Time-stamped Generalized Predictive Control of Networked Control Systems with Random Delay

Yi Liu
Liaoning University of Technology / Computer Center, Jinzhou, China
Email: jzly71@126.com

Abstract—The key point of the networked control systems is that conventional generalized predictive control algorithm cannot deal with random delays, packet losses and vacant sampling. In this paper, the time-stamped generalized predictive control (TSGPC) with fuzzy rectification is proposed to deal with this problem. The TSGPC use time-stamped to calculate the real delay and the LMS adaptive recursive algorithm of weight coefficient are used to estimate the induced delay. In order to reduce the transmission times between the controller and the actuator, a threshold that is in advance would be used to decide whether the control information are sent to the actuator or not. The control information is ensured optional or sub-optimal by using this method. The fuzzy rectification is used to control the increasing of the prediction error. The simulation results show that the TSGPC with fuzzy rectification can trace the desired output precisely.

Index Terms—networked control systems, time-stamped, TSGPC, fuzzy rectification

I. INTRODUCTION

Networked control systems (NCS) are distributed feedback control systems closed via a shared band limited digital communication network, which connects sensor nodes, actuator nodes and controller nodes together to exchange information and control signals [1-3]. Such NCS have received increasing attentions in recent years due to their low cost, simple installation and maintenance, and high reliability. The study of NCS is an emerging interdisciplinary research area, combining among others control theory, communication theory and computer science. Significant research work has been done within the last decade on NCS, making available systematic stability analysis and design tools from a different point of view [4]. Despite lots of advantages the network brings to the control system, potential issues such as network induced time delays and packet dropouts arise that may degrade system performance and even cause system instability [5-7].

Three main issues raised in NCS are network-induced delay, data transmission dropout, and bandwidth and packet size constraints. As is known, network induced delay can significantly degrade the performance and even lead to instability of NCS, so the maximum allowable size of network-induced delay are an important indexes in the sense of guaranteeing system stability. Many researchers have studied stability analysis and controller design for NCS with linear controlled plant (namely, linear NCS) in the presence of network-induced delay [8-10]. The stability problem of NCS with short time delays is studied and the constant stabilizing state feedback gain is obtained [11]. Time based time delay analysis of the NCS is provided to explain how it affects network systems and an adaptive Smith predictor control scheme is designed [12]. Time delay is considered in an independent layer to design a stabilizing controller based on model predictive control approach [13]. The maximum allowable delay bounds are obtained for the stability of NCS and are used as the basic parameters for a scheduling method for NCS [14]. So far, the stability synthesis for the NCS with time delays has not been fully investigated.

Fuzzy control is a useful approach to solve the control problems of nonlinear systems. Takagi-Sugeno (T-S) fuzzy system [15] is a popular and convenient tool to approximate nonlinear systems because of its simple structure with local dynamics. Descriptor system, which are also referred to as singular systems, implicit systems, generalized state-space systems, differential algebraic systems, have been extensively studied for many years. A fuzzy model [16] in the descriptor form is introduced, and stability and stabilization problems for the system are addressed. Recently, more and more attention has been paid to the study of fuzzy descriptor systems [17-19]. Therefore, it is meaningful to employ fuzzy descriptor model in NCS systems design. However, to the best of our knowledge, the fuzzy descriptor NCS with time-delay has not yet been fully investigated.

Generalized predictive control algorithm that based on model predictive control is another more successful application of control algorithms [20]. Since a larger sampling period is used, GPC particularly suit to approximate nonlinear systems because of its simple structure with local dynamics. Descriptor system, which are also referred to as singular systems, implicit systems, generalized state-space systems, differential algebraic systems, have been extensively studied for many years. A fuzzy model [16] in the descriptor form is introduced, and stability and stabilization problems for the system are addressed. Recently, more and more attention has been paid to the study of fuzzy descriptor systems [17-19]. Therefore, it is meaningful to employ fuzzy descriptor model in NCS systems design. However, to the best of our knowledge, the fuzzy descriptor NCS with time-delay has not yet been fully investigated.

Generalized predictive control algorithm that based on model predictive control is another more successful application of control algorithms [20]. Since a larger sampling period is used, GPC particularly suit to the constraint conditions of communications. However, the traditional generalize predictive control algorithm can not effectively deal with the random delays and packet losses in the networked control systems [2, 21], so in this paper, the generalized predictive control algorithm based on the time-stamped function with fuzzy rectification is proposed. Using this algorithm, TSGPC is extended to handle random delays, packet losses and vacant sampling in networked control systems.

