Finite Difference Migration Imaging of Magnetotellurics

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Abstract—we put forward a new migration imaging technique of Magnetotellurics (MT) data based on improved finite difference method, which increased the accuracy of difference equation and imaging resolution greatly. We also discussed the determination of background resistivity and reimaging. The processing results of theoretical model and case study indicated that this method was a more practical and effective for MT imaging. Finally the characteristics of finite difference migration imaging were summarized and the factors which can affect the migration imaging were analyzed.

Index Terms—Magnetotellurics, Migration imaging, Finite difference

I. INTRODUCTION

With the deepening of oil and gas exploration, the exploration blocks have become complex increasingly. MT sounding method with the restrictions of other geophysical exploration methods has become an increasingly important exploration method [1-3]. The range of MT method is much wider than seismic exploration and the probing depth also deeper than seismic and other methods. Therefore, in deep and complex lithology region, MT has certain advantages, especially for deep earth structure and the distribution of igneous rocks. It can also benefit to searching for a high impedance strata of deposited depression with oil and gas structures in the overthrust fault zone and volcanic rocks [4-5].

MT imaging techniques have been improved considerably in recent years on the basis of propagation similarity between the EM wave in the conductive media and the seismic wave in the elastic media. The PSI (Pseudo Seismic Interpretation) method can provide the average velocity of the EM wave depending on the similar process of reflective seismic method. The phase-shift migration method realized the downward continuation of the separated MT and used the U/D theory to image the subsurface conductivity boundaries by importing the advanced seismic wave equation theory such as phase-shift migration and PSPI [6-7]. Take the seismic reverse time concept as reference, Zhdanov put forward a new method of resistivity imaging based on frequency-domain electromagnetic migration [8-10]. Instead of transforming the diffusive field into wave field, this method transferred the principles of wave field analysis to interpretation of the EM field and the diffused time-reversed fields were called migrated fields. This method could be applied for determining both the position and resistivity imaging of the anomalous structures or interfaces [11-13].

The methods have been significantly improved in the recent years. However, there are still no fully developed approaches to image the resistivity property itself, which is the key problem in the inversion of EM data [14-15]. We present a new method of the resistivity imaging based on frequency-domain electromagnetic migration.

II. MIGRATION IMAGING OF MT

Zhdanov put forward the idea of electromagnetic field offset. The imaging problems of electromagnetic field offset and elastic wave seismic exploration offset is similar. It puts the analysis principle of seismic wave field method into the interpretation of the electromagnetic field under the conditions of the diffusion equation, instead of putting the electromagnetic wave meeting the conditions of the diffusion field into a wave field. The elastic wave field satisfies the general wave equation, while the electromagnetic field in our study accords with the diffusion equation. There is a similar mechanism for electromagnetic field and the elastic wave field. The imaging function named map can characterize the interface of different mediums [16], that is

$$\text{map}(x,z) = \frac{u(x,z,t_u)}{d(x,z,t_d)}$$

where $u$ and $d$ denote the up going and down going wave functions respectively, $x$, $z$ indicate the horizontal and vertical coordinates respectively, and $t_u$, $t_d$ represent the travel time of up going and down going waves. The same phase of incident wave and reflected wave indicates that the time for the initial incidence is equivalent to the beginning of reflection. The electromagnetic field received on the earth surface is the superimposed result of
the up going and down going fields. The up going field shows a kind of back propagation that goes from the observation surface into the earth. In other words, it is a superimposed field of various kinds of source fields with different mediums under the ground. The migration imaging method can relocate the up going field to each source field successively, i.e., the interface of different mediums in a reverse time transmission[17-19].

A. TE polarization mode

Suppose \( y \) indicates the landscape orientation of underground target in 2D (with variable property of electricity). When the variation of medium resistivity is not obvious, we can express the component \( y \) of electromagnetic field approximately as follows

\[
E_y(x, z, \omega) = G_E^d(x, z, \omega) \exp(ik_n z)
\]

\[
+G_E^u(x, z, \omega) \exp(-ik_n z)
\]

where \( G_E^d, G_E^u \) are unknown coefficients relating to depth, \( k_n = \sqrt{i\omega\mu_r\sigma_r(x, z)} \), \( \sigma_r(x, z) \) is background conductivity. Down going wavefield and up going wavefield of electric field are represented as

\[
H_y^d(x, z, \omega) = G_M^d(x, z, \omega) \exp(ik_n z)
\]

\[
H_y^u(x, z, \omega) = G_M^u(x, z, \omega) \exp(-ik_n z)
\]

The up going and down going fields satisfy Helmholtz equation,

\[
\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z^2} H_y^{u,d}(x, z, \omega)
\]

\[
+k_n^2(x, z, \omega)H_y^{u,d}(x, z, \omega) = 0
\]

