An Isomorphic New Algorithm for Finding Convex Hull with a Maximum Pitch of the Dynamical Base Line Guided by Apexes Distributing Characteristics

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Abstract—In this paper, a representative algorithm convex hull with half-dividing and recurrence is commented; and according to the isomorphic fundamental theorem of the convex hull construction, and guided by the isomorphic distributing characteristics of a convex hull’s the apexes, a more efficient new algorithm to find a convex hull based on the dynamical base line with a maximum pitch of the dynamical base line is given. The general characters of the new algorithm are: 1) find out the outside-most poles which are the leftmost, rightmost, topmost and bottommost points of the given 2D point set, i.e. the four initial poles which have the maximum or the minimum coordinate value of the X or Y axis among all the points in the given 2D point set; 2) divide the original distributed domain into four sub-domains with the initial poles; 3) in every sub-domain, construct a current pole with a maximum pitch to its base line based on the last pole got just dynamically and sequentially, and draw the rims of this convex polygon with these poles for intelligent approximating for a convex hull of the given 2D point set step by step.

Index Terms—isomorphic, convex hull algorithm, maximum pitch, dynamical base line, apexes distributing characteristics

I. INTRODUCTION

A convex hull can be applied in intelligent information technology (e.g. cluster analysis in data mining, as well as security informatics). Many literatures explained the vital significance and many studies of the research, improvement and enhancement about the convex hull algorithms of 2D point set or a set of segments (hereafter called convex hull for short) and its efficiency [1-8]. This paper will comment on another representative convex hull algorithm with half-dividing recurrence, then a new more efficient algorithm to find a convex hull based on the dynamical base line with a maximum pitch is given by us according to the isomorphic fundamental theorem of the convex hull construction.

II. THE DESCRIPTIONS OF THE PROBLEM OF 2D CONVEX HULL AND THE CONVEX HULL ALGORITHM

DEFINITION 1: Suppose that Q is the polygon in given plane, Q1(x1,y1), Q2(x2,y2), ..., Qn(xn,yn) are the spots of Q. If any line segment QiQj (i \neq j, 1 \leq i \leq n, 1 \leq j \leq n < +\infty) is all not outside Q, then Q is called a convex polygon. Suppose that the 2D point set S=\{P_i(x_i,y_i) | 1 \leq i \leq m, 3 \leq m < +\infty\} is composed by the spots which are in the given plane. If the apexes of polygon Q belong to S, and Q is the least Convex Polygon which covers all points in S, then Q is called the convex hull of the given 2D point set S. How to seek the convex hull in given 2D point set S=\{P_i(x_i,y_i) | 1 \leq i \leq m, 3 \leq m < +\infty\} is called 2D Convex Hull problem. An algorithm which could produce 2D convex hull of the given 2D point set is called 2D Convex Hull algorithm.

III. THE COMMENTARY OF 2D CONVEX HULL ALGORITHM RESEARCH
Since the 20th century 70's, the 2D Convex Hull problem's complexity and its application importance had caused the domestic and foreign experts quite to pay attention to the convex hull algorithms, and proposed such as the Gift Wrapping convex hull algorithm, the Graham scan convex hull algorithm, the half-dividing convex hull algorithm, and so on[5]. Here, this paper only comments on a representative algorithm convex hull with half-dividing.

The Synopsis of the half-dividing Convex Hull Algorithm

The literature [1], explained the half-dividing convex hull algorithm which is based on half-dividing technical and the recurrence method, the algorithm’s efficiency is higher than the Gift Wrapping convex hull algorithm and the Graham Scan convex hull algorithm, its total running time is \( t(m) = O(m \log m) \).

The method of half-dividing convex hull algorithm could be summarized as follows:

Step 0: The point \( P(x_i, y_i) \) in 2D point set \( S \) satisfies \( x_i \leq x_{i+1} \) and \( 1 \leq i \leq m-1 \), \( 3 \leq m < +\infty \), makes the leftmost point \( P_{\text{left}} \) and the rightmost point \( P_{\text{right}} \) of the initial 2D point set as the end points of the line of demarcation, which divides \( S \) into two sub-domains \( \text{Sup} \) and \( \text{Slow} \). (For example in Figure 1: when \( m = 11 \), the points with minimum or maximum x-coordinate are the leftmost point \( P_{\text{left}} \), rightmost point \( P_{\text{right}} \), and it also divides the point set \( S \) into the upper sub-domain \( \text{Sup} = \{P_2, P_5, P_8, P_9\} \) and the lower sub-domain \( \text{Slow} = \{P_1, P_3, P_7, P_{10}\} \).

