Adaptive Tracking Control for Nonaffine Nonlinear Systems with Zero Dynamics

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Abstract—A direct adaptive neural network tracking control scheme is presented for a class of nonaffine nonlinear systems with zero dynamics. The method does not assume boundedness on the time derivative of a control effectiveness term. Parameters in neural networks are updated using a gradient descent method which designed in order to minimize a quadratic cost function of the error between the unknown ideal implicit controller and the used neural networks controller. The final updated law is a nonlinear function of output error. No robust control term is used in controller. The convergence of parameters and the uniformly ultimately bounded of tracking error and all states of the corresponding closed-loop system are demonstrated by Lyapunov stability theorem. Simulation results illustrate the availability of this method.

Index Terms—nonaffine nonlinear, neural network, tracking control, gradient descent method, zero dynamic

I. INTRODUCTION

Modern mechanical or electrical systems that are to be controlled become more and more complicated and, thus, their mathematical models are often hard to be established. In recent years, adaptive neural network [1, 2, 3, 4, 7, 8, 9, 10, 12, 15] that model the functional mechanism of the human brain and fuzzy logic control [5, 6, 11, 13, 14, 16, 17, 18] that can cooperate with human expert knowledge have been successfully applied to many control problems because they need no accurate mathematical models of the system under control. These methodologies become especially more helpful if control of highly uncertain, nonlinear and complex systems is the design issue. The main philosophy that is exploited heavily in system theory applications is the universal function approximation property of neural networks or fuzzy logic. Benefits of using neural networks or fuzzy logic for control applications include its ability to effectively control nonlinear plants while adapting to unmodeled dynamics.

In fact, most of the works [1-4, 6, 9, 11, 12, 13, 15-18, 22-25] are devoted to the control problem of the affine-in-control nonlinear systems, i.e., systems characterized by inputs appearing linearly in the system state equation. Few results are available for nonaffine nonlinear systems where the control input appears in a nonlinear fashion [5, 7, 8, 10, 14, 19, 21]. In general, a two-step procedure is taken in nonaffine nonlinear systems. First, based on implicit function theorem an ideal controller is developed to stabilize the underlying system and makes the tracking approach a neighborhood of zero. Then, a neural network or fuzzy logic to approximate this ideal controller is designed. Based on the Lyapunov stability analysis, an adaptation law is devised to update the adjustable parameters. However a bounding controller may also be added for more performance robustness.

In the above most methods the parameter adaptation laws are designed based on a Lyapunov approach, where an error signal between the desired output and the actual output is used to update the adjustable parameters and the control laws are composed of three control terms: a linear control term, an adaptive neural network control term and a robust control term used to compensate for disturbances and approximation errors. On the other hand, almost all of the above works don’t consider the zero dynamics, though it plays an important role in nonlinear system control. Considering that zero dynamics exist in many practical systems, including isothermal continuous stirred tank reactors, aircraft trajectory tracking control systems and others, it is necessary to investigate their influence on nonlinear system.

In the paper, according to [5], we introduce a direct adaptive neural network control approach for a class of nonaffine nonlinear systems with zero dynamics. The basic idea is to use neural network to adaptively construct an unknown ideal controller and the parameter adaptive laws is designed, based on the gradient descent method, to directly minimizing the error between the unknown ideal controller and the neural network controller. And no robust control term is used in controller. This paper proves the availability of the method in both theory and simulation experiment.

The paper is organized as follows. First, the problem is formulated in Section II. Zero dynamics is given in III. Designing a control law with on-line tuning of neural network weighting factors is given in Section IV. In Section V, convergence and stability analysis of control
system is given. In Section VI, simulation results are presented to confirm the effectiveness and applicability of the proposed method. Finally, conclusions are included.

