Realization of Quadrilateral Mesh Partition and Optimization Algorithm Based on Cloud Data

Lijuan Wu
College of Physics Science and Technology Shenyang Normal University Shenyang china
Wulijuan1964@163.com

Abstract—Algorithms of quadrilateral mesh partition and optimization based on cloud data are proposed in the paper. Quadrilateral meshes are partite by means of dynamic edge-extending according to the types of the nodes. The mesh optimization algorithms are designed, and topology optimization of degree 2 and 3 node and region mesh optimization processing based on viewpoint are programmed and debugged as well. Realizations of the algorithm are introduced in details. The algorithm improves mesh quality and examples are shown to illustrate its feasibility.

Index Terms—quadrilateral mesh, mesh partition, topology optimization, viewpoint, cloud data.

I. INTRODUCTION

With the development of laser scan equipment, cloud data including more details become possible, and computer reconstruction of cloud data become more significant in many applications. According to the verification of the theory and the practicality, the precision of quadrilateral mesh element is higher than triangular mesh element, so we must obtain an algorithm of how to build a quadrilateral mesh.

At present, the algorithm of how to build triangular mesh is mature enough, but the algorithm of how to partition quadrilateral mesh is very few, some times in order to advance precision analysis and satisfy some special demands, we must build quadrangular mesh[1,2].In CAD, when redesigning the parameters of bent surface(such as NURBS,B-Spline and so on),we need also build the corresponding quadrilateral mesh beforehand[3].The paving algorithm presented by Blacker and some other people has the potential one extremely, and extended by ROGER J.Cass and some other people later, and applied to building quadrilateral mesh on three-dimensional bent surface[4], Zheng zhiwen also put forward a advanced automatic quadrangular mesh generation algorithm on 3D surface[5],the algorithm can adapt to divide complicated bent surface, but it is the finite meshes, and require the boundary points must be even, so this method restricts the agility of boundary. In this paper, different mesh element generation methods, collision detecting, boundary optimization and edges process are come up with, and the algorithm not only adopt to build the finite meshes, but also generate quadrilateral meshes based on cloud data. Besides, the algorithm complexity is lower and the agility is advanced.

During quadrilateral mesh partition, because the surface original cloud data set is not distributing regularly, and veracity problems of boundary node confirming as well as the algorithm focus on mesh generation, it is difficult to control mesh shape any more during quadrilateral mesh partition, and results of some cases will be presented, such as concave quadrilateral mesh and triangle mesh. It is necessary to adjust mesh topology structure to improve quality of meshes. The aims of mesh optimization are satisfied for certain quality requirements, but appraising mesh quality does not have a uniform standard [7].

Generally speaking, optimization of surface quadrilateral mesh should satisfy two requirements as following:

(1) Mesh topology structure is reasonable, and the shape is regular;
(2) Meshes approach body surface in valid error scale.

So mesh optimization becomes an important part of improving mesh quality.

II. THE BASIC PRINCIPLE OF ALGORITHM

A. Basis concept definition.

Edge: Edge is defined as the boundary between partitioned area and partitioning area in the process of mesh partition. As edges 12,23 are shown in Fig.1.

Edge point: Edge point is defined as the point on the boundary, and called node too. As edge points 1,2,3,4 are shown in Fig.1.

Fixed edge: Fixed edge is composed of lines that connect the boundary points of non-closed surface.

Edge ring: During the process of mesh partition, edges are linked by means of end to end to form edge ring according to anticlockwise or clockwise direction. The same end point belongs to the start edge and the end edge. In other words, the edge ring is a closed loop. In Fig.1,
the edge ring makes up of 12-23-34-……-101 in anticlockwise direction.

B. Generation of edge points.

Known three points \( p_1, p_2, p_3 \), and the front edge point \( p_{pp1} \) and the back edge point \( p_{pp3} \), and calculate the quadrilateral mesh vertexes \( p_1, p_2, p_3 \).

