Depth from Defocus Based on Geometric Constraints

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Abstract—This paper proposes a Depth from Defocus (DFD) model based on geometric constraints. The two measured defocused images match with each other with this method including geometric constraints, which bypasses estimation of the radiance. These geometric constraints vary with different relative position of image plane and image focus. The experimental results on the synthetic and real images show that this method is accurate and efficient. The experimental results on the synthetic images with noise show that this method is robust to the images with Salt &Pepper and Poisson noise.

Index Terms—depth from defocus, relative spread of point spread function, geometric constraints

I. INTRODUCTION

Depth measurement is an important research field in computer vision, and it has been one of the key techniques in many fields, such as medicine, robotics and remote-sensing [1-2]. This paper focuses on the method to recover the depth map from multiple defocused images (typically two) with different camera parameters (i.e. focal length or radius of the lens) from a single viewpoint, which is so-called Depth from Defocus (DFD). Compared with other image-based depth measurement approaches, e.g., Depth from Stereo (DFS) and Depth from Motion (DFM), DFD can effectively avoid correspondence problems [3].

Since the introduction of DFD into depth measurement [2], various DFD approaches have been extensively researched and greatly developed in recent years. In order to obtain effective depth estimation from defocused images, the depth and radiance of the scene are simultaneously retrieved with earlier approaches. For example, some adopted Markov random fields to model both depth and radiance, and then minimized energy function to retrieve depth and radiance of the scene [4-5]; others formulated DFD as the problem of minimization of the discrepancy between the measured images and the model images [7-10]. These methods above can be accurate and effective since the depth and radiance of the scene were simultaneously retrieved, but they may not be suitable for practical and real-time purposes since they were based on minimization techniques, which require extensive computations. In order to avoid estimating the additional radiance, some operated DFD in the frequency domain [11-13]; others formulated them as the discriminative learning-based problem [14-15]. However, these methods have some defects in the estimation, for example, artifacts due to noise and windowing.

This paper poses depth estimation as the problem of matching the two measured defocused images with each other, which has been done by Favaro [16], rather than the discrepancy between the measured defocused images and the defocused model images. It is not necessary to estimate an additional unknown, the radiance. This paper derives geometric constraints on the relative spread of Point Spread Function (PSF) according to different relative position of image plane and image focus, unlike the work of Favaro [16], where the method can be accurate and effective by the introduction of smoothness regularization term and neighborhood regularization term, but required extensive computations. In addition to accuracy and effectiveness, this proposed method is efficient owing to these simple geometric constraints proposed. An extended enumeration method is proposed to minimize the discrepancy with the geometric constraints between the two measured defocused images. This method offers an advantage of computation and simplicity in the implementation (see Section II). In Section III, the experimental results are shown on the synthetic and real defocused images.

II. FORMALIZATION OF DEPTH FROM DEFOCUS

A. Formalized Depth from Defocus

In this subsection, we introduce the image formation model, and how to match the two measured defocused images with each other. At last, the relationship between depth and relative spread of PSF is given.

The geometry of the basic image formation process in real aperture camera is shown in Fig.1 [17-18]. When the object point is in focus, the formula \( f_D + f_v - f_F \) indicates, by the lens law, the relationship between object distance \( D \), focal length \( F \), and image focus-to-lens distance \( v \); when the object point is not in focus, its image is no longer a point but a blurred circle whose radius \( r \) is described by a blur parameter \( \sigma \) defined as [16]

\[
\sigma = \rho v_0 \sqrt{1 - \frac{1}{F} - \frac{1}{v} - \frac{1}{D}}
\]  

(1)
where $\sigma$ is also called the spread of PSF, $v_0$ is the radius of lens aperture, $v_0$ is the image plane-to-lens distance, and $\rho$ is a camera constant that depends on the sampling resolution on the image plane. According to (1), if $v_0 < v$, then $1/F < 1/D + 1/v_0$, else $1/F \geq 1/D + 1/v_0$.