Integrate equation (2) and (4)

\[
\frac{\partial^3 G_E^d}{\partial x^2 \partial z^2} + \frac{\partial^3 G_E^u}{\partial z^2} + (2i\frac{\partial k_n}{\partial x}) \frac{\partial G_E^d}{\partial x} + (2i\frac{\partial k_n}{\partial x} + 2ik_n) \frac{\partial G_E^d}{\partial z} - (2k_n z - 2i) \frac{\partial k_n}{\partial z} G_E^d = 0
\]

The above equation to \( z \) for partial derivative can get

\[
\frac{\partial^3 G_E^d}{\partial x^2 \partial z^2} + \frac{\partial^3 G_E^u}{\partial z^2} + (2i\frac{\partial k_n}{\partial x}) \frac{\partial G_E^d}{\partial x} + (2i\frac{\partial k_n}{\partial x} + 2ik_n) \frac{\partial G_E^d}{\partial z} - (2k_n z - 2i) \frac{\partial k_n}{\partial z} G_E^d = 0
\]
\( \frac{\partial^3 G_{E,u}^{du}}{\partial x^2 \partial z} = \pm (2ik_n + 2iz) \frac{\partial^2 G_{E,u}^{du}}{\partial x^2} \) \\
\[ \pm (2iz \frac{\partial k_n}{\partial x} \frac{\partial^2 G_{E,u}^{du}}{\partial x \partial z} + (4k_n z \pm 2i) \frac{\partial k_n}{\partial x} + 2k_n \frac{\partial k_n}{\partial z} + 4z^2) \frac{\partial G_{E,u}^{du}}{\partial z} ] \\
\[ - (4k_n^2 z + 2 \frac{\partial k_n}{\partial x} + 2k_n \frac{\partial k_n}{\partial z} + 4z^2) \frac{\partial G_{E,u}^{du}}{\partial x} \] \\
\[ \frac{\partial^3 G_{E,M}^{du}}{\partial x^2 \partial z} = \pm \frac{\partial^2 G_{E,M}^{du}}{\partial x \partial z} \] \\
\[ \pm (2iz \frac{\partial k_n}{\partial x} \frac{\partial^2 G_{E,M}^{du}}{\partial x \partial z} + (4k_n z \pm 2i) \frac{\partial k_n}{\partial x} + 2k_n \frac{\partial k_n}{\partial z} + 4z^2) \frac{\partial G_{E,M}^{du}}{\partial z} ] \\
\[ - (4k_n^2 z + 2 \frac{\partial k_n}{\partial x} + 2k_n \frac{\partial k_n}{\partial z} + 4z^2) \frac{\partial G_{E,M}^{du}}{\partial x} \] \\
\[ \frac{\partial^3 G_{M,u}^{du}}{\partial x^2 \partial z} = \pm (2ik_n + 2iz) \frac{\partial^2 G_{M,u}^{du}}{\partial x^2} \]

(9)

(10)

(16)

(17)

(18)

(19)

(20)

(21)

\( \frac{\partial^3 G_{E,M}^{pu}}{\partial x^2 \partial z} = (2k_n + 2z) \frac{\partial^2 G_{E,M}^{pu}}{\partial x^2} \) \\
\[ \pm (2ik_n + 2iz) \frac{\partial^2 G_{E,M}^{pu}}{\partial x \partial z} \] \\
\[ - (2z \frac{\partial k_n}{\partial x} \frac{\partial^2 G_{E,M}^{pu}}{\partial x \partial z} + (4k_n z + 2i) \frac{\partial k_n}{\partial x} + 2k_n \frac{\partial k_n}{\partial z} + 4z^2) \frac{\partial G_{E,M}^{pu}}{\partial z} ] \\
\[ - (4k_n^2 z + 2 \frac{\partial k_n}{\partial x} + 2k_n \frac{\partial k_n}{\partial z} + 4z^2) \frac{\partial G_{E,M}^{pu}}{\partial x} \] \\
\[ \frac{\partial^3 G_{M,u}^{pu}}{\partial x^2 \partial z} = (2k_n + 2z) \frac{\partial^2 G_{M,u}^{pu}}{\partial x^2} \] \\
\[ \pm (2ik_n + 2iz) \frac{\partial^2 G_{M,u}^{pu}}{\partial x \partial z} \] \\
\[ - (2z \frac{\partial k_n}{\partial x} \frac{\partial^2 G_{M,u}^{pu}}{\partial x \partial z} + (4k_n z + 2i) \frac{\partial k_n}{\partial x} + 2k_n \frac{\partial k_n}{\partial z} + 4z^2) \frac{\partial G_{M,u}^{pu}}{\partial z} ] \\
\[ - (4k_n^2 z + 2 \frac{\partial k_n}{\partial x} + 2k_n \frac{\partial k_n}{\partial z} + 4z^2) \frac{\partial G_{M,u}^{pu}}{\partial x} \]