1) When the line between the new vertex and the current point \( p \) is on the mesh edge, shown as edge \( pp_{pp} \) in Fig.2, then set \( d(p,p_{pp})=L \); When the line between the new vertex and the current point \( p \) is on the mesh diagonal, shown as edge \( pp_{diagonal} \) in Fig.2, then set \( d(p,p_{pp})=L/sin(\alpha) \), \( d(p,p_{pp3})=L/sin(\alpha) \).

2) Project \( p_{pp3} \) on the tangent plane \( M \) determined by the \( k \) neighborhood points of \( p \), and calculate angle between boundary edge \( pp_{pp3} \) and \( pp_{pp} \), and divide from the middle of the angle \( \alpha \) on the plane \( M \), and intercept distance \( d \), because the return value of function \( arcos() \) is \( 0 \sim \pi \), when \( \alpha > \pi \), a error value is returned, it is necessary that judge availability of points \( pp_{21}, pp_{22} \) and edge meshes \( m_1, m_2 \), which point \( p \) belongs to, intersect or not, and select no intersecting point \( pp_{21} \), and look for the nearest point \( pp_{22} \) of point \( pp_{21} \), then accord to mesh optimization factor, find the best point to replace point \( p \) in the \( k \) neighborhood points of \( p \). The determinative method of point \( p_1 \) and \( p_3 \) is analogy to point \( p_2 \).

C. The storage of edge points.

Firstly, select four points to make them be close to square, and compose starting quadrilateral meshes, and record mesh vertex points in turn. Take first point for edge point of the current processing, and the next point for back edge point, and the last one for the front edge point to generate new mesh vertexes, and record edge points from the front edge point to the back edge point in turn, as shown in Fig.2. The order of recording is \( p_1, p_2, p_3 \), the processing of the rest edge point may be deduced by analogy, as shown in Fig.3.4.

III. NEW MESH ELEMENT GENERATION

The two edges intersecting at the node are projected onto its tangent plane. Generating new nodes on the plane according to the internal angle type, its \( k \) neighborhood points are calculated, and a corresponding point in its \( k \) neighborhood is found. The new point is composed of quadrilateral mesh with the point and three old edge points, here the quality factor[6] of the mesh is better. The point is considered as the new edge point of the mesh, as shown in Fig.5.

According to the degree of internal angle[8], the classification of nodes is the following:

End node: \( \alpha \leq 120^\circ \);
Edge node: \( 120^\circ < \alpha \leq 240^\circ \);
Angle node: \( 240^\circ < \alpha \leq 360^\circ \);

For different type of nodes, the generation algorithm of new nodes goes like the following:

The two boundary edges \( OP, OQ \) are projected on the tangent plane \( M \), their projection are \( OP', OQ' \). New vertex \( F' \) was generated according to type of the internal angle on the tangent plane \( M \), then \( k \) neighborhood of point \( F' \) is gotten. Point \( F \) is regarded as a new node, a quadrilateral mesh that the quality factor is much better composed by point \( F, O, P, Q \). Collision check and optimization processing of the vertex point \( F \) was carried, as shown in Fig.5.

A. The generation of new mesh.

According to the degree of internal angle (See Fig.5), for different types of nodes, the generation algorithm of new nodes is the following.

End nodes as basis points. Create one new point \( N_j \) from three old nodes \( N_{j-1}, N_{j}, \) and \( N_{j+1} \), and connect the new
and old nodes to form one new mesh element, as shown in Fig.6. The angle is $\alpha/2$ between $V_j$ and edge $N_jN_{i-1}N_{i+1}$, then

$$|V_j| = d / \sin(\alpha/2)$$

Where $d$ is length of an edge.

Edge nodes as basis point. Create new nodes $N_j$, $N_k$ and $N_l$ from three old nodes $N_{i-1}, N_i, N_{i+1}$, connect new and old nodes to form two new mesh elements, the angles between $V_j, V_k$ and $V_l$ and edge $N_{i-1}N_i$ are $\alpha/4, \alpha/2, 3\alpha/4$ respectively. Where $|V_k| = d, |V_j| = d / \sin(\alpha/4), |V_l| = |V_j|$. As shown in Fig.7.