A defocused image $I: \mathbb{Z}^2 \rightarrow \{0,1\}$ is described with the linear model as

$$I(y) = \int_{\Omega} h_{\sigma}(y, x) f(x) dx$$  \hspace{1cm} (2)$$

where $f: \Omega \rightarrow \{0,1\}$ is the radiance of the scene, $h_{\sigma}$ denotes the PSF of the camera that depends on camera parameters and the depth of scene $D: \mathbb{Z}^2 \rightarrow [0, \infty]$, $y = [y_1, y_2]^{T}$ lies on the image plane and $x = [x_1, x_2]^{T}$ parameterizes point in 3D space. More specifically, the PSF in (2) is often approximated by a Gaussian kernel [17]:

$$h_{\sigma}(y, x) = \frac{1}{2\pi\sigma^2} e^{-\frac{\|y-x\|^2}{2\sigma^2}}$$  \hspace{1cm} (3)$$

Note that other common PSF (e.g. Pillbox function) may be chosen besides the Gaussian kernel in (3).

In DFD, the two measured defocused images $I_1$ and $I_2$ are obtained with different camera parameters respectively. Notice that, in this paper, the camera parameters (the radius of lens aperture and focal length) are invariant except for image plane-to-lens distances. Correspondingly, $\sigma_1$ and $\sigma_2$ denote the spreads of PSF in the two measured defocused images $I_1$ and $I_2$ respectively.

Generally, the problem of DFD can be formulated as the minimization of the discrepancy between the measured defocused images and the defocused model images in (2) [5-8]. However, this requires the estimation of an additional unknown, the radiance. To avoid estimating the radiance, this paper follows the work of Favaro [16] that one defocused image is blurred with a kernel to match with the other defocused image. The idea is to further blur with a kernel one defocused image until it matches the other.

When $y \in \Sigma = \{y : \sigma_1^2 > \sigma_2^2\}$, the defocused image $I_2$ is blurred with a kernel until it matches the defocused image $I_1$, the approximation model is written by

$$I_1(y) = \int h_{\sigma_1}(y, x) f(x) dx$$

$$= \int h_{\sigma_2}(y, \overline{y}) I_2(\overline{y}) d\overline{y}$$  \hspace{1cm} (4)$$

When $y \in \Sigma^c = \{y : \sigma_1^2 < \sigma_2^2\}$, the defocused image $I_1$ is blurred with a kernel until it matches the defocused image $I_2$, the approximation model is written by

$$I_2(y) = \int h_{\sigma_2}(y, x) f(x) dx$$

$$= \int h_{\sigma_1}(y, \overline{y}) I_1(\overline{y}) d\overline{y}$$  \hspace{1cm} (5)$$

where (4) holds in $\Sigma = \{y : \sigma_1^2 > \sigma_2^2\}$, (5) holds in the complementary domain $\Sigma^c = \{y : \sigma_1^2 < \sigma_2^2\}$, and the relative spread $\Delta \sigma$ is defined as $\Delta \sigma = \sqrt{\sigma_1^2 - \sigma_2^2}$ for all $y \in \Sigma$ and as $\Delta \sigma = -\sqrt{\sigma_2^2 - \sigma_1^2}$ for all $y \in \Sigma^c$.

To simplify the notation, we define

$$\hat{I}_1(y) = \int h_{\sigma_1}(y, \overline{y}) I_2(\overline{y}) d\overline{y}$$

$$\hat{I}_2(y) = \int h_{\sigma_2}(y, \overline{y}) I_1(\overline{y}) d\overline{y}$$  \hspace{1cm} (6)$$