Step 1: Execute the sub-algorithm \( \text{UH}(\text{Sup}) \), in order to produce the upper convex hull.

\( \text{Sup} = \{P_2, P_5, P_8, P_9\} \)

1. If \( m \leq 3 \), then all of the points in point set \( \text{Sup} \) are the convex hull’s apexes, and return these points; otherwise, carry out ② of Step 1.

② According to the principle of dividing these points almost equally, it divides the point set \( \text{Sup} \) into two sub-domains \( \text{Sup}_1 \) and \( \text{Sup}_2 \).

③ Taking \( \text{Sup}_1 \) as the new \( \text{Sup} \), executes the upper convex hull sub-algorithm \( \text{UH}(\text{Sup}) \) recursively, and produces \( \text{Sup}_1 \)’s convex hull \( \text{Sup}_1 \).

④ Taking \( \text{Sup}_2 \) as the new \( \text{Sup} \), executes the upper convex hull sub-algorithm \( \text{UH}(\text{Sup}) \) recursively, and produces \( \text{Sup}_2 \)’s convex hull \( \text{Sup}_2 \).

⑤ Draw the common tangent of convex hull \( \text{Sup}_1 \) and convex hull \( \text{Sup}_2 \) (For example the imaginary line \( Q_8Q_7 \) shown in Figure 2), in order to merge convex hull \( \text{Sup}_1 \) and convex hull \( \text{Sup}_2 \) into convex hull \( Q_{\text{up}} \).

Step 2: Transfer recursion sub-algorithm \( \text{UH}(\text{Slow}) \), in order to produce the lower Convex Hull.

① If \( m \leq 3 \), then the points in the point set \( \text{Slow} \) are the convex hull’s apexes, and return to the points; Otherwise, carryout ② of Step 2.

② According to the principle of dividing these points almost equally, it divides the point set \( \text{Slow} \) into two sub-domains \( \text{Slow}_1 \) and \( \text{Slow}_2 \).

③ Take \( \text{Slow}_1 \) as the new \( \text{Slow} \), transfer the lower convex hull recursion sub-algorithm \( \text{UH}(\text{Slow}) \) recursively, and produce the \( \text{Slow}_1 \)’s convex hull \( \text{slow}_1 \).

④ Take \( \text{Slow}_2 \) as new \( \text{Slow} \), transfer the lower Convex Hull recursion sub-algorithm \( \text{UH}(\text{Slow}) \) recursively, and produce the \( \text{Slow}_2 \)’s convex hull \( \text{slow}_2 \).

⑤ Structure the common tangent of convex hull \( \text{slow}_1 \) and convex hull \( \text{slow}_2 \) (For example, the imaginary line \( Q_2Q_3 \) shown in Figure 2), in order to merge convex hull \( \text{slow}_1 \) and convex hull \( \text{slow}_2 \) into convex hull \( Q_{\text{low}} \).

Step 3: Merge the upper convex hull \( Q_{\text{up}} \) and the lower convex hull \( Q_{\text{low}} \) into final convex hull \( Q \) (For example the convex polygon \( Q_1Q_2Q_3Q_4Q_5Q_6Q_7Q_8 \) shown in Figure 2).
The Weakness of the half-dividing Convex Hull Algorithm

It is not difficult to see that the main shortcomings of half-dividing convex hull algorithm are:
1st, because of the recursion, the algorithm’s efficiency is not high. 2nd, only speaking of the upper convex hull production processing, in the trapezoid which take the common tangent $Q_iQ_{i+1}$ as a waist, and select the line of the direction of $X$ coordinate of the points $Q_i$ and $Q_{i+1}$ as the upper and lower underside (in fact, it may expand to: produce the quadrangle $P_{left}P_{right}Q_{i+1}Q_i$ which takes the upper common tangent $Q_iQ_{i+1}$ as one side, the line of demarcation $P_{left}P_{right}$ as its opposite side, while $Q_{i+1}P_{left}$ as another side, and $Q_iP_{right}$ as its opposite side), the more points $S_{up1}$, $S_{up2}$ have, the more invalid sides of sub-convex hull the algorithm have; So it may reduce the algorithm efficiency. (As shown in Figure 2: Many invalid sides and redundancy processing in non-convex hull Q.)

IV. ISOMORPHIC CHARACTERISTICS ANALYSIS OF CONVEX HULL’S Apexes Distributing

In order to improve and raise further the algorithm efficiency, it is necessary to research on the important natures which is about that an algorithm of convex hull has closed relation to the distributing characteristics of the distributed domain of 2D point set.