![Fig.7 Edge nodes](image)

Angle nodes as base point. Create five new points $N_j, N_k, N_l, N_m, N_n$ from three old nodes $N_{i-1}, N_i, N_{i+1}$ and connect the new and old nodes to form three new elements, as shown in Fig. 8. The angles between $V_j, V_k, V_m, V_n$ and edge $N_{i-1}N_i$ are $\alpha/6, \alpha/3, \alpha/2, 2\alpha/3, 5\alpha/6$ respectively, then

$$|V_k| = d, |V_j| = d / \sin(\alpha/6), |V_m| = |V_k|, |V_n| = |V_l|$$

Create new nodes based on the current edge nodes to form new elements, and update list of edge nodes.

![Fig.8 Angle nodes](image)

B. The computation of quadrilateral quality factor.

There are many methods to define the quality factor of a quadrilateral mesh, in this paper the quality factor is defined by four internal angles among four edges, edge length and concavo-convex. The quality factor $\gamma$ is the following:

$$\gamma = k_1\gamma_1 + k_2\gamma_2 + k_3\gamma_3 + k_4\gamma_4$$  \hspace{1cm} (1)

where $\gamma_1, \gamma_2, \gamma_3, \gamma_4$ are quality factors of $\gamma$, $k_1, k_2, k_3, k_4$ are weighted factors.

Project the four edges of spatial quadrilateral to the plane defined by three points of the quadrilateral, as shown in Fig. 9. Suppose the four internal angles are $\alpha_1, \alpha_2, \alpha_3, \alpha_4$ respectively and arrange them by descending order, that is $\alpha_1 \geq \alpha_2 \geq \alpha_3 \geq \alpha_4$. The quality factor $\gamma_1$ is defined as follows:

$$\gamma_1 = \frac{\alpha_3 \times \alpha_4}{\alpha_1 \times \alpha_2}$$  \hspace{1cm} (2)

The four edges are $l_1, l_2, l_3, l_4$ respectively. Arrange them by descending order and set $\gamma_2$ as follows:

$$\gamma_2 = \frac{l_4}{l_1}$$  \hspace{1cm} (3)

If the quadrilateral degenerates to the triangle, then $\gamma_1, \gamma_2$ are equal to 0. $\gamma_1, \gamma_2$ are bigger, the mesh quality is better and nearer to a square.

![Fig.9 Computing quality factor of a quadrilateral](image)

![Fig.10 Concave quadrilaterals](image)

Concave quadrilaterals are the worst conditions, now $\gamma_3$ is equal to 0. If the quadrilaterals are convex, $\gamma_3$ is equal to 1.

For the quadrilateral mesh $ABCD$, connect the diagonal $BD$ or $AC$ to get four edges of the quadrilateral and the four angles $\beta_1, \beta_2, \beta_3, \beta_4$ are formed by the diagonals.

![Data Model of CTH](image)

![Fig.11 An example of meshes generation](image)
respectively. If $\beta_1 + \beta_2 \geq 180^\circ$ or $\beta_3 + \beta_4 \geq 180^\circ$, the quadrilateral is concave.

Quality factor $\gamma$ in the quadrilateral meshes is between 0 and 1. The value of $\gamma$ is more, the quality of the quadrilateral is better and the quadrilateral is nearer to a square. The values of $k_1, k_2, k_3$ in this algorithm are 0.3, 0.2, 0.5 respectively, which will guarantee that the quadrilateral is convex and nearer to the square. The CTH model Surface mesh partition example based on above designing ideas is shown in Fig.11

IV. DESIGN OF MESHES TOPOLOGY OPTIMIZATION ALGORITHM

Topology structure is a connection relationship between mesh nodes. Adjusting of topology structure is that change connection relationship between nodes, including increasing or removing nodes of meshes. It is processed according to the node degree. The best shape of quadrilateral meshes is square, and its internal angle is $90^\circ$, hereby, a node surrounded by four adjacent elements is the best.

During quadrilateral meshes topology optimizations, major processing problems come into appear:

1) Angles processing on fixed edge;
2) Nodes and mesh elements processing;
3) Degenerating triangle or concave quadrilateral meshes processing.

