviously, it satisfies the trend that roles in grids are increasing dramatically and makes the market model of grids computing much more practical.

The detailed user broker algorithm, resource broker algorithm and ESPSA prediction algorithm are described in section A, B and C respectively.

A. Algorithm of Resource Broker

As described above, ESPSA considers a resource-limited grid model where resource brokers need to act as auctioneers who invoke auctions among user brokers. For a resource broker, if it makes a higher expected profit for resources, it will be favored by the resources. Thus, the resource broker should limit the number of the bidders in order to fulfill most of the bidders’ expected victorious probability in one round auction.

We propose a method which classifies the \( S_{\text{hide}} \) into two sets: \( S_{\text{hide}}^{\text{win}} \) and \( S_{\text{hide}}^{\text{los}} \) represent users with an expected victory probability which is higher or lower than the basic victory probability set by the model respectively.

The number of bidders in each round should be calculated by Eq.(1).

\[
N_{\text{bidder}} = \begin{cases} 
S_{\text{win}}^{\text{bidder}}, & \text{in the 1st round auction and } \min(Pr_i^{\text{win}}) > Pr_{RB}^{\text{win}} \\
\frac{1}{Pr_{RB}^{\text{win}}}, & \text{else cases in 1st round} \\
S_{\text{los}}^{\text{bidder}}, & \text{in the 2nd round auction} 
\end{cases}
\]

where \( Pr_i^{\text{win}} \) specifies the expected victorious probability of user broker \( UB_i \), and \( Pr_{RB}^{\text{win}} \) represents the basic victorious probability that resource broker \( RB_i \) assures.

The specific algorithm of resource broker \( RB_i \) is described in Table I.

B. Algorithm of User Broker

User brokers need to represent users to join in the auction for resources. Apparently, user agents with higher victorious probability will get more users. The analyst entity provides predicting services. User brokers can

![Figure 1. ESPSA model](image-url)
predict other user brokers’ bids to get a high victorious probability by using the predicting services provided by an analyst.

Specifically, a user broker calculates its primitive bid by Eq.(2)[1].

\[ b'_i = \frac{y_i}{L} \]  

(2)

Where \( b'_i \) is a user broker’s primitive bid, \( L \) is the load status of the resource and \( y_i \) is a weighting factor \((i = 1, 2, \ldots , i, \ldots n)\).

**TABLE II. USER BROKER’S ALGORITHM**

<table>
<thead>
<tr>
<th>Algorithm: User Broker’s algorithm</th>
<th>User Broker’s algorithm</th>
<th>A task set ( T ) from user.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Input:</td>
<td></td>
<td>Output: Bid ( b ).</td>
</tr>
<tr>
<td>1. Firstly, user broker should apply to join in the auction. Send the applying request along with its expected victorious probability and deadline.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2. If the user broker is divided into the urgent one, then join auction, go to step 3. Else, wait for the second auction, go to step 3.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3. Calculate the primitive bid by Eq.(2)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4. Send predicting request to an analyst.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5. Waiting until received the predicting information ( B'_{\text{obs}} ) from the analyst.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>6. Adjust the bid by Eq.(3).</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Then, \( UB_i \) sends a predicting request to the analyst and gets a set of other user brokers’ bids \( B'_{\text{prev}} \). After that, \( UB_i \) adjusts its bid according to Eq.(3).

\[ b_j = \begin{cases} \max \left\{ \left\{ b'_i \right\} \cup B'_{\text{prev}} \right\} + \alpha, & \text{if} \quad \alpha > \max \left\{ \left\{ b'_i \right\} \cup B'_{\text{prev}} \right\} \\ b'_i, & \text{else} \end{cases} \]  

(3)

Where \( \alpha \) is a user broker’s budget in one round of the auction. \( b'_i \) is the calculated bid of a user broker.

The user agents’ algorithm is listed in Table II.

C. ESPSA Model’s Prediction Algorithm

In ESPSA model, the entity of analyst based on HMM prediction model is introduced. In our HMM prediction model, we use the Viterbi Algorithm[26].