Definition 2: The domain where all point in a given 2D point set $S = \{P_i(x_i,y_i) \mid 1 \leq i \leq m, 3 \leq m < +\infty\}$ distribute is called the distributed domain of 2D point set S.

Definition 3: In a given 2D point set $S = \{P_i(x_i,y_i) \mid 1 \leq i \leq m, 3 \leq m < +\infty\}$, the points which have the maximum (or minimum) x-coordinate, or maximum (or minimum) y-coordinate where if there are more than one points have the same maximum (or minimum) x-coordinate then only the points which have maximum (or minimum) y-coordinate, are called the outside-most points. Among these outside-most points (in general speaking as shown in Fig. 3, there are four outside-most points which could be recorded as the point $P_{1,0}$, $P_{2,0}$, $P_{3,0}$ and $P_{4,0}$ in 2D point set S), the bottommost one, the rightmost one, the topmost one and the leftmost one are just the bottommost apex, the rightmost apex, the topmost apex and the leftmost apex, of a convex hull $Q$ of 2D point set S separately, so they could be still recorded as the apex $Q_{1,0}$, $Q_{2,0}$, $Q_{3,0}$ and $Q_{4,0}$ separately. The line segment $Q_{1,0}Q_{3,0}$ and $Q_{2,0}Q_{4,0}$ are called the diameters of a convex hull $Q$ of S; the point of intersection $Q_0$ of the diameters of a convex hull $Q$ of a 2D point set S is called the center of a convex hull Q. The 4 parts of distributed domain of 2D point set S is divided by the diameter line $Q_{1,0}Q_{3,0}$ and $Q_{2,0}Q_{4,0}$ are called distributed sub-domains recorded as sub-domain S1 (to which apex $Q_{1,0}$ and $Q_{2,0}$ belong), S2 (to which apex $Q_{2,0}$ and $Q_{3,0}$ belong), S3 (to which apex $Q_{3,0}$ and $Q_{4,0}$ belong) and S4 (to which apex $Q_{4,0}$ and $Q_{1,0}$ belong). The line segment $Q_{1,0}Q_{2,0}$, $Q_{2,0}Q_{3,0}$, $Q_{3,0}Q_{4,0}$ and $Q_{4,0}Q_{1,0}$ are called the initial base lines which belong to a convex hull Q.

It is important that we must pay attention to the cases of the outside-most points should be divided into many kinds. For example: the general case with 4 outside-most points is shown as Figure 3; the more general cases with 4~8 outside-most points are shown as Figure 4~8; while the degenerated cases with 3 outside-most points or less than 4 sub-domains are shown as Figure 9~18. So the upper limit case is that there are 8 outside-most points (which may be recorded as the apex $Q_{1,0}$, $Q_{1,1}$, $Q_{2,0}$, $Q_{2,1}$, $Q_{3,0}$, $Q_{3,1}$, $Q_{4,0}$, $Q_{4,1}$) and 4 sub-domains (which may be recorded as the apex $S_1$, $S_2$, $S_3$, $S_4$) separately as shown as Figure 5. But the number of the distributed sub-domains

![Figure 4. The case of the 8 outside-most points and 4 sub-domains](image)

![Figure 5. The case of the 7 outside-most points and 4 sub-domains](image)
The convex hull problem was proposed in the 20th century, and its algorithm research began in the 70’s, spreads in the 80’s, extremely in the 90’s. However since the 21st century, the traditional convex hull algorithm research appeared the awkward predicament which bogs down. In order to change the case, guided by Isomorphic Fundamental Theorem of a Convex Hull Construction and based on Isomorphic Apexes Distributing Characteristics, some new algorithms were given out by me and my researching team continuously since 2005 [9-10]. In this paper, a new convex hull algorithm is given.