The paper is organized as follows. After the Introduction, Section II formulates the problem. Section III describes the new algorithm for NCS with random
delay. The main results are presented in section III. Number examples are included to demonstrate the power of this method in Section IV. And finally in the section V some conclusion remarks are given.

II. PROBLEM STATEMENTS

Figure 1 gives the networked control systems structure with a time-stamped of generalized predictive control function. where \( \tau^{ec} \) is the delay from the sensor to the controller and \( \tau^{ca} \) is the delay from the controller to the actuator, the computational delay of the processor is usually small when compared to network-induced delays since processor coded with the control algorithm works at a higher speed; hence, for the sake of simplicity, it is usually ignored in the analysis of NCS. Sensors, actuators and controllers keep time synchronization, single packet is transmitted. It is assumed that the sensor node, the actuator node, and the controller have an identical sampling period \( h \). \( T \) represents time-driven, \( E \) represents event-driven.

![Figure 1. The structure of Networked Control Systems.](image)

The sensor node samples the outputs, combines them into one data packet (Note: sensor-to-controller data packets contain the \( \tau^{ec} \) information, which is determined by the actuator, and then \( \tau^{ca} \) which is passed on to the sensor node is to be sent back to the controller node), and then transmits the data packet to the controller node. Each data packet is time stamped so that the delay information can be extracted at the controller node so as to indicate how old the received measurement is. At the controller, the received sensory data are stored in a shared memory. The model predictive controller and the identifier use these stored data history. At each sampling interval, \( \tau^{ec} \) is determined first, the number of measurements that need to be estimated by the minimum effort estimator to fill in the missing sensory data up to the current \( k \)-th sampling instant, is determined using \( \tau^{ec} \) and the age of the latest sensor data. Then the output of the estimated system is updated into the shared memory. Besides using in the subsequent computations of control actions, the predicted output can be used in the next sample instant if there is an event of vacant sampling or data losses. Figure 2 gives the timing diagram of the networked control systems. Where \( \tau'^{+} = 2h, \tau'^{-} = 2h \)

![Figure 2. The timing diagram of Networked Control Systems.](image)

III. DESCRIPTION OF THE ALGORITHM

The predictive networked control strategy developed in this paper assumes the packet losses will happen if the delay exceeds a pre-set time interval. It should be noted that with sensory data estimation and actuator buffering, the developed networked control strategy treats the event of out-of-order data and the event of vacant sampling the same as packet losses. This is because at a particular time instant, older data that arrive at the controller is used to replace the data histories for use in prediction and estimation. On the other hand, older data that arrive at the actuator will be discarded if newer data are available. This is true as long as the sequential occurrences of out-of-order data, vacant sampling, or packet losses, are within the worst-case delay.

Generalized predictive control use a polynomial type estimator and employs a multivariable input-output model of the CARIMA [20, 22, 23] form to describe the object that interfered by the random events:

\[
A(z^{-1})y(k) = B(z^{-1})u(k-1) + \frac{1}{\Delta}c(z^{-1})e(k).
\]

Where \( y, u, e \) is the control input vector, and the output vector, and zero-mean white noise, \( z^{-1} \) is the postpone operator, \( \Delta = 1 - z^{-1} \) is the different operator, \( A(z^{-1}) \) and \( C(z^{-1}) \) are \( p \times p \) Matrix polynomial matrices, \( B(z^{-1}) \) is a \( p \times m \) polynomial matrix.

A. Delay Estimator Design

AR model and the LMS [24-26] algorithm are used to track the random time delays in the networked control systems. \( \Gamma_i \) can be defined as:

\[
\Gamma_i = \sum_{j=0}^{\infty} \delta_i r_{i-j},
\]

where \( \delta_i \) is a weighting coefficients for \( i > 0 \). \( \epsilon_i \) can be defined as \( \epsilon_i = \tau_i - \Gamma_k \), where \( \epsilon_i \) is a fitting residual. In order to make the variance \( \epsilon_i \) is very small, \( \delta_i \) need to be dynamically adjusted. At the \( k \)-th sampling instant, the weighting coefficients vector form is expressed
as \( \delta_k = [\delta_{k1}, \delta_{2k}, \ldots, \delta_{nk}]^T \), n of delayed data up to the k-th instant is expressed as:

\[
T_k = [\tau_{k1}, \tau_{k2}, \ldots, \tau_{kn}]^T.
\]  