The discrepancy between one measured defocused image and the defocused model image obtained by the other measured defocused image is denoted by

$$\Phi(\Delta \sigma) = \int \hat{I}_1(y) - I_1(y) dy + \int \hat{I}_2(y) - I_2(y) dy$$

$$= \int H(\Delta \sigma(y)) \hat{I}_1(y) - I_1(y) dy$$

$$+ \int (1 - H(\Delta \sigma(y))) \hat{I}_2(y) - I_2(y) dy$$  \hspace{1cm} (7)$$

where $H$ denotes the Heaviside function. The function (7) is minimized with extended enumeration method. In this paper, the camera parameters (the radius of lens aperture and focal length) are invariant except for image plane-to-lens distances. Therefore, the estimation of depth $D$ can be obtained from the relative spread $\Delta \sigma$ via

$$[D(y)]^{-1} = \frac{1}{\frac{1}{v_1 + v_2} - \frac{1}{v_1 + v_2}}$$

$$\times \left[1 + \frac{\Delta \sigma(y)\Delta \sigma(y)}{\rho^2v_0^2} \frac{1}{v_1 + v_2} \right] \frac{1}{v_1 + v_2}$$  \hspace{1cm} (8)$$

More details on the formula (8) are reported in [19].
Figure 2. Geometry of image formation at different position relationship of distance of image focus \( v \), the first image plane \( v_1 \), and the second image plane \( v_2 \) and the focal length \( F \). (a) Geometry of image formation, if \( F < v < v_1 \). (b) Geometry of image formation, if \( v_1 < v < 2F \). (c) Geometry of image formation, if \( v_1 < v < (v_1 + v_2)/2 \). (d) Geometry of image formation, if \( (v_1 + v_2)/2 < v < v_2 \).

**B. Geometric constraints on relative spread of PSF**

In order to obtain effective depth estimation and improve the efficiency of searching algorithm, this paper discusses a series of constraints on \( \Delta \sigma \) according to different relative position of image plane and image focus. According to convex imaging law, if camera acquires the images that are inverted, reduced and real, the relationship between \( v \) and \( F \) satisfies \( F < v < 2F \). Therefore, different position relationship between distance of image focus \( v \), the first image plane \( v_1 \), the second image plane \( v_2 \) and the focal length \( F \) determines the geometric constraints of the relative spread of PSF, which can be stated as the following.

(i) As is shown in Fig.2(a), the distance of image focus \( v \) satisfies \( F < v < v_1 \), so the constraint of the relative spread of PSF is denoted by

\[
\rho^2 r_0^2 \frac{v_1 - v_2}{v_1 + v_2} \left( \frac{v_1^2 + v_2}{v_1 + v_2} - \frac{v_1^2}{F} \right) \leq \Delta \sigma |\Delta \sigma| <
\]

\[
\rho^2 r_0^2 \frac{v_1 - v_2}{v_1 + v_2} \left( \frac{v_1^2 + v_2}{v_1 + v_2} - \frac{v_1^2}{v_1^2} \right) < \Delta \sigma |\Delta \sigma| < \rho^2 r_0^2 \frac{v_1 - v_2}{v_1 + v_2} \left( \frac{v_1^2}{v_1^2} - 1 \right)
\]

(ii) As is shown in Fig.2(b), the distance of image focus \( v \) satisfies \( v_1 < v < 2F \), so the constraint of the relative spread of PSF is denoted by

\[
\rho^2 r_0^2 \frac{v_1 - v_2}{v_1 + v_2} \left( \frac{v_1^2}{v_1 + v_2} - \frac{v_1^2}{2F} \right) < \Delta \sigma |\Delta \sigma| <
\]

\[
\rho^2 r_0^2 \frac{v_1 - v_2}{v_1 + v_2} \left( \frac{v_1^2}{v_1 + v_2} - \frac{v_1^2}{2F} \right) < \Delta \sigma |\Delta \sigma| < \rho^2 r_0^2 \frac{v_1 - v_2}{v_1 + v_2} \left( \frac{v_1^2}{v_1^2} - 1 \right)
\]

(iii) As is shown in Fig.2(c), the distance of image focus \( v \) satisfies \( v_1 < v < \frac{v_1 + v_2}{2} \), so the constraint of the relative spread of PSF is denoted by

\[
\rho^2 r_0^2 \frac{v_1 - v_2}{v_1 + v_2} \left( \frac{v_1^2}{v_1 + v_2} - \frac{v_1^2}{2F} \right) < \Delta \sigma |\Delta \sigma| < \rho^2 r_0^2 \frac{v_1 - v_2}{v_1 + v_2} \left( \frac{v_1^2}{v_1^2} - 1 \right)
\]