A theoretical Basis of the Convex Hull New Algorithm with a Maximum Pitch of the Dynamical Base Line

The Isomorphic Fundamental Theorem of a Convex Hull Construction, proposed by Professor Zhou Qihai, is the theoretical basis of the new convex hull algorithm. The Isomorphic Fundamental Theorem of a Convex Hull Construction: The 2D point set is $S = \{P(x_i, y_i) \mid 1 \leq i \leq m, 3 \leq m < +\infty\}$, the apex set of convex polygon $Q$ is

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**Figure 6.** The case of the 6 outside-most points and 4 sub-domains

**Figure 7.** The case of the 5 outside-most points and 4 sub-domains

**Figure 8.** The case of the 4 outside-most points and 3 sub-domains

**Figure 9.** The case of the 3 outside-most points and 3 sub-domains

**Figure 10.** The case of the 4 outside-most points and 2 sub-domains

**Figure 11.** The case of the 3 outside-most points and 2 sub-domains

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If convex polygon $Q$ is the convex hull in 2D point set $S$, then:

i. (Apex to interior point) Interior Point of Convex Hull Irrelevant Theorem: All apexes $Q_i(x_i,y_i)$ of convex hull $Q$ must not be in the difference set of $S$ and $R$, namely $Q_i(x_i,y_i) \notin S-R$, $1 \leq i \leq n < \infty$, where $3 \leq n$, $3 \leq m$.

ii. (Apex to apex) Apex of Convex Hull Independent Theorem: Any apex $Q_k(x_k,y_k)$ of convex hull $Q$ must not be in the set $R-\{Q_k\}$, namely $Q_k(x_k,y_k) \notin R-\{Q_k\}$, where $1 \leq k \leq n < \infty$, $3 \leq n$.

They could be proved as follows:

Figure 12. The case of the 3 outside-most points and 1 sub-domain

Figure 13. The case of the 3 outside-most points and 1 sub-domain

Figure 14. The case of the 8 outside-most points and 1 sub-domain

Figure 15. The case of the 8 outside-most points and 0 sub-domain

Figure 16. The case of the 8 outside-most points and 0 sub-domain (all interior points are inside of the convex hull of the outside-most points)

Figure 17. The case of the 8 outside-most points and 0 sub-domain (all interior points are at the sides of the convex hull of the outside-most points)
i. Suppose any apex of convex hull $Q$ satisfied $Q_k(x_k, y_k) \in S-R$, $1 \leq k \leq n$; Then the apex $Q_k(x_k, y_k) \notin R$ (it indicates that the apex $Q_k(x_k, y_k)$ should be the interior point of the convex hull $Q$). But $R$ is the set of apexes, therefore we may know the apex $Q_k(x_k, y_k) \in R$. Obviously, both are contradictory. Therefore the first proposition is right. (It shows that any interior point of the convex hull $Q$ must not be the apex of the convex hull.)

ii. Suppose the point $Q_k$ is one of the apexes of the convex hull $Q$, then it has two most neighboring apexes at two sides of the point $Q_k$ at least, there are $Q_{k-1}$ and $Q_{k+1}$ ($Q_k \in R$; if $k-1=0$, then set the value of $k-1$ be $n$; if $k+1=n+1$, then set the value of $k+1$ be $1$). Obviously, the three points compose a triangle $Q_{k-1}Q_kQ_{k+1}$, but the position of the apex $Q_k$ is invariable, and it only relates to its coordinate position, while has nothing with the coordinate position of the points $Q_k$ and $Q_{k+1}$. Analogizes by this, we may know that the position of the apex only relate to its coordinate position, not the coordinate position of the neighboring points $Q_i$ and $Q_j$ ($i \neq k, j \neq k, i \neq j, Q_k \in R, Q_i \in R, Q_j \in R$). So any apex of the convex hull is independent in other apexes.

Therefore, the above fundamental theorem is right.

Based on the Isomorphic Fundamental Theorem of a Convex Hull Construction, the authors believed that the isomorphic improvement and optimization trend of the convex hull algorithm, which will be the main short cut to enhance the efficiency of the Convex Hull algorithm in the future without doubt, must be:

1) According to the Interior Point Irrelevant Theorem, on the one hand we should make the distributed domain of pole (actually apex) minimizing, namely makes the distributed domain of apex as small as possible to reduce the invalid process load when judging the poles; on the other hand determine object directly, namely let the object approach the poles as far as possible to largely enhances the direct focalization of the Convex Hull’s poles object.

2) According to the Convex Hull Apex Independent Theorem, on the one hand we could embark from the different initial object in order to improve, optimize and innovate new parallel Convex Hull algorithms; On the other hand, to different view of the processing, transform and create new parallel Convex Hull algorithm. Therefore, in the process of producing Convex Hull, we could reduce the distributed domain of poles as far as possible. The Convex Hull algorithm only finds out every pole (namely each end point of every side of Convex Hull) quickly and directly in as small as possible distributed domain.

Basic Definition and Structure of the New Algorithm

The following will summarize a new algorithm to find a convex hull based on dynamical base line with a maximum pitch which is according to the Isomorphic Fundamental Theorem of Convex Hull Construction.