In HMM prediction model, hidden states couldn’t be got directly, but they can be deduced by observation sequences. In general, \( \lambda = (A, B, \pi) \) can be used to represent a HMM, where \( A \) represents a transition probability between initial states, \( B \) represents observation sequence, \( \pi \) represents initial states matrix. Given the definitions of \( A, B \) and \( \pi \), the HMM can generate a hidden sequence: \( O = \{ O_1, O_2, O_3, \ldots , O_n \} \).

In ESPSA model, the resource demand quantities and bids that are submitted by user brokers correspond to observation sequence and hidden states respectively. When an auction launched, each resource broker publishes its attributes, such as process elements’ computing capacity, load status and so on. Each user broker can get the attributes. And then, user brokers use observed resource demand quantities \( D_j = \{ d_{j1}, d_{j2}, \ldots , d_{jn} \} \) to predict their rivals’ bids in next round auction: \( b_j = \{ b_{j1}, b_{j2}, \ldots , b_{jn} \} \). \( d_j \) is the demand quantity that user broker \( i \) of resource \( j \), \( b_j \) is the bid price of user broker \( i \) for resource \( j \). We take five bids for examples, written as level1_bidding, level2_bidding, level3_bidding, level4_bidding and level5_bidding respectively, which correspond to the zones of \( \{ 0, b_{1i} \} , \{ b_{1i}, b_{2i} \} , \{ b_{2i}, b_{3i} \} , \{ b_{3i}, b_{4i} \} , \{ b_{4i}, b_{5i} \} \), where \( b_k \) is a bid, \( k = 1, 2, 3, 4, 5 \).

At time \( t \), for user \( i \), its bid for resource \( j \) is \( b'_{ij} \), and its resource demand quantity for \( j \) is \( d_{ij} \). Suppose that there were \( n \) resources in grid system, according to HMM, the initial states at time \( t \) are:

\[ \pi = \text{states} = \{ b'_{1i}, b'_{2i}, \ldots, b'_{ni} \} \]  

(4)

And at time \( t \), the resource demand quantity matrix that user broker \( i \) observes other user brokers’ is:

\[ B = \text{observations} = \{ d_{i1}, d_{i2}, \ldots, d_{in} \} \]  

(5)

The transition matrix from hidden states at time \( t-1 \) to hidden states at time \( t \) is:

\[ A = \text{transition \_ probability} = \begin{pmatrix} P_{11}^i & \cdots & P_{1n}^i \\ \vdots & \ddots & \vdots \\ P_{n1}^i & \cdots & P_{nn}^i \end{pmatrix} \]  

(6)

The emission matrix between observation sequences and hidden states is:

\[ \text{emission \_ probability} = \begin{pmatrix} P_{11} & \cdots & P_{1n} \\ \vdots & \ddots & \vdots \\ P_{n1} & \cdots & P_{nn} \end{pmatrix} \]  

(7)

where \( P_{ij} \) represents the probability of \( \text{obs}_j \) when in state \( \text{sta}_i \) at time \( t \), \( i = 1, 2, \ldots , n \), \( j = 1, 2, \ldots , n \).

As stated above, in ESPSA model, the hidden states are correlated with observation states by emission matrix. Specifically, given the hidden and observation states corresponding to bidding prices and resource demand quantity respectively, the ESPSA can predict the user brokers’ bids.
IV. Simulation and Results Analysis

The GridSim toolkit [23] is utilized in the simulation of this paper. Resource entities, user entities, broker entities and information about resource demand are considered.

A. Experiment Setup

It is necessary to make some initial setting reasonable by training especially the parameters used in HMM. There exist 300 resources in the resource queue, which means the user brokers may take part in at most 300 auctions. Table III lists the properties of the four resources.