A. Basic meshes topology optimization.

Angles processing on fixed edge. If $\alpha < 120^\circ$ and fixed vertex A belong to two mesh element, as shown in Fig.12(a), edge exchange operation is processed. Method is employed in Fig.12(c) or Fig.12(d) according to quality factor of mesh element $M_1$ and $M_2$ as well as degree of nodes 1 and node 2. If $Ne_2 \geq Ne_1$, select the method as shown in Fig.12(a) or select the method as shown in Fig.12(d), where Nei is degree of node i.

Node and mesh element processing. If a edge is not fixed boundary, and the total of degree of two end nodes is large than or equal to nine, as shown nodes 2 and 3 in Fig.13(a), Namely, if $Ne_2 + Ne_3 \geq 9$, it carries through edge exchange operation, and make degree of any node tend to 4.

If $(Ne_2 + Ne_3)(Ne_1 + Ne_0) \geq 3$ and $(Ne_2 + Ne_3)(Ne_1 + Ne_4) \geq (Ne_2 + Ne_3)(Ne_0 + Ne_5)$, change edge 2-3 to 0-5, as shown in Fig.13(b).

If $(Ne_2 + Ne_3)(Ne_1 + Ne_0) \geq 3$ and $(Ne_2 + Ne_3)(Ne_1 + Ne_4) \geq (Ne_2 + Ne_3)(Ne_0 + Ne_5)$, change edge 2-3 to 0-5, as shown in Fig.13(c). Nei is degree of node i.

Fig.12 Angle nodes processing on fixed edges

Fig.13 Mesh edges processing

Fig.14 Edges exchange strategy of 5/3 topology structure

Fig.15 Edges moving processing strategy of 5/3 topology structure
B. Complex meshes topology optimization.

Edges exchange strategy of 5/3 topology structure, what is called 5/3 topology structure is one of diagonal nodes that hold three adjacent mesh elements, and the other holds five adjacent meshes. As shown in Fig.14(a), node P0 and P1 is 5/3 structure, degree of P0 is 5, degree of P1 is 3. If edge e0 is exchanged, the structure is moved along, and it makes node P2 and P1 possess 5/3 structure characterized and changed as following:

1. Degrees of nodes P0 and P1 became 4;
2. Degrees of nodes P2 is changed from 4 to 5;
3. Degrees of nodes P1 is changed from 4 to 3;

As shown in Fig.14(b). Edges exchange strategy in Fig.14(c) is also adopted.

Edge moving strategy of 5/3 topology structure. The moving end condition is as following:

If there is a node of degree 5 on Line3, as shown in Fig.15(a), after edge exchange operation is finished, the original nodes of degree 3 and 5 become nodes of degree 4, but still remain a node of degree 5, as node P4 is shown in Fig.15(b).

If there is a node of degree 3 on Line5, edge operation to improve quality of mesh topology is finished. Edge moving stops when one of the above conditions is fulfilled. If none of them can be met, the associate path stops at an edge of the domain and mesh topology structure cannot be improved. Moving process stops on fix boundary edge for surface mesh possessing boundary edges.

C. Optimizations region Determining based on viewpoint.

The viewpoint is obtained by the left button down in the window screen, and the optimization region is determined according to mesh belonging to the transformed viewpoint. As shown in Fig.16.

V. REALIZATION OF MESHES TOPOLOGY OPTIMIZATION ALGORITHM

A. Degree 2 nodes optimization processing.

The degree of the current node CurP is 2, and points NP1 and NP2 are adjacent with point CurP in two meshes, NNP1 and NNP2 are two non-adjacent with CurP. As shown in Fig.19(a).

Optimization method. The original mesh cell NNP1-NP1-CurP-NP2 and NP1-NNP2-NP2-CurP merged into a new mesh NNP1-NP1-NNP2-NP2, the optimization results are showed in Fig. 19(b).

Mesh nodes data structure processing. The current node CurP processing: the meshes amount possessed by the current node CurP is set 0; the array contents are set -1, which store mesh number. Boundary node flag is set 0 and flag the node inside new mesh.

Adjacent nodes NP1, NP2 processing: the amount of meshes belongs to the node CurP minus 2; the array contents that store mesh numbers are set -1.