DEFINITION 2: The distributed domain of every point in 2D point set $S=\{P(x,y) \mid 1 \leq i \leq m, 3 \leq m \leq +\infty \}$ is called the S distributed domain. In set S, the outside-most points which have the maximum or minimum x-coordinate, y-coordinate are recorded:
maximum (Namely satisfies convex hull Q). That is:

\[ P_{ij}(x_i, y_i = \min \{ y_j \mid 1 \leq i \leq m, \ 3 \leq m < +\infty \}, y_j \}, \ P_{ji}(x_j, y_j = \min \{ y_i \mid 1 \leq j \leq m, \ 3 \leq m < +\infty \}, y_i \} \], \ P_{i0}(x_0, y_0 = \max \{ x_i, \ y_i \mid 1 \leq i \leq m, \ 3 \leq m < +\infty \}, y_0 \), \ P_{0j}(x_0, y_0 = \max \{ y_j \mid 1 \leq j \leq m, \ 3 \leq m < +\infty \}) \}

The outside-most points \( P_{11}, P_{12}, P_{13} \) and \( P_{14} \) are called initial poles of convex hull Q and recorded as \( Q_{1,0}, Q_{2,0}, Q_{3,0} \) and \( Q_{4,0} \). The line segment \( Q_{1,0}Q_{3,0} \) and \( Q_{2,0}Q_{4,0} \) are called the line of demarcation of the convex hull. The intersection of the two line segments is called the centrifugal point of convex hull. The two lines of demarcation divide the original distributed domain into four sub-domains which are sub-domain I, II, III and IV called as sub-domains of 2D point set (shown as in Figure 22).

According to the above, this paper proposes a more efficient new algorithm to find a convex hull based on dynamical base line with a maximum pitch that is guided by the Isomorphism Fundamental Theorem of the Convex Hull Construction. The algorithm’s thought may be structured as follows:

Step 0: Initialization Processing.

1. Structure the line of demarcation and sub-domain of the distributed domain processing: Produce the convex hull Q’s line of demarcation \( Q_{1,0} \) and \( Q_{2,0} \), and divide the initial 2D point set \( S \) into four sub-domains I, II, III and IV, make the points \( P_i(x_i, y_i) \) satisfy \( x_i \leq x_{i+1} \) in every sub-domain.

![Figure 22. The Initial apexes, sub-domains of convex hull](image)

2. Minimize the distributed domain and its sub-domain processing: Delete the centrifugals which are in the quadrilateral \( Q_{1,0}Q_{2,0}Q_{3,0}Q_{4,0} \) (Attention: It may degenerate to triangle \( Q_{1,0}Q_{2,0}Q_{3,0} \) and the new domain is still expressed as \( S \).

Step 1: Mark the sub-domain I as \( S_1 \). Take the point \( Q_{1,0} \) as the initial pole (i.e. initial apex), take the line \( Q_{1,0}Q_{4,0} \) as the initial base line. Seek for the poles (i.e. apex) in the sub-domain \( S_1 \). Namely:

\[ P_{ij}(x_i = \min \{ x_j \mid 1 \leq j \leq m, \ 3 \leq m < +\infty \}, y_j \}, \ P_{ji}(x_j, y_j = \max \{ x_i, \ y_i \mid 1 \leq i \leq m, \ 3 \leq m < +\infty \}, y_j \} \]

In \( S_1 \), delete all interior points of triangle \( Q_{2,0}Q_{3,0}Q_{4,0} \), and the new sub-domain \( S_1 \) is still expressed as \( S_1 \). Take the line \( Q_{2,0}Q_{4,0} \) as the new base line recorded as \( Q_{2,0}Q_{4,0} \). Return to 1 of Step 1.

Step 2: Mark the sub-domain II as \( S_1 \). Take the point \( Q_{2,0} \) as the initial pole, take the line \( Q_{2,0}Q_{1,0} \) as its initial base line. Seek for the poles in the sub-domain \( S_1 \). Namely:

\[ P_{ij}(x_i = \min \{ x_j \mid 1 \leq j \leq m, \ 3 \leq m < +\infty \}, y_j \} \]

In \( S_1 \), delete all interior points of triangle \( Q_{3,0}Q_{4,0}Q_{3,0} \), and the new sub-domain \( S_1 \) is still expressed as \( S_1 \). Take the line \( Q_{3,0}Q_{4,0} \) as the new base line recorded as \( Q_{3,0}Q_{4,0} \). Return to 1 of Step 2.