(iv) As is shown in Fig.2(d), the distance of image focus \( v \) satisfies \( \frac{v_1 + v_2}{2} < v < v_2 \), so the constraint of the relative spread of PSF is denoted by
\begin{align}
0 < \Delta \sigma | \Delta \sigma | < \rho^2 r_0^2 \frac{v_1 - v_2}{v_1 + v_2} \left( \frac{v_1^2}{v_2^2} - 1 \right) \tag{12}
\end{align}

Since set \( v_i < v_j \) in all discussion, (9) and (11) determine \( \Delta \sigma < 0 \), while (10) and (12) determine \( \Delta \sigma > 0 \). Additionally, notice that geometry of imaging process for any object may be decomposed into arbitrary combination of four kinds of geometry of imaging processes above in Fig.2, as is shown in Fig.3.

**Step 2:**

- **Step 3:** To obtain the optimal solution \( \Delta \sigma \), solve the optimization problem with (7) and the interval constraint determined in **Step 2** with the extended enumeration method proposed.

**Step 4:**

According to the relationship of estimation of the depth \( D \) and the relative spread \( \Delta \sigma \) in (8), estimate the depth of the scene.

### III. Experimental Results

This section describes the results from a series of experiments designed to validate the new proposed DFD algorithm. We use two groups of synthetic defocused images and two groups of real defocused images to test it. In simulated experiments, we reconstruct depth information of synthetic stair scene and cosine plane. Furthermore, we compute the mean and standard deviation of the estimated depth of stair scene with and without various noises (such as Gaussian, Salt & Pepper, and Poisson noise) at different depth level. In real experiments, we reconstruct depth information of two groups of real defocused images.

#### A. Experimental Results on Simulated Images without Noise

To evaluate the performance of the proposed DFD algorithm, we reconstruct the depth information of synthetic piecewise smooth surface (stair scene) and continuous smooth surface (cosine plane) without noise.

In first simulated experiment, the scene was composed of 21 horizontal stripes of 21×210 pixels, which are placed from 650mm to 850mm in equidistantly ascending depths as we move from top to the bottom of the scene. Every stripe was generated by the same random radiance but with different equifocal planes. Two defocused images were captured by bringing the plane at 650mm and 850mm into focus in front of a camera with a 35mm lens and F-number 4 respectively, which are shown in Fig.4(a.1) and Fig.4(a.2) respectively; Fig.4(b.1) and Fig.4(b.2) show true depth map and estimated depth map of stair scene respectively; Fig.4(c.1) and Fig.4(c.2) show true mesh of depth and estimated mesh of depth of cosine plane respectively. From Fig.4(b.1-b.2) and Fig.4(c.1-c.2), we can see that the estimated depth is very close to the true depth and it is hard to see the difference between them except for edges in images.

In second simulated experiment, the scene was obtained by the cosine plane of 257×257 pixels such that depth = 750 + 10 \cos (\pi x / 64) \), in which depth variation is only related to x-direction and not related to y-direction. Two defocused images were captured by bringing the plane at 650mm and 850mm into focus in front of a camera with a 35mm lens and F-number 4 respectively, which are shown in Fig.5(a.1) and Fig.5(a.2) respectively; Fig.5(b.1) and Fig.5(b.2) show true depth map and estimated depth map of cosine plane respectively; Fig.5(c.1) and Fig.5(c.2) show true mesh of depth and estimated mesh of depth of cosine plane respectively. From Fig.5(b.1-b.2) and Fig.5(c.1-c.2), we can see that...
the estimated depth is very close to the true depth and it is hard to see the difference between them.

The experimental results show that the continuous smooth surface (cosine plane) is superior to the piecewise smooth surface (stair scene) in the depth estimation by using the proposed DFD algorithm, because the numbers of edge in cosine plane are less than those in stair scene.