D. Joint imaging of TE and TM polarization modes

For the TE and TM polarization modes, the apparent electric reflection coefficient is the ratio of up going wave and down going wave according to the U/D imaging principle

\( \beta_{E_u}(x, z, \omega) = \frac{E^u(x, z, \omega)}{E^d(x, z, \omega)} \) \\
\( \beta_{M_u}(x, z, \omega) = \frac{E^u(x, z, \omega)}{E^d(x, z, \omega)} \)

(16)

(17)

Normalizing electrical and magnetic reflection coefficients

\( \beta_{E_u}^{n}(x, z, \omega) = \frac{\beta_{E_u}(x, z, \omega)}{||\beta_{E_u}(x, z, \omega)||} \) \\
\( \beta_{M_u}^{n}(x, z, \omega) = \frac{\beta_{M_u}(x, z, \omega)}{||\beta_{M_u}(x, z, \omega)||} \)

(18)

(19)

The superimposed equation is

\( \beta_{E,M,u}^{n}(x, z, \omega) = \frac{1}{N} \sum_{n=1}^{N} \beta_{E,M,u}^{n}(x, z, \omega) \)

(20)

Realize the resistivity imaging based on the normalized migrated reflectivity function \( \beta_{E,M,u}^{n}(x, z, \omega) \) and the apparent reflectivity function \( \beta_{E,M,u}(x, z, \omega) \). For inhomogeneous background resistivity \( \rho_n(x, z) \), we can calculate the migrated apparent resistivity \( \rho_n(x, z) \) as

\( \rho_n(x, z) = \left[ \frac{1 + \beta_{E,M,u}^{n}(x, z) \beta_{E,M,u}^{n}(x, z)}{1 - \beta_{E,M,u}^{n}(x, z) \beta_{E,M,u}^{n}(x, z)} \right]^{\frac{1}{2}} \rho_n(x, z) \)

(21)

Furthermore, another improvement also has been made by joining the TE and TM polarized mode together using
weighted coefficient \((0 < \lambda_1, \lambda_2 < 1, \lambda_1 + \lambda_2 = 1)\) to achieve a joint imaging result

\[
\rho_m(x, z) = \left[ \frac{1 + (\lambda_1 \beta_{En}^n(x, z) + \lambda_2 \beta_{Ma}^n(x, z))(\lambda_1 \beta_{En}^p(x, z) + \lambda_2 \beta_{Ma}^p(x, z))}{1 - (\lambda_1 \beta_{En}^n(x, z) + \lambda_2 \beta_{Ma}^n(x, z))(\lambda_1 \beta_{En}^p(x, z) + \lambda_2 \beta_{Ma}^p(x, z))} \right] \times (\lambda_1 \rho_{En}(x, z) + \lambda_2 \rho_{Ma}(x, z))
\]

(22)

Two modes joint imaging gives prominence to the consistency of the same geoelectric model and reinforces the single information.

**III. MODEL TEST AND REAL OBSERVED MT DATA IMAGING**

Devise a geoelectric model as shown in Figure 1: the resistivity of first layer is \(100 \, \Omega \cdot m\) and thickness is 1000m. Oblique bottom is at 1400m. The resistivity of second layer is \(10 \, \Omega \cdot m\) and thickness is 1000m. The resistivity of the third layer is \(100 \, \Omega \cdot m\).

Figure 2 gives the result of TE, TM joint imaging. We can see that the migrated result is better and clearly reflects the syncline structure.

![Figure 1. Schematic Diagram of Syncline](image1)

![Figure 2. The Output of the Two Modes of TE, TM Joint Imaging](image2)

Figure 1. Schematic Diagram of Syncline

Figure 2. The Output of the Two Modes of TE, TM Joint Imaging

We use the method to deal with the MT data of NE region of the Junggar Basin, China. The gobi and desert are the main terrain in this region.

![Figure 3](image3)

Figure 3

a: Geological profile interpretation chart. b: The result of continuous rapid inversion. c: The result of Two Modes of TE, TM joint imaging.

From the second figure with the third one, the result of two modes of TE, TM joint imaging is more clear and closer to the real geoelectric model, indicating the method of two modes of TE, TM joint imaging better than continuous rapid inversion. However, if using conventional inversion to obtain the preliminary estimate of the underground structure firstly, the resistivity values are similar to the actual resistivity distribution, with the result as the background conductivity offset imaging effect will be better. So the two methods complement each other in practice in order to serve better for MT interpretation.

**IV. CONCLUSION**

This paper put forward a new multi-parameter MT migration imaging technique using the improved finite difference method, which had greatly increased the imaging accuracy and resolution. It is a multi-parameter, multi-mode imaging method and can simultaneously obtain information on the interface and resistivity. In comparison with conventional inversion, two modes of TE, TM joint imaging can get better vertical resolution with less false information.

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