Step 3: Mark the sub-domain III as \( S_1 \). Take the point \( Q_{3,0} \) as the initial pole, take the line \( Q_{3,0}Q_{2,0} \) as the initial base line. Seek for the poles in the sub-domain \( S_1 \). Namely:

\[ P_{ij}(x_i = \min \{ x_j \mid 1 \leq j \leq m, \ 3 \leq m < +\infty \}, y_j \} \]

In \( S_1 \), delete all interior points of triangle \( Q_{4,0}Q_{3,0}Q_{4,0} \), and the new sub-domain \( S_1 \) is still expressed as \( S_1 \). Take the line \( Q_{4,0}Q_{3,0} \) as the new base line recorded as \( Q_{4,0}Q_{3,0} \). Return to 1 of Step 3.

Step 4: Mark the sub-domain IV as \( S_1 \). Take the point \( Q_{4,0} \) as the initial pole, take the line \( Q_{4,0}Q_{3,0} \) as the initial base line. Seek for the poles in the sub-domain \( S_1 \). Namely:

\[ P_{ij}(x_i = \min \{ x_j \mid 1 \leq j \leq m, \ 3 \leq m < +\infty \}, y_j \} \]

In \( S_1 \), delete all interior points of triangle \( Q_{4,0}Q_{4,0}Q_{4,0} \), and the new sub-domain \( S_1 \) is still expressed as \( S_1 \). Take the line \( Q_{4,0}Q_{4,0} \) as the new base line recorded as \( Q_{4,0}Q_{4,0} \). Return to 1 of Step 4.

Finally, the convex polygon Q which is composed by the line segments which link the apexes orderly in sub-domains I, II, III and IV must be the convex hull Q in 2D limited point set.

The Mathematical Proof of the Key Technology and Core Foundation of the New Algorithm

The key technology and the core foundation of the new algorithm are the following two important propositions:

1. The apex of the convex hull Q has nothing to do with the minimizing processing of distributed domain and its sub-domain.
ii. The point $P_i$ must be an apex of convex hull $Q$ in 2D point set.

Its mathematic certification as follows:

i. In the minimizing processing of the distributed domain and its sub-domain, we only delete the interior points of the convex hull $Q$; therefore according to the Interior Point Irrelevant Theorem and the Apex Independent Theorem, we know that the apexes of convex hull $Q$ must be in the distributed domains or its sub-domain after the minimizing processing.

ii. For the purpose of unified narrating, take $Q_{1,0}$ as $Q_{5,0}$ in Figure 22, and record the initial base line $Q_{5,0}Q_{5,0}$ at the same time. As shown in Figure 23, set $Q_{i,0}$ be the initial apex of current sub-domain $S_i$ (Attention: $Q_{i,0}$ is certainly the outside most point in 2D limited point set), the next outside most point is $Q_{i+1,0}$, the initial base line as $Q_{i,0}Q_{i+1,0}$, find out the point $P_i$ of which the pitch from the base line to $Q_{i+1,0}Q_{i,k}$ is maximum. Because $P_i$ satisfies $P_i = \max\{\angle Q_{i+1,0}Q_{i,k}P_i | P_j \in S_i\}$, and also satisfies "If the points of the maximum pitch is not only one, then only leave the farthest one". Thus, any point $P_j$ in $S_1$ must satisfies $\angle Q_{i+1,0}Q_{i,k}P_j \geq \angle Q_{i+1,0}Q_{i,k}P_i$, namely the point $P_i$ must be in $\triangle Q_{i+1,0}Q_{i,k}P_i$, or just in the rim $Q_{i,k}P_i$ at the most. (Attention: To the next case, their length must be satisfied $Q_{i,k}P_i < Q_{i,k}P_i$). Therefore, the point $P_i$ must be an apex of convex hull $Q$.

VI. CONCLUSIONS

The algorithm that is proposed in this paper not only in the running time and space complexity but also the efficiency, obviously surpasses the Gift Wrapping convex hull algorithm, the Graham scan convex hull algorithm, the half-dividing convex hull algorithm and so on. Moreover, it is very easy to be transformed into its parallel algorithm. Therefore, it will enhance the speed of constructing 2D convex hull effectively, and could improve and enhance the application level and the working efficiency of the 2D convex hull in the imagery processing, the writing decomposes, the pattern recognition, the object classification, the computation graph, the fingerprint recognition, the telemeter remote control, the thing recognizes, the geological prospecting, the space & sky using, and so on.

REFERENCE