<table>
<thead>
<tr>
<th>Name</th>
<th>Resource characteristics</th>
<th>PE number</th>
<th>A PE MIPS rating</th>
<th>Transmitting Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>R_1</td>
<td>Sun Ultra, Solaris</td>
<td>2</td>
<td>320</td>
<td>2000MB/s</td>
</tr>
<tr>
<td>R_2</td>
<td>Sun Ultra, Solaris</td>
<td>4</td>
<td>350</td>
<td>2000MB/s</td>
</tr>
<tr>
<td>R_3</td>
<td>Intel Pentium Linux</td>
<td>1</td>
<td>370</td>
<td>2500MB/s</td>
</tr>
<tr>
<td>R_4</td>
<td>Intel Pentium Linux</td>
<td>3</td>
<td>390</td>
<td>2500MB/s</td>
</tr>
</tbody>
</table>

Each grid user in this simulation has 2~4 tasks to execute on the grid and each task is with at least 2000MI (million instructions). In order to assure the variety of tasks, the size of each task is set to vary from -10%~+10% of 2000MI.

B. Simulation Methodology

The simulation process consists of two phases: parameter training, iterative rounds of auctions.

1) Parameter training. As the quality of the predictions depend on the model's parameters, these parameters have to be carefully chosen in order to maximize the prediction performance. We adopt Vertibi Training [26] which is an iterative training procedure that derives new parameter values from the observed counts of emissions and transitions in the Viterbi paths of the training sequences for the HMM parameters. In each iteration, a new set of parameter values is derived from the transitions and emissions in the sampled state paths for all sequences in the training set.

When HMM is initialized, the Viterbi segmentation is replaced by a uniform observation sequence. That is, the segmentation is divided into N equal parts. Each iteration, the means and variances are estimated. The training is stopped after a fixed number of iterations or as soon as the change in the log-likelihood is sufficiently small.

2) Equilibrium and winning probability. The winning probability is adopted to estimate the Nash equilibrium. Iterative auctions are simulated in repeated manner, and each round’s winning probability is recorded for future analysis. When the winning probabilities at Nash Equilibrium are available, the results are analyzed by payoff matrices in subsection D.

C. Results

We compare the HMM prediction model with random bidding model and average prediction model. And numerous results in predicting of bids, actual bids and victorious probability are stated in this section.

1) Comparison between prediction models: After the training of data, the maximum bid value is limited to 20. And the amount of resources is set randomly in [0,100]. After 40 rounds of auctions, from Figure 2, we can envisage the comparison between the actual bids and prediction bids. It is obvious that the trend of HMM prediction bids approximates the actual bids most of the time. However, the mean value prediction [2] bids fluctuate without any connection with the actual bids. Thus, we can conclude from the results that the prediction model based on HMM can approximate the actual bids better than the prediction method adopted by [2].

2) Victorious probability results of a two-player game. Suppose that two user brokers (UB_1 and UB_2) are competing in the game. Specifically, if UB_1 adopts no prediction algorithm and UB_2 utilizes the HMM prediction method, the results after 300 rounds of auctions are shown in Figure 3. It can be seen from Figure 3(a) that UB_2 can achieve a higher victorious probability.

If UB_1 adopts the mean value prediction method, and UB_2 still uses the HMM prediction method, the results is depicted in Figure 3(b). It is apparent that UB_2’s victorious probability still exceeds the UB_1’s.

3) Victorious probability results of a multi-player game. On the analogy of the results of the two-player game, we can conclude that if all the players in the n-players game adopts the HMM prediction method, the victorious probabilities will converge at around 1/n. It means the expected victorious probabilities will be a very small value if a large amount of players join the auction.

![Figure 2. Comparison among real value, HMM prediction bids and Mean Value prediction bids](image)
Thus, to avoid this problem, our algorithm of resource broker sets the number of bidders according to Eq.(1).

Specifically, in ESPSA model, the method that we propose classifies the $S_{\text{bidder}}$ into urgent set $S'_{\text{bidder}}$ and the less urgent set $S''_{\text{bidder}}$. Then, according to each set, we propose two different rounds of auctions. The number of bidders in each round should be calculated by Eq.(1). So the resource broker limits the number of user brokers in each auction by this way and guarantees the high victorious probability.