Non-adjacent-nodes NNP1, NNP2 processing: A new mesh replaces original two meshes belonging to the deleted node CurP. Original two meshes will be set
deleting flag. That is processFlag = 2 in QuadMesh data structure [8].

B. Degree 3 nodes optimization processing.

To the current node CurP of degree 3, nodes P1, P2, P3 belong to only one mesh, nodes P12, P13, P23 belong to the two meshes. No1, No2, No3 are owned by the current node CurP. As shown in Fig.20.

Optimization method.
Step1: Find the current node CurP and numbers of meshes it owns, that is No1, No2, No3;
Step2: Find nodes that belong only to one mesh, that is P1, P2, P3;
Step3: Find the nodes that belong to two meshes, that is P12, P23, P13
Step4: Create two new meshes
P12-P1-P13-P3-P12
P12-P3-P23-P2-P12, as shown in Fig.21 (a). Or
P13-P2-P3-P1-P13
P13-P2-P1-P3-P1, as shown in Fig.21 (b). Or
P1-P23-P3-P12-P1
P1-P23-P1-P12-P1, as shown in Fig.21 (c).
Step5: Calculate the sum of each new meshes quality factor, that is γ1, γ2, γ3, taking two meshes of a large quality factor as the optimized meshes.

Mesh nodes data structure processing. The current node CurP processing: the current node CurP is set inside new mesh; mesh node flag is set 0; the mesh amount owned by the node is set 0. The array contents are set -1, which stores meshes number.

Nodes P1, P2, P3 processing: the mesh amount reduces 1, which belongs to nodes P1, P2, P3, and delete meshes owned by three nodes. The array contents are set -1, which store mesh number.

Nodes P13, P23, P12 processing: the amount of meshes reduces 2, which belongs to nodes P13, P23, P12, and delete meshes owned by three nodes. The array contents are set -1, which store meshes number, and move them to end of the array.

C. Region Meshes optimization processing.

The current node CurP of degree 3, which is regarded as viewpoint, belongs to three meshes. Meshes No1, No2 and No3 are owned by the current node CurP, as shown in Fig.17.

Optimization method.
Step1: Determine the current node CurP viewpoint by human-computer interaction, and confirm the numbers of meshes it owns, such as No1, No2 and No3;

Step2: Determine optimization region according to mesh edges contacted with viewpoint.
Step3: Decide the result of meshes optimized by computing sum of quality factor of meshes optimized in Fig.17 and Fig.18.

Mesh nodes data structure processing. The viewpoint CurP processing: First, the node CurP is set inside the new mesh, its mesh node flag is set 0, the mesh amount owned by the node is set 0, and deletes meshes owned by node Curp. Second, the corresponding array value which store mesh number are set -1. Last, the amount of meshes is minus 1.

Nodes from P1 to P5 processing: The amount of belongs to nodes P1 to P5 are newly recorded according to optimization result, and adjusted the array contents, which store mesh number owned by node.

Meshes No1, No2, No3 processing: In the QuadMesh data structure, the original meshes No1, No2, No3 set mesh deleting flag, that is processFlag = 2.

VI. CONCLUSION

The original surface was partitioned into quadrilateral meshes by means of dynamic edges extending, and carried out program debugging for the quadrilateral mesh generation algorithm, feasibility of algorithm shown by an generation example in Fig11, and realized topology optimization of degree 2, 3 nodes and viewpoint region according to the above design ideas. As running results shown in Fig.22~26, an optimization processing example of degree 2 nodes is shown in Fig.22, and an optimization processing example of degree 3 nodes is shown in Fig.23, and an optimization processing example based on viewpoint is shown in Fig.24. The original quadrilateral mesh surface is shown in Fig.25, and meshes surface is shown in Fig.26 after the simple operation of topology optimization. The algorithm is run iteratively, reducing amount of nodes of degree 2 and 3, and the surface meshes close to the whole quadrilateral mesh. The results show that the algorithm is feasible.

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REFERENCES


[6]. Barry Joe. Quadrilateral mesh generation in polygonal regions. CAD, 1995, 27 (3) : 194~199