A. Notations and Preliminaries

The following notations and definitions will extensively be used throughout the paper. Let $\mathbb{R}$ be the real number, $\mathbb{R}^n$ and $\mathbb{R}^{n \times m}$ represent the real $n$-vectors and the real $n \times m$ matrices, respectively. $|y|$ denotes the usual Euclidean norm of a vector. In the case where $y$ is a scalar, $|y|$ denotes its absolute value and if $Y$ is a matrix, $|Y|$ means Frobenious norm defined as $|Y| = \sqrt{tr\{Y^TY\}}$, where $tr\{\cdot\}$ stands for trace operator.

Implicit Function Theorem: Assume that $h: \mathbb{R}^r \times \mathbb{R}^n \rightarrow \mathbb{R}^n$ is continuously differentiable at each point $(a,b)$ of an open set $S \subset \mathbb{R}^r \times \mathbb{R}^n$. Let $(a_0, b_0)$ be a point in $S$ for which $h(a_0, b_0)$ and for which the Jacobian matrix $\left[ \frac{\partial h}{\partial a} \left| \begin{array}{c} \frac{\partial h}{\partial b} \end{array} \right. \right](a_0, b_0)$ is nonsingular. Then there exist neighborhoods $U \subset \mathbb{R}^r$ of $a_0$ and $V \subset \mathbb{R}^n$ of $b_0$ such that for each $b \in V$ the equation $h(a, b) = 0$ has a unique solution $a = g(b)$ in $U$. Moreover, the solution can be given as $a = g(b)$ where $g$ is continuously differentiable at $b = b_0$.

II. PROBLEM FORMULATION

Consider the following SISO non affine nonlinear system [8]:

$$
\begin{align*}
\frac{d\xi_i}{dt} &= \xi_{i-1}, \quad i = 1, \ldots, r-1 \\
\frac{d\xi_r}{dt} &= h(\xi_r, \eta, u) \\
\frac{\partial h}{\partial u} &= g(\xi_r, \eta, u) \\
y &= \xi_i,
\end{align*}
$$

where $\xi = [\xi_1, \ldots, \xi_r]^T \in \mathbb{Z}_k \times \mathbb{R}^r$, $\eta \in \mathbb{Z}_k \subset \mathbb{R}^{n-\gamma}$ are system states and $u \in \Omega_e \subset \mathbb{R}$, $y \in \mathbb{R}$ are system input and output respectively. $h(\xi_r, \eta, u)$ is a smooth partially known function, and $g(\xi_r, \eta, u)$ is a smooth partially known vector field.

The control objective is to design an adaptive neural network controller for a class of SISO non affine nonlinear systems (1) such that the system output follows a desired trajectory while all signals in the closed-loop system remain bounded.

Assumption 1: The function $h(\xi_r, \eta, u) = \frac{\partial h(\xi_r, \eta, u)}{\partial u}$ is nonzero and bounded for all $(\xi_r, \eta, u) \in \mathbb{Z}_k \times \mathbb{Z}_k \times \mathbb{R}$. This implies that $h(\xi_r, \eta, u)$ is strictly either positive or negative for all $(\xi, \eta, u) \in \mathbb{Z}_k \times \mathbb{Z}_k \times \mathbb{R}$. Without loss of generality, it is assumed that it exists a constant $c$ such that $\lambda h(\xi, \eta, u) \geq c > 0$.

Define the reference vector:

$$
\tilde{y}_d = (y_d, \dot{y}_d, \ldots, y_{d(r-1)})^T \in \mathbb{R}^r
$$

The reference signal $y_d$ and its time derivative are assumed to be smooth and bounded. We also define the tracking error as

$$
e = y_d - y
$$

and corresponding error vector as

$$
\tilde{e} = (e, \dot{e}, \ldots, e^{(r-1)})^T \in \mathbb{R}^r.
$$

Assumption 2: When the desired output $y_d$ and its r-order derivative are of known bound, there exists a positive constant $b_2$ to satisfy

$$
\|y_d - y_d^{(j)} \| \leq b_2
$$

Then the error equation is as follows:

$$
\tilde{e} = A_2 \tilde{e} + b\left[ y_d^{(r)} - h(\xi, \eta, u) \right]
$$

where $A_2 = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \end{bmatrix}_{r \times r}$, $b = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 1 \end{bmatrix}_{r \times 1}$. Obviously, $(A_2, b)$ is controllable, then there will exist a constant matrix $K = [k_0, k_1, \cdots, k_{r-1}]^T$ which makes eigenvalues of matrix $A_2 + bK^T$ all have negative real part. Thus, for any given positive definite symmetric matrix $Q$, there exists a unique positive definite symmetric solution $P$ to the following Lyapunov algebraic equation:

$$
A_2^TP + PA_2 = -Q
$$

Define a signal

$$
\nu = \rho(y_d) + K^T \xi + \lambda \tanh \left( \frac{b^T \xi}{\Xi} \right)
$$

where $\tanh(\bullet) \in (-1,1)$ is the hyperbolic tangent function, $\Xi, \lambda$ are the positive design parameters, when error $e \rightarrow +\infty$, the value of $\tanh \left( \frac{b^T \xi}{\Xi} \right)$ $\rightarrow +\infty$, and when error $e \rightarrow -\infty$ the value of $\tanh \left( \frac{b^T \xi}{\Xi} \right)$ $\rightarrow -\infty$.

When $e \rightarrow 0$, $\tanh \left( \frac{b^T \xi}{\Xi} \right) \rightarrow 0$. The term $\lambda \tanh \left( \frac{b^T \xi}{\Xi} \right)$ is a smooth approximation of the discontinuous term $\lambda \sign(\frac{b^T \xi}{\Xi})$ usually used in robust
control. So, $\lambda$ is selected larger than the magnitude of the uncertainty and it will affect the convergence rate of the tracking error, and $\Xi$ is chosen very small to be small approximate the sign function and it will affect the size of the residual set to which the tracking error will converge. The sign function is not used here to avoid problems associated with it as chattering and solutions existence.

By adding and subtracting $\nu$ in (3), we obtain

$$\ddot{\varepsilon} = \left(A_b - bK^T \right)\varepsilon - b\lambda \tanh \left( b^TP\frac{\varepsilon}{\Xi} \right) - b\left[ h(\xi, \eta, u) - \nu \right]$$  \hspace{1cm} (5)

From the fact that the signal $v$ does not explicitly depend upon the control input $u$ and Assumption 1, the partial derivative of $h(\xi, \eta, u) - \nu$ with respect to the input $u$ satisfies

$$\frac{\partial}{\partial u} [h(\xi, \eta, u) - \nu] = \frac{\partial h(\xi, \eta, u)}{\partial u} > 0$$  \hspace{1cm} (6)

Thus according to the implicit function theorem, there exists some ideal controller $u'(\xi, \eta, u, \nu)$ satisfying the following equality for all $(\xi, \eta, u) \in Z_k \times Z_k \times \mathbb{R}$ :

$$h(\xi, \eta, u')(\xi, \eta, u, \nu) - \nu = 0$$  \hspace{1cm} (7)

Therefore, if the control input $u$ is chosen as the ideal controller $u'(\xi, \eta, u, \nu)$, the closed-loop error dynamic (5) is reduced to

$$\ddot{\varepsilon} = \left(A_b - bK^T \right)\varepsilon - b\lambda \tanh \left( b^TP\frac{\varepsilon}{\Xi} \right)$$  \hspace{1cm} (8)

Considering the following positive function to the error dynamic:

$$V = \varepsilon^T P \varepsilon$$  \hspace{1cm} (9)

Using (4) and (8), the time derivative of (9) becomes

$$\dot{V} = -\varepsilon^T Q\varepsilon - 2\lambda b^TP\varepsilon \tanh \left( b^T P \frac{\varepsilon}{\Xi} \right)$$  \hspace{1cm} (10)

Because the term $b^TP\varepsilon$ and $\tanh \left( b^T P \frac{\varepsilon}{\Xi} \right)$ always have same sign, we conclude that $\dot{V} \leq 0$, and only when $e = 0$, $\dot{V} = 0$, which means $\lim_{t \to +\infty} |e| = 0$.