In order to prove the superiority of the ESPSA algorithms with user broker number restrictions, we made a comparison among actual victorious probability, expected victorious probability under user broker number restrictions and expected victorious probability without user broker number restrictions. The comparison results are depicted as Fig.4. For easy reference, we use ESPSA-NNres to represent the ESPSA algorithms without user broker number restrictions. It can be seen from Figure 4 that the difference between expected victorious probability and actual victorious probability is much smaller in the ESPSA algorithm. In the 10th round of the auction, the average expected victorious probability is 0.4 and the result by ESPSA algorithm is 0.38. However, the ESPSA-NNres algorithm only gets a victorious probability of 0.34. Moreover, in the 30th round, user brokers’ expected victorious probability is 0.58, the ESPSA’s and ESPSA-NNres’s victorious probabilities are 0.5 and 0.18 respectively.

4) Resource utilization rate and execution time. By increasing the user brokers that take part in the GVP [2] and ESPSA game, the recorded resource utilization rate of the two algorithms are illustrated as Figure 5. It is worth to note that each data in Figure 5 is obtained through 200 rounds of auctions. From it, we can see that ESPSA in this paper can guarantee a higher resource utilization rate than GVP.
From Figure 6, the algorithm execution time comparison between GVP and ESPSA algorithm is illustrated. It is obvious that the ESPSA algorithm runs more quickly at the most of circumstances.

Thus, according to the above analysis, we can see that the ESPSA model has higher superiority than GVP.

D. Equilibrium Analysis

Reference [2] utilized a mean value prediction method. Specifically, a player calculates his rival’s bid in the current round of auction by Eq.(8),

$$\bar{b}^j = \frac{1}{j-1} \sum_{i=1}^{j-1} b^i$$  \hspace{1cm} \text{(8)}

Then, the player only needs to bid a litter higher than the prediction bid.

We adopt the Cobb-Douglas production function to calculate the actual value (As Eq.(9) shows).

$$v(T, R, \alpha) = \eta ((1-\alpha) \ln T + \alpha \ln R)$$  \hspace{1cm} \text{(9)}

$T$ represents the expected time that the user want to hold the resource. $R$ represents the amount of resources that users want to have. $\alpha$ is a weighted factor which represents the preference of $T$ and $R$. $\eta$ is the profit that brought by the unit payoff. In the experiment, we set $\eta = 20$ and $\alpha = 0.95$. Given one user broker’s historical resource demand quantities, through Eq.(9), the actual bid can be carried out.

By Eq.(8) and Eq.(9), the results are shown in Figure.2. The ESPSA algorithm performs better than the algorithm in [2] in approximating the actual value.

Figure 3 depicts the victorious probability results of random bidding, mean value prediction bidding and ESPSA prediction bidding strategies. Thus, the payoff matrixes can be given as follows.

$$\begin{array}{|c|c|}
\hline
\text{User} & \text{Broker1} \\
\hline \text{ESPSA Prediction} & 0.5, 0.5 \\
\text{Random Prediction} & 0.3, 0.7 \\
\text{Strategy} & 0.5, 0.5 \\
\hline
\text{User} & \text{Broker2} \\
\hline \text{ESPSA Prediction} & 0.5, 0.5 \\
\text{Mean Value} & 0.6, 0.4 \\
\text{Prediction} & 0.5, 0.5 \\
\text{Strategy} & 0.5, 0.5 \\
\hline
\end{array}$$

From Figure 7 and Figure 8, it is obvious that the strict dominant strategy of user brokers is the ESPSA prediction strategy.