(2)

The AR model about time delayed on network is obtained as:

\[
\Gamma_k = \sum_{i=1}^{n} \delta_k \tau_{ki} = T_k^T \delta_k. 
\]  

(3)

In which \( \varepsilon_k = \tau_k - \delta_k^T T_k \)

\[
E[\varepsilon_k^2] = E[\varepsilon_k^2] - 2R_{TT}^T \delta_k + \delta_k^T R_{TT} \delta_k. 
\]  

(4)

In which

\[
E[\tau_k^{-1} \tau_{k-1} \tau_{k-2} \ldots \tau_{k-n} \tau_k]^T = \\
E[\bar{T}_k^T \bar{T}_k]^T = \\
E \begin{bmatrix}
\tau_{k-1} & \tau_{k-1} & \tau_{k-2} & \ldots & \tau_{k-n} \\
\tau_{k-2} & \tau_{k-2} & \tau_{k-2} & \ldots & \tau_{k-n} \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
\tau_{k-n} & \tau_{k-n} & \tau_{k-n} & \ldots & \tau_{k-n}
\end{bmatrix}
\]

Using (4), the mean square error of delay is a quadratic function of weights coefficients and a parabolic shape surface function with a unique minimum value, hence, gradient method can be used to seek the minimum value. According to the steepest descent method, \( \delta_{k+1} \) can be defined as \( \delta_{k+1} = \delta_k - \mu \hat{\nabla}(k) \), \( \mu \) is a very small number named convergence factor, \( 0 < \mu < \frac{1}{\lambda_{\text{max}}} \), in which \( \lambda_{\text{max}} \) is the maximum eigenvalue of \( R_{TT} \). It is very difficult to calculate \( \nabla(k) \) accurately, so the actual calculation is replaced with its estimate of \( \hat{\nabla}(k) \):

\[ \hat{\nabla}(k) = \hat{\nabla}E[\varepsilon_k^2] = \nabla[E[\varepsilon_k^2] = 2\varepsilon_k^T \nabla[\varepsilon_k] = -2e_k^T T_k, \]

the LMS adaptive recursive algorithm of weight coefficient is obtained as:

\[
\delta_{k+1} = \delta_k - \mu \hat{\nabla}(k) = \delta_k + 2\mu e_k T_k.
\]  

(5)

Once the networked delay has been estimated, the value is rounded. The estimate value can be used to control the NCS, at the same time, the new measurement of time delay is used to replace the data histories which has been stored in the controller memory.

B. TSGPC Design

In order to predict the required j-step-ahead future control signal that will drive the system to track a desired trajectory, an extension to the multivariable Generalized Predictive Control [27] is used for this purpose. Here an optimal set of current and future changes in control signal: \( \Delta u(k+j) \) for \( j = 0,1,2,\ldots, H_z \) is sought to continuously minimize the quadratic cost function:

\[
V_z(H_1, H_2, H_z) = \\
\sum_{j=0}^{H_z} \sum_{i=0}^{n} ||y'(k+j|k) - y_i(k+j)||_q^2 + \\
\sum_{j=0}^{H_z} \sum_{i=0}^{n} ||\Delta u(k+j-1)||_x^2.
\]  

(6)

Where \( y'(k+j|k) \) is the j-steps ahead-predicted system outputs based on the history up to the time instant k, and \( y_i(k+j) \) are the reference future trajectories. \( H_1, H_2, H_z \) are, respectively, the minimum and maximum prediction horizons, and the control horizon, where \( 1 \leq H_1 \leq H_2 \leq H_z \). The weighting sequence matrices \( Q \) and \( R \) are diagonal and positive definite.