### III. ZERO DYNAMICS

If system (1) is controlled by the input $u$, the state vector $\eta$ is completely unobservable from the output, then the subsystem

$$\frac{\partial \eta}{\partial t} = q(0, \eta, u(0, \nu, v(0, \eta)))$$  \hspace{1cm} (11)

is addressed as the zero dynamic.

**Assumption 3**: Zero dynamics (11) is exponentially stable, and the function $q(\xi, \eta, u)$ is Lipschitz in $\xi$. There exists Lipschitz constants $L_z$ and $L_v$ such that

$$\|q(\xi, \eta, u) - q(0, \eta, u)\| \leq L_z \|\xi\| + L_v$$  \hspace{1cm} (12)

where $u = u(0, \eta, v(0, \eta))$.

By Lyapunov converse theorem, there is a Lyapunov function $V_u(\eta)$ which satisfies

$$\sigma_i \|\eta\| \leq V_u(\eta) \leq \sigma_i \|\eta\|^2$$  \hspace{1cm} (13)

$$\frac{\partial V_u(\eta)}{\partial \eta} q(0, \eta) \leq -\sigma_i \|\eta\|^2$$  \hspace{1cm} (14)

$$\left\| \frac{\partial V_u(\eta)}{\partial \eta} \right\| \leq \sigma_i \|\eta\|^2$$  \hspace{1cm} (15)

Where $\sigma_i, i = 1, 2, 3, 4$ are positive constant.

### IV. DESIGN OF CONTROLLER

In control engineering, radial basis function (RBF) NNs are usually used as a tool for modeling nonlinear functions because of their good capabilities in function approximation. RBFNN represents a class of linearly parameterized approximations and can be replaced by any other linearly parameterized approximations such as spline functions or fuzzy systems. Moreover, nonlinearly parameterized approximations, such as multilayer neural network (MNN), can be linearized as linearly parameterized approximations, with the higher order terms of Taylor series expansions being taken as part of the modeling error.

In this paper, the following RBF NN based on GGAP-RBF [20] algorithm which can avoid to select initial neural network parameters and nodes number of hidden layer artificially $u(z) = \phi^T(z) \theta$ is used to approximate the ideal controller $u'(z)$ , where $z = [\xi^T, \eta^T, v^T]^T$, $\phi(z) = (\phi_1(z), \ldots, \phi_M(z))^T$ is the basic function vector, and $\theta = (\theta_1, \ldots, \theta_M)^T$ is the adjustable parameter. It has been proven that neural network can approximate any smooth function over a compact set $\Omega_z \subset \mathbb{R}^n$ to arbitrarily any accuracy as

$$u'(z) = \phi^T(z) \theta' + \delta(z)$$  \hspace{1cm} (16)

with bounded function approximation error $\delta(z)$ satisfying $|\delta(z)| \leq \bar{\delta}$. Where $\theta'$ is an ideal parameter vector which minimizes the function $|\delta(z)|$. In this paper, we assume that the used neural network does not violate the universal approximation property on the compact set $\Omega_z$, which is assumed large enough so that the variable $z$ remains inside it under closed-loop control.

Let us define the control error between the controllers $u(z)$ and $u'(z)$ as

$$u(z) - u'(z) = \delta(z)$$  \hspace{1cm} (16)

with bounded function approximation error $\delta(z)$ satisfying $|\delta(z)| \leq \bar{\delta}$. Where $\theta'$ is an ideal parameter vector which minimizes the function $|\delta(z)|$. In this paper, we assume that the used neural network does not violate the universal approximation property on the compact set $\Omega_z$, which is assumed large enough so that the variable $z$ remains inside it under closed-loop control.
\[
e_v = u^*(z) - u(z) = \phi^*(z)\hat{\theta} + \delta(z