Figure 4. Performance test for the proposed algorithm with synthetic piecewise smooth surface (stair scene). (a.1) defocused image in near focus. (a.2) defocused image in far focus. (b.1) the true depth map. (b.2) the estimated depth map. (c.1) the true mesh of depth. (c.2) the estimated mesh of depth.

Figure 5. Performance test for the proposed algorithm with synthetic continuous smooth surface (cosine plane). (a.1) defocused image in near focus. (a.2) defocused image in far focus. (b.1) the true depth map. (b.2) the estimated depth map. (c.1) the true mesh of depth. (c.2) the estimated mesh of depth.

B. Experimental Results on Simulated Images with Noise

To evaluate the robustness of the proposed DFD algorithm, we compare the estimated depth information from defocused images without noise with that from defocused images with noises (such as Gaussian, Salt & Pepper, and Poisson noise). In this subsection, all experiments are tested on the stair scene.
Tab. 1 shows that Root Mean Square (RMS) of estimated depth from defocused images without noise is compared with that from defocused images with noise (such as Gaussian, Salt & Pepper, and Poisson noise). Tab. 1 also shows that RMS of estimated depth without noise approximates that with Salt & Pepper and Poisson noise, but that without noise differ greatly from that with Gaussian noise. Additionally, RMS of estimated depth with Salt & Pepper noise slightly varies at different levels.

### Table 1.

<table>
<thead>
<tr>
<th>Noise</th>
<th>No Gaussian</th>
<th>Salt &amp; Pepper</th>
<th>Poisson</th>
</tr>
</thead>
<tbody>
<tr>
<td>Level</td>
<td>0.01</td>
<td>0.02</td>
<td>0.05</td>
</tr>
<tr>
<td>RMS mm</td>
<td>2.1154</td>
<td>13.2228</td>
<td>19.6054</td>
</tr>
</tbody>
</table>

Fig. 6 shows that mean and standard deviation of estimated depth from defocused images without noise is compared with that from defocused images with noise (such as Gaussian Salt & Pepper, and Poisson noise). From Fig. 6, we can see that mean and standard deviation of estimated depth without noise is very close to that with Salt & Pepper and Poisson noise and it is hard to see the difference between them, but that above are distinct from mean and standard deviation of estimated depth with Gaussian noise.

In summary, these above show that the proposed DFD algorithm is robust to Salt & Pepper and Poisson noise except for Gaussian noise.

Figure 6. Robustness test for the proposed DFD algorithm. (a) Mean and standard deviation of the estimated depth from defocused images without noise. (b) Mean and standard deviation of the estimated depth from defocused images with Gaussian noise at the value of variance 0.02. (c) Mean and standard deviation of the estimated depth from defocused images with Salt & Pepper noise at the 0.02 level. (d) Mean and standard deviation of the estimated depth from defocused images with Poisson noise.

Figure 7. Detail of two 240×320 defocused images and estimated depth map after median filtering by the proposed DFD algorithm. (a) Defocused image in near focus. (b) Defocused image in far focus. For more details on the scene and camera settings, please refer to [20]. (c) Estimated depth map after median filtering.
These geometric constraints vary with different relative constraints, which bypasses estimation of the radiance. The two measured defocused images match can be considered for future work.

The two datasets, the two camera parameters (the radius of lens aperture and focal length) are invariant except for image plane-to-lens distance.

In the two subsections, we test the proposed DFD algorithm on real images that are publicly available [12, 17], where also specifications and settings of the camera can be found. In the two datasets, the two camera parameters (the radius of lens aperture and focal length) are invariant except for image plane-to-lens distance.

V. Conclusion

This paper proposes DFD model based on geometric constraints. The two measured defocused images match with each other with this method including geometric constraints, which bypasses estimation of the radiance. These geometric constraints vary with different relative position of image plane and image focus. The experimental results on synthetic defocused images without noise show that this method is more applicable to the continuous smooth surface (cosine plane) than piecewise smooth surface (stair scene). The experimental results on the synthetic images with noise show that this method is robust to Salt &Pepper and Poisson noise except for Gaussian noise. Research on piecewise smooth surface (stair scene) and the image with Gaussian noise can be considered for future work.

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