In a game, if a rational user agent has a strict dominant strategy, it will choose that strategy. In such way, the game reaches Nash equilibrium. For the reason that strict dominant strategy equilibrium must be Nash equilibrium [2], the two-player game above has Nash equilibrium:

$$S^* = (\text{ESPSA Strategy}, \text{ESPSA Strategy})$$

From the above analysis, we can conclude that the ESPSA predict strategy is superior to not only random bidding strategy but also mean value prediction strategy. Thus, all the rational players will choose the ESPSA strategy in the auction.

E. Computational Complexity of ESPSA

When weighing a model or algorithm is whether advanced or not, the computational complexity is an essential factor. In ESPSA model, the whole computational load depends on the HMM algorithm, that is the load of the analyst’s calculation and prediction algorithm (Viterbi Algorithm).
The Viterbi Algorithm provides a computationally efficient way of analyzing observation of HMMs to recapture the most likely underlying state sequence [26]. It exploits recursion to reduce computational complexity and takes advantage of the context of the whole sequence to make judgments. In conclusion, using the Viterbi algorithm to decode an observation sequence (the resource demand quantities), the computational load will only be linear in T [26].

Thus, we can get that any users who participate the game using the ESPSA model will acquire the grid or cloud resources with high victorious probability within a very short time.

F. Proof of Nash Equilibrium in ESPSA

1) Nash Equilibrium of a Two-user Game

In this paper’s setting, the users evaluate the value of an item according to the resource’s load information. If we assume that each resource’s load variation l is a normal distribution, the payoff function of each user can be given by Eq.(10).

\[
u_i(b_1,\ldots,b_m;l) = \begin{cases} \frac{\gamma_i}{b_i} - b_i, & \text{if } b_i \text{ wins} \\ \frac{1}{m} \left( \sum_{i=1}^{m} \left( \frac{\gamma_i}{b_i} - b_i \right) \right), & \text{if } b_1 = \ldots = b_m \\ 0, & \text{if } b_i \text{ loses} \end{cases}
\]

where \(\gamma_i\) is a constant, \(m\) is the total number of participants in the game, \(b_i\) is the bid price that user \(i\) competes for a resource, and \(i=1,2,\ldots,m\). It is common a same load may bring different benefit for different users, so the constant \(\gamma_i\) varies according to each user’s profile. \(L\) is the load vector.

Without loss of generality, initially, a game, in which two users (that is \(m=2\)) compete for one resource, is considered in the proof of Nash Equilibrium. Suppose that the strategies set of the users is \(\{B_1, B_2\}\), and each user’s bidding function is strictly increasing. If given user 1’s bid, user 2’s payoff function is

\[
u_2(b_1,b_2;l) = \begin{cases} \frac{\gamma_2}{b_2} - b_2, & \text{if } b_2 > b_1 \\ \frac{1}{2} \left( \sum_{i=1}^{2} \left( \frac{\gamma_i}{b_i} - b_i \right) \right), & \text{if } b_1 = b_2 \\ 0, & \text{if } b_2 < b_1 \end{cases}
\]

To prove the existence of Nash Equilibrium, firstly, user 2’s best response function need to be derived if given user 1’s bid.

**Lemma 1.** Given user 1’s bid strategy \(b_1 \in B_1\), user 2’s best response is \(b_2(l) = c_2 \frac{\gamma_2}{l} \).

Proof. Suppose that user 2’s probability density function is \(f \left( \frac{\gamma_i}{l} \right)\) (for the ease of proof, we suppose \(\eta_i = \frac{\gamma_i}{l}, i=1,2\) ), user 2’s winning probability is

\[
P_{2_{win}}(b) = P(b_i(\eta_i) \leq b) = P(\eta_i \leq b^{-1}(b)) = \int_{b^{-1}(b)} f(\eta_i) d\eta_i
\]

where \(b^{-1}\) is the reverse function of user 1’s bidding function. In order to win, user 2 has to find an optimal strategy in strategy set \(B_2\). So, this problem can be modeled by a non-linear programming problem represented by Eq.(13).

\[
\max_{\eta} \int_{\eta \leq b} f_2(\eta_2) P_{2_{win}}(b_2(\eta_2)) d\eta_2
\]

subject to

\[
\int_{\eta \leq b} f_2(\eta_2) f(\eta_1) d\eta_1 d\eta_2 \leq \alpha_2
\]

where \(\alpha_2\) represents the budget of user broker 2 in a round of one auction.

