Object Tracking via Tensor Kernel Space Projection

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Abstract—Although there has been significant progress in the past decade, object tracking under complex environment is still a very challenging task, due to the irregular changes in object appearance. To alleviate these problems, this research developed an object tracking algorithm via tensor kernel space projection. In the initial stage of tracking, a template matching algorithm was used to obtain a priori images of the appearance of the object. The steps taken were as follows: define the tensor kernel function based on a multi-linear singular value decomposition, view the object appearance color image as tensor data, calculate the kernel matrix for the priori appearance image samples, use KPCA to obtain the projection matrix of the image samples in kernel space, and finally, obtain an optimal estimate of the object state through Bayesian sequence interference. Meanwhile, the projection matrix in kernel space was updated on-line. Experiments on two real video surveillance sequences were conducted to evaluate the proposed algorithm against two classical tracking algorithms both qualitatively and quantitatively. Experimental results demonstrate that the proposed algorithm is robust in handling occlusion and object scale changes.

Index Terms—object tracking, tensor kernel, SVD, kernel pace, projection

I. INTRODUCTION

Object tracking as an important research area in computer vision and pattern recognition, has many applications, such as: object recognition, video surveillance, traffic monitoring, human-computer interaction, video compression, weapon automatic tracking, etc. [1-3].

The chief challenge in object tracking is how to adapt to appearance variability in the object. The appearance variability includes the effects of: camera motion, camera viewpoint changes, illumination change, pose variation, shape deformation and occlusions [4]. To solve this problem, there are two types of methods based on different appearance models: generative and discriminative methods.

Generative methods model appearance of targets use gray value or other features, and predict the target state by find the image patch most similar to the target. The classical mean-shift [5-6] tracking method using the kernel function forms a weighted model of the color histogram, and defining the similarity measure by its Bhattacharyya coefficient, seeks the most similar region to the reference template by a first-order gradient descent algorithm. The CBWH tracker [7] transforms only the object model but not the object candidate model. Ross [4] uses an incremental sub-space model combined with a particle filter to adapt to appearance changes. Gao [8] uses a Mexican hat wavelet to change the mean-shift tracking kernel and embedded a discrete Kalman filter to achieve satisfactory tracking. The \( \ell \)-tracker [9] uses a visual tracking problem as a sparse approximation, obtains sparse representation by solving \( \ell \)-regularisation, and finally finds the minimum reconstruction error region. Gao [10] fused multi-feature weights through DS evidence theory and combined with a particle filter achieves better results when considering the particle degeneration phenomenon.

Discriminative methods formulate object tracking as a binary classification problem for separates the target from the background. The Support Vector Tracking [11] integrates SVM into optical flow classifier for object tracking. Avidan [12] combined weak classifiers into a strong classifier to distinguish the object with background. The online boosting tracker [13] selects features using boosting classifier. The MIL tracker [14] using Multiple Instance Learning to overcome slight inaccuracies in labeled training examples can cause drift, the samples are considered within positive and negative bags.

The authors proposed an algorithm that uses tensor kernel space projection for object tracking to estimate the object state in real-world scenarios. For the purpose of highlighting the real differences between color images, a tensor kernel based on multi-linear singular value decomposition was defined. The Bayesian sequence interference framework was combined therewith to project the observe image samples in kernel space using a kernel matrix. Experimental results showed that the...
The proposed algorithm could achieve object tracking in real-world and occlusion environments, it was also adaptive to object scale changes.

II. TENSOR KERNEL BASED ON MULTI-LINEAR SINGULAR VALUE DECOMPOSITION

A. Tensor Algebra

A tensor can be seen as a multi-order array which exists in multiple vector space, meanwhile, tensor algebra forms the mathematical basis of the multi-linear analysis [15]. A scalar is a zero-order tensor, a vector is a first-order tensor and a matrix is a second-order tensor. It is obviously that a color image is a third-order tensor. An N-order tensor can be denoted by $X^{I_1 \times I_2 \times \cdots \times I_N}$, the element of $X$ is $x_{i_1 \cdots i_N}$, where $1 \leq i_n \leq I_n$.

**Definition 1:** The n-mode unfolding matrix of a tensor data $X$ is denoted as $X^{(n)} \in \mathbb{R}^{I_n \times (I_1 \times I_2 \times \cdots \times I_{n-1} \times I_{n+1} \times \cdots \times I_N)}$.