The Diophantine equation:

\[
I_w = E_j(z^{-1}) \Delta A(z^{-1}) + z^{-1} F_j(z^{-1})
\]

Combining the Diophantine equation and (1) yield,

\[
y'(k+j|k) = F_j(z^{-1}) y(k) + E_j(z^{-1}) B(z^{-1}) \Delta u(k+j-1).
\]  

(7)

Where

\[
\text{deg}(E_j(z^{-1})) = j-1 \\
\text{deg}(F_j(z^{-1})) = \text{deg}(A(z^{-1})).
\]

This term can be separated into two parts by introducing the new equation:

\[ T(z^{-1}) = \Delta A(z^{-1}) E_j(z^{-1}) + z^{-1} F_j(z^{-1}). \]  

(8)

In which:

\[ E_j(z^{-1}) = 1 + \sum_{i=1}^{j-1} e_{ij} z^{-i} \]

\[ F_j(z^{-1}) = \sum_{i=0}^{n} f_{ij} z^{-i} \]

\( T(z^{-1}) \) is defined as filter polynomial, and generally the first order.

Here

\[ \hat{G}_j(z^{-1}) = G_j'(z^{-1}) + z^{-1-\degree} E_j(z^{-1}). \]  

(9)

In which:

\[ \hat{G}_j(z^{-1}) = B(z^{-1}) E_j(z^{-1}) \]

\[ \text{deg} \hat{G}_j = n_b + j - 1 \]

\[ \text{deg} G_j = j - d \]

\[ \text{deg} E_j = n_b - 2 \]

Combining (8), (9) and (1) yield,

\[ y_j'(k+j|k) = G_j'(z^{-1}) \Delta u_j(k+j-1) + E_j(z^{-1}) \Delta u_j(k-1) + F_j(z^{-1}) y_j(k) \]  

(10)

In which

\[ y_j(k) = y(k)/T(q^{-1}) \]

\[ u_j(k) = u(k)/T(q^{-1}) \]

\[ F_j(z^{-1}) = E_j(z^{-1}) \Delta u_j(k) + F_j(z^{-1}) y_j(k) \] is the free response term. This term can be easily computed recursively by utilizing (1) as:
is positive number, and $(\epsilon(k))$ and $\Delta \epsilon(k)$ are both positive numbers, it indicates that the prediction error trend will remain unchanged, and then the input control can be properly reduced, the output control ($\Delta U_f$) is middle center.

If $\epsilon(k)$ is positive number, and $\Delta \epsilon(k)$ is negative number, it indicates that the prediction error trend will reduce, and then the input control can be reduced to the minimum, the output control ($\Delta U_f$) is small.

IV. SIMULATION

In order to demonstrate the validity of the proposed methods, a numerical example is involved to illustrate the effectiveness of the proposed criteria. TrueTime1.5 is the simulation software.

A. Time Delay Prediction

TrueTime Network Module is adopted for randomly generating a ground of $N = 500$ delay data samples. The values thereof fluctuate at 0-6 sampling periods. The sampling period is 0.01s. The LMS adaptive recursive algorithm of weight coefficient is adopted for forecasting delay. The comparison between sample delay and delay after forecast is shown in figure 4 and 5. (Vertical axis is the size of the delay, unit: millisecond). It is observed that the LMS adaptive recursive algorithm of weight coefficient prediction method can effectively predict delay variation trend.
The networked control system consists of the sensor nodes, the actuator nodes, the controller nodes and an interference node. Fig.6 is a good example of a platform design.

The transfer function of controlled plant is defined as \( G(s) = \frac{400}{s^2 + 5s + 200} \), the discrete model is \( A(z^{-1})y(k) = B(z^{-1})u(k-1) \), in which

\[
A(z^{-1}) = 1 - 1.355z^{-1} + 0.7788z^{-2}
\]

\[
B(z^{-1}) = 0.4422 + 0.4063z^{-1}
\]

The reference signal is square signal. The square signal simulation results are shown in figure 7 and figure 8.

We conclude that using the present method, the capacity of tracking the reference signal has been remarkably improved.

The ACKNOWLEDGMENT

This work was supported in part by the School Foundation from Liaoning University of Technology of China (Grant NO. X201218).
REFERENCES


Yi Liu was born in China in 1971. She received the B.S. degree in Systems Engineering from Shanghai University of Technology in 1994, M.S. degree in Computer Application Technology from Liaoning Project Technology University in 2004 and Ph.D. degree in Control Theory and Engineering from Hebei University of Technology in 2011, respectively.

Since 1994, she served as a teacher at Department of Computer Center at Liaoning University of Technology. Her current research interests are mainly in Networked Control Systems, tracking control, optimal and stochastic control.