It can be solved by a Lagrange multiplier. Then, we get Eq.(14).

\[
\int_{\eta \leq b} f_2(\eta_2) f(\eta_1) d\eta_1 d\eta_2 - \lambda_2 \left( \int_{\eta \leq b} b_1(\eta_1) f_2(\eta_2) f(\eta_1) d\eta_1 d\eta_2 - \alpha_2 \right) = \int_{\eta \leq b} f_2(\eta_2) (\eta_2 - \lambda_2 \cdot b_1(\eta_1)) f(\eta_1) d\eta_1 d\eta_2
\]

It is obvious that, if the Eq.(14) gets maximum value, the objective of Eq.(13) gets its optimal value. Therefore, when \(\eta_2 - \lambda_2 \cdot b_1(\eta_1) = 0\), the maximum value can be achieved. So, \(b_2^{opt} = \frac{\gamma_2}{\lambda_2}\). From \(\eta_2 = \frac{\gamma_2}{l}\), we get \(b_2^{opt} = \frac{\gamma_2}{l \lambda_2}\). Suppose \(c_2 = \frac{1}{l \lambda_2}\), it is easy to get \(b_2(l) = c_2 \frac{\gamma_2}{l} \).

Given user 2’s best response, the existence of Nash Equilibrium can be derived.

**Theorem 1.** In the ESPSA scheduling model with two users, there exists Nash Equilibrium.

Proof. According to the Nash Existence Theorem [10, 11, 12], if each user’s strategy space is non-empty, closed and bounded convex set in Euler space and the payoff function is continuous, quasi concave function, there exists a pure Nash Equilibrium.

From the assumptions of ESPSA model, the strategy sets \(B_1\) and \(B_2\) are non-empty, closed and bounded. According to Eq.(10), the payoff function is continuous in its domain. From Lemma 1, it is obvious that the best response, \(b(l) = c_2 \frac{\gamma_2}{l} \), is convex under its domain. Thus, payoff function is quasi concave [13].

From the above prove, Nash Equilibrium exists in ESPSA model of two users.

2) Nash Equilibrium of N-winners Game

In the above section, we proved the existence of Nash equilibrium of two users for one auction. However, a
more realistic grid or cloud scheduling scenario setting is that multiple winners are allowed in one auction. It is because one resource always consists of several machines. Thus, in this section, we give a definition of the \( n \)-winner game and then propose a similar method of proving the Nash Equilibrium existence of a game. 

Given user broker \( i \)'s bidding strategy \( b_i \in B \), and the strategy of user brokers except user broker \( UB_i \) is \( b_{-i} \in B_{-i} \), \( i = 1, 2, \ldots, m \). User brokers aim to maximize their own expected benefit, while satisfying their budgets. Thus, the payoff function of user broker \( i \) can be written as Eq. (15).

\[
    u_i(b_i,b_{-i};l) = \begin{cases} 
    \gamma - b_i, b_i \in \max_n (b_1,b_2,\ldots,b_n) \\
    0, b_i \notin \max_n (b_1,b_2,\ldots,b_n)
    \end{cases}
\]  

(15)

where \( n \) is the number of winners, and we use \( \max_n (b_1,b_2,\ldots,b_n) \), \( 0 < n \leq m \), to represent the top \( n \) bids in \( \{b_1,b_2,\ldots,b_n\} \).

**Lemma 2.** Given the payoff function of each user broker and the bidding strategy of other user brokers \( b_{-i} \), in the \( n \)-winners ESPSA game, the best bidding strategy of a user broker \( i \) is in the form of \( b_i = c \gamma/l \), where \( c \) is a constant.