The element $x_{i_1 \cdots i_n \cdots i_N}$ of $X$ in $X^{(n)}$ at $i_n$-th row and at $[(i_{n+1} - 1)I_{n+2} \cdots I_N I_1 \cdots I_{n-1} + \cdots + (i_N - 1)I_1 \cdots I_{n-1} + (i_1 - 1)I_2 \cdots I_{n-1} + \cdots + i_{n-1}]$-th column.

![Fig.1 The schematic diagram of tensor kernel based on multi-linear SVD.](image)

- The original data can be embedded in high-dimensional feature space through a nonlinear mapping $\varphi$. The traditional kernel function $k(x, y)$ for vector data is known as: $R^N \times R^N \to R$ and
satisfied \( k(\hat{X}, \hat{Y}) = \langle \phi(\hat{X}), \phi(\hat{Y}) \rangle \), so the kernel function \( k(x, y) \) for tensor data must obey the rule:
\[
R^{k_1 \times k_2 \times \cdots \times k_n} \times R^{k_1 \times k_2 \times \cdots \times k_n} \rightarrow R
\]
Kernel methods for non-linear models have proven successful in many computer vision fields[16]. So it is feasible to improve the discriminative power of supervised tensor-based models using a tensor kernel. The most recent tensor-based techniques are based on matrix unfolding. The multi-linear singular value decomposition of tensor data \( \tilde{X} \) can be reduced to the matrix unfolding of \( \hat{X} \), where \( \hat{X} \) is the matrix unfolding of \( X \), \( n \leq l_{ij} \),
\[
\hat{X}(\alpha) = U_{X(\alpha)} \sum_{\alpha} V_{X(\alpha)}^T
\]
where \( U_{X(\alpha)} \), \( \sum_{\alpha} \), \( V_{X(\alpha)}^T \) are the three components of this SVD, and can be stated in block-partitioned form,
\[
U_{X(\alpha)} = \begin{pmatrix} U_{1X(\alpha)}^1 & U_{1X(\alpha)}^2 \\ \sum_{\alpha} & 0 \\ 0 & 0 \end{pmatrix}
\]
\[
\sum_{\alpha} = \begin{pmatrix} \sum_{\alpha} \end{pmatrix}
\]
\[
V_{X(\alpha)}^T = \begin{pmatrix} V_{1X(\alpha)}^1 & V_{1X(\alpha)}^2 \\ V_{2X(\alpha)}^1 & V_{2X(\alpha)}^2 \end{pmatrix}
\]

**Definition 4:** Given a dataset \( X = \tilde{X}_1, \tilde{X}_2, \cdots, \tilde{X}_m \in X \), the kernel matrix of this set is an \( N \times N \) matrix \( K \), and the element \( k_{ij} \) is
\[
k_{ij} = k(\hat{X}_i, \hat{X}_j)
\]

A simple template matching object tracking method used in the first \( m \) frame images, then we can get the prior acquisition object sample tensor dataset \( \{\hat{X}_1, \hat{X}_2, \cdots, \hat{X}_m\} \).
Because \( k_{ij} \geq 0 \), the kernel matrix \( K \) is non-negative, and positive semi-definite. \( k(\hat{X}, \hat{Y}) \) correspond to the kernel function in mode \( i \) of the two tensors, and can be calculated by:
\[
k_i(\hat{X}, \hat{Y}) = \exp \left( -\frac{\|V_{1X(\alpha)}^1 \hat{X}_1 - V_{1X(\alpha)}^1 \hat{Y}_1 \|^2}{2\sigma^2} \right)
\]
Where \( \sigma \) indicates an appropriate bandwidth. Then the kernel function between tensors is defined as:
\[
k(\hat{X}, \hat{Y}) = k_1(\hat{X}, \hat{Y})k_2(\hat{X}, \hat{Y})k_3(\hat{X}, \hat{Y})
\]
The kernel function of colour image tensor data is showed as Figure1.

### III. Tensor Kernel Space Projection

Principle component analysis can only handle the linear information in a dataset, a kernel trick may be used in kernel space to distinguish object and background[17].