**Proof:** The strategies for user broker \( i \) can be represented as the set of bidding functions \( B_i = \{b_1,b_2,b_3\cdots b_n\} \). In order to prove the existence of Nash Equilibrium, the form of response correspondence of each user broker \( b(\cdot) \) has to be derived.

Given all the other user brokers’ (except user broker \( i \) ) bid \( b_{-i} \), the expected payoff of user broker \( i \) is:

\[
    u_i = \gamma/l - b_i \Pr(b_i \in \max_n (b_1,b_2,\ldots,b_n))
\]  

(16)

where \( \Pr(b_i \in \max_n (b_1,b_2,\ldots,b_n)) \) is the probability of \( b_i \) in the set of top \( n \) bids. \( \gamma/l - b_i \) represents the net payoff of \( UB_i \) when it wins the game. Let \( \omega_i \) be the set of winners, and \( \omega_{-i} \) be the \( n \) winners without the participation of \( UB_i \). It is obvious that the bid of \( UB_i \) only needs to be greater than the least one in the set \( \omega_{-i} \). Thus, we use \( b_{\text{least}(\omega_i)} \) to represent that user broker. Thus, to win the game, user broker \( i \)'s bid only need to be greater than \( b_{\text{least}(\omega_i)} \). For the sake of simplicity, that user broker is represented by \( UB_k \) and its bid as \( b_k \). Thus, \( UB_k \)'s optimal bidding function can be represented by

\[
    \max_{b_k} u_k = \gamma/l - b_k \Pr(b_k < b_i)
\]

s.t.

\[
    b_k \cdot \Pr(b_k < b_i) \leq w_i
\]  

(17)

ESPISA is an extended version of second sealed price sealed auction. It allows multiple winners and the payment of a winner \( k \) is the least winner’s bid without the participation of the user broker \( k \). Then, the payment of user broker \( i \) is \( b_2 \) as the constraint term of Eq. (17) shows.

By applying the similar method, we can derive the result. The analysis is omitted for briefness.

**Theorem 2:** A Nash Equilibrium exists in the ESPSA game \( G = [m,\{S_i\},\{u_i(\cdot)\}] \) with \( \{S_i\} \) and \( u_i(\cdot) \) defined above.

**Proof:** The proof can follow the steps of Theorem 1 to show the existence of Nash equilibrium. For the sake of briefness, we omitted them here.

V. CONCLUSION AND FUTURE WORK

The ESPSA model is proposed in this paper and the model focuses on the resource allocation problem in grids and cloud computing. Firstly, we designed the interactions procedures between different entities in the ESPSA. Then, the algorithms of user and resource brokers are proposed. Specifically, in order to assure the victorious probability of each user broker, we introduce a bidder number restriction method in the resource broker’s algorithm. Moreover, the user broker gets prediction information from the analyst who adopts the HMM prediction model. Then, based on the prediction results, the user broker carries out its bid. Simulation results show that, the ESPSA algorithms bids performs better in the approximation to actual bids than random bidding and mean value predict strategy. What’s more, we analyzed the existence of Nash equilibrium based on the simulation results and the computational load of the ESPSA model. Lastly, we proved the existence of Nash equilibrium of the ESPSA model with two participants and multi-participant theoretically.

It is worthwhile to note that the ESPSA model is much more applicable than some basic auction models. However, the network delay, fraud user, reliability of resources problems are not considered in this paper. Thus, how to make the model realistic, fulfill the QoS requirements of users and improve the resource scheduling algorithms form the next step of our work.

ACKNOWLEDGMENT

This paper is partially supported by NSFC-JST under grant No.: 51021140004; Nature Science Foundation of China under grant No.: 60673046, 90715037; University Doctor Subject Fund of Education ministry of China under grant No.: 200801410028; National 973 Plan of China under grant No.: 2007CB7124025; Natural Science Foundation Project of Chongqing, CSTC under grant No.: 2007BA2024; Natural Science Foundation of China under Grant No. 69093153; the Fundamental Research Funds for the Central Universities; Nature Science Foundation of China under grant No.61070181.

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