Given a training dataset \( \alpha_1, \alpha_2, \cdots, \alpha_n \), using kernel mapping \( \phi \) projecting them into kernel space, we get \( \phi(\alpha_1), \phi(\alpha_2), \cdots, \phi(\alpha_n) \). Let \( N = [\alpha_1, \alpha_2, \cdots, \alpha_n] \), and \( \psi(\phi(\alpha_1), \phi(\alpha_2), \cdots, \phi(\alpha_n)) \). Projecting \( \phi(\alpha) \) in kernel space along projection direction \( p \),
\[
\beta_i = p^T \phi(\alpha_i)
\]
The total dispersion of the projected training dataset is
\[
D = \frac{1}{m} \sum_{i=1}^{m} (\beta_i - \bar{\beta})(\beta_i - \bar{\beta})^T
\]
\[
= p^T \left[ \frac{1}{m} \sum_{i=1}^{m} (\phi(\alpha_i) - \bar{\alpha})(\phi(\alpha_i) - \bar{\alpha})^T \right] p
\]
Where \( \bar{\alpha} = \frac{1}{m} \sum_{i=1}^{m} \phi(\alpha_i) \) is the mean value of the dataset in kernel space. The purpose of this kernel space projection is to find the optimal projection direction \( p \) by maximizing the total dispersion of the projected training dataset \( D \). Thus,
\[
J(p) = \text{arg max} \ p^T S p
\]
To maximize the objective function, we need to find the Eigen-value decomposition to \( S \).
\[
\lambda p = Sp = \frac{1}{m} \sum_{i=1}^{m} (\phi(\alpha_i) - \bar{\phi})^T (\phi(\alpha_i) - \bar{\phi}) p
\]
\[
= \sum_{i=1}^{m} \left[ \frac{1}{m} (\phi(\alpha_i) - \bar{\phi})^T p \right] (\phi(\alpha_i) - \bar{\phi})
\]
Here \( p \) can be regarded as the feature vector of \( S \). It can also be seen that the projection direction \( p \) is the linear combination of \( (\phi(\alpha_i) - \bar{\phi}) \), Namely that
\[
\psi(N) = \left[ (\phi(\alpha_1) - \bar{\phi}), (\phi(\alpha_2) - \bar{\phi}), \cdots, (\phi(\alpha_n) - \bar{\phi}) \right]
\]
That is
\[
p = \sum_{i=1}^{m} c_i (\phi(\alpha_i) - \bar{\phi}) = \psi(N)C
\]
Where \( C = [c_1, c_2, \cdots, c_m]^T \) is the matrix which comprising the linear combination coefficients. Now, solving for the optimal projection direction was transformed into a problem whereby solving the combination coefficient matrix was required, the objective function became:
\[
J(C) = \text{arg max} \ C^T \left[ \psi(N)^T \psi(N) \right] C
\]
Let \( A_m = m \times m \) matrix with all entries equal to \( 1/m \),
\[
\hat{K} = \psi(N)^T \psi(N)
\]
Then,
\( \tilde{K} = K - A_m K - KA_m + A_m KA_m \)  \hspace{1cm} (18)

Eigen-value decomposition to \( S \),

\[ S_p = (\lambda/m)p \]  \hspace{1cm} (19)

Then

\[ \psi(N) = (\lambda/m)KC \]  \hspace{1cm} (20)

\[ \tilde{K}C = (\lambda/m)C \]  \hspace{1cm} (21)

It is can be seen that \( C \) is the matrix composed of the Eigen-vectors corresponding to the first \( l \) maximum Eigen-values of \( \tilde{K} \).

The tensor kernel space projection algorithm works as shown in Table 1:

**TABLE 1.**

**TKSP ALGORITHM**

1. Define tensor kernel function \( k(\alpha, \beta) \).
2. Compute kernel matrix \( K \) and matrix \( \tilde{K} \).
3. Eigen-value decomposition to \( K \) to obtain Eigen-values and Eigen-vectors, and sort Eigen-values in descending order \( \hat{\lambda}_1 \geq \hat{\lambda}_2 \geq \cdots \hat{\lambda}_l \geq \cdots \hat{\lambda}_k \). \( v_i \) is the eigenvector corresponding to \( \hat{\lambda}_i \). Take the first \( l \) Eigen-values, let \( \epsilon_i = \sqrt{\hat{\lambda}_i} (i = 1,2,\cdots,l) \). Then, the projection matrix \( C = [v_1,v_2,\cdots,v_l] \).
4. The projected data in kernel space are \( \beta = C^T k(N, \alpha) \).

**IV. BAYESIAN SEQUENCE INTERFERENCE FRAMEWORK FOR OBJECT TRACKING**

**A. Bayesian Sequence Interference**

In the problem of object tracking, the object state in the current frame only related to the object state in the prior frame. Define \( o_t \) as the state variables of the object at time \( t \). Let \( o_t = (x_t, y_t, w_t, h_t) \), where \( x_t, y_t, w_t, h_t \) denotes the center coordinate, the width and height of the tracking rectangle.

Given a set of targets \( Z_t = \{z_{t1}, z_{t2}, \cdots, z_{tn}\} \), the objective of object tracking is to obtain the optimal estimate value of the hidden state variables \( o_t \). According to Bayesian theorem, it can be obtained in a similar fashion as that of the object state:

\[
P(o_t | Z_t) \propto P(z_t | o_t) \prod P(o_{t-1} | o_{t-1}) P(o_{t-1} | Z_{t-1}) do_{t-1}
\]  \hspace{1cm} (22)

Where \( P(o_{t-1} | o_{t-1}) \) refers to the state transition model and \( P(z_t | o_t) \) refers to the observation model. It can be seen that the observation model \( P(z_t | o_t) \) determine the tracking results.

**State transition model:** This was used to model the object motion between consecutive frames. Because of the irregular movement of object, the state \( o_t \) is modeled by independent Gaussian distribution around its counterpart in state \( o_{t-1} \). Described as

\[
P(o_t | o_{t-1}) = N(o_t; o_{t-1}, \Delta)
\]  \hspace{1cm} (23)

Where \( \Delta \) means the diagonal covariance matrix of \( x_t, y_t, w_t, h_t \), and the element is \( \sigma^2, \sigma^2, \sigma^2, \sigma^2 \). Point to Gaussian distribution, \( N \) particles can be randomly generated. According to the particle can obtain multiple states \( \{o'_i, i = 1,2,\cdots, N\} \). During the computing process, with the increase in the number of particles, the \( a \) posteriori probability estimate was more accurate, but at the same time, the computational efficiency was low, so a balance was sought between these factors.

**Observation model:** this was used to measure the similarity between the appearance observation and the object appearance model. Given a drawn particle state \( o'_i \) and a cropped version of the corresponding image patch \( z'_t \) from the frame image \( I_t \). The probability of an image patch being generated from the kernel space is inversely proportional to the difference between image patch and the appearance model, and could be calculated between the negative exponential distance of the projected data and the center of dataset

\[
p(z'_t | o'_i) = \exp \left\{ -\|s'_t - \tilde{s}\| / \sigma \right\}
\]  \hspace{1cm} (24)

Where \( \sigma \) indicates an adjustment factor, \( \|\cdot\|_F \) is the Frobenius norm, \( s'_t = C^T k(X, z'_t) \), \( \tilde{s} = C^T k(X, \tilde{x}) \), \( C \) is the projection matrix in kernel space, and \( \tilde{x} \) is the center of dataset in kernel space. The state \( o'_i \) corresponding to the maximum \( p(z'_t | o'_i) \) is the optimal object state at time \( t \).

**B. Projection Matrix Online Update**

Along with the movement of the object in scenarios, the object appearance also changed. The projection matrix in kernel space must be updated on-line so that at time \( t \), we have obtained the former \( t - I \) time object states. When \( t - I \) is the times used as initial training dataset numbers, we recalculated the projection matrix in kernel space using the newly acquired \( t - I \) object tensor data points.

The flow chart of whole proposed object tracking algorithm is shown as figure 2.
V. COMPARATIVE EXPERIMENTS AND ANALYSIS

All the experiments were carried out in the MATLAB® 2010a environment running on a Pentium® Dual-Core E5200 processor with a clock-speed of 2.50 GHz.

In these experiments, we use Portuguese shopping center video monitoring data to test. The video frame image size was $388 \times 284 \times 3$, and the experimental data were compared with classical CBWH, and MIL, tracking algorithms. The CBWH tracking algorithm select $16 \times 16 \times 16$ bin histogram. In MIL tracking algorithm, the feature pool is 250, weak classifiers number is 50 and the learning rate is 0.85. The variance of Gaussian distribution is $[2, 2, 0.5, 0.5]$ in the proposed tracking algorithm. In the initial frame, the object tracking rectangle was manually/artificially marked, and a template matching tracking algorithm was used to get the previous twenty frame object states.

A. Experiment A

We first selected scale changes in an environment of complex surveillance scenario, meanwhile the movement of object was irregular. The object was walking toward the camera. The length of this video was 200 frames.

The tracking results of experiment A are shown as Figure 3. It can be seen from the pictures that when a similar object appeared around the target, the CBWH and MIL tracking algorithms cannot achieve well tracking; our tracking algorithm could adapt thereto. In the tracking progress, the size of the object changed from $28 \times 79$ to $44 \times 111$. 

![Fig.2 The flow chart of the proposed visual tracking algorithm.](image)
B. Experiment B

The second video had scale changes and occlusion in the surveillance scenarios studied. The object was walking away from the camera. The length of this video was 300 frames.

The tracking results from experiment B are shown in Figure 4. It can be seen from the pictures that when the object was almost occluded by other moving objects, the CBWH and MIL tracking algorithms suffered interference from the other objects, and could not have tracked the target. Our tracking algorithm overcame the problem of occlusion and tracked the target.

In the tracking progress, the size of object changed from $39 \times 112$ to $21 \times 64$. 
C. The Evaluation of Tracking

To quantitatively compare the experimental results of the tracking algorithms, we used two metrics to describe it: Tracking errors and Tracking correct ratio. We initially hand-labelled the object state in each experimental scenario.

The error in X-axis \( e_x \) is

\[
e_x = |x_e - x_g|
\]

(25)

Where \( x_e, x_g \) are the X-axis coordinates of the center of the experiment tracking rectangle and the ground truth rectangle.

The error in Y-axis \( e_y \) is

\[
e_y = |y_e - y_g|
\]

(26)

Where \( y_e, y_g \) are the Y-axis coordinates of the center of the experiment tracking rectangle and the ground truth rectangle.

The error in center coordinate \( e_c \) is

\[
e_c = \sqrt{(x_e - x_g)^2 + (y_e - y_g)^2}
\]

(27)

The error in experiment A and B are shown as Table 2 and 3. The data in bold refer to optimal results.

<table>
<thead>
<tr>
<th>Items</th>
<th>x-errors</th>
<th>y-errors</th>
<th>c-errors</th>
</tr>
</thead>
<tbody>
<tr>
<td>Proposed Method</td>
<td>1.7145</td>
<td>2.3905</td>
<td>3.2788</td>
</tr>
<tr>
<td>CBWH</td>
<td>4.0725</td>
<td>15.6325</td>
<td>16.9349</td>
</tr>
<tr>
<td>MIL</td>
<td>4.2025</td>
<td>5.0400</td>
<td>6.8776</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Items</th>
<th>x-errors</th>
<th>y-errors</th>
<th>c-errors</th>
</tr>
</thead>
<tbody>
<tr>
<td>Proposed Method</td>
<td>1.8276</td>
<td>2.4318</td>
<td>3.2209</td>
</tr>
<tr>
<td>CBWH</td>
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<td>21.0142</td>
</tr>
<tr>
<td>MIL</td>
<td>12.7800</td>
<td>19.4817</td>
<td>23.5559</td>
</tr>
</tbody>
</table>

The tracking correct ratio \( r \) is

\[
r = \frac{\text{area}(R_e \cap R_g)}{\text{area}(R_e \cup R_g)}
\]

(28)

Where \( R_e \) is the experiment tracking rectangle, \( R_g \) is the ground truth rectangle, \( \text{area}() \) means the area of the region. It was thought that if the tracking correctly ratio was greater than 0.5, the tracking in this frame was successful. The tracking correctly ratios in two scenarios are shown as Figures 5 and 6. The red line indicates the results of the proposed tracking algorithm, the green line indicates the CBWH tracking algorithm and the blue line indicates the MIL tracking algorithm. It can be seen that our proposed tracking algorithm had better results than the other two tracking algorithms. Also, the tracking correctly ratio of our algorithm was almost always greater than 0.5, implying that the algorithm was successful.
VI. CONCLUSION

In this work, we proposed the use of a tensor kernel space projection for object tracking in video surveillance scenarios. Considering the spatial structure of the object image, we defined a tensor kernel function based on multi-linear singular value decomposition. We projected the object image tensor data into kernel space, and combined this with a Bayesian sequence interference tracking framework to achieve object tracking. To adapt to the object appearance changes, we updated the projection matrix on-line and also analyzed the performance of our proposed tracking algorithm in an assessment against challenging real-world video surveillance scenarios and compared the output with two classical tracking algorithms. The experimental results demonstrated the accuracy and robustness of the proposed tracking algorithm.

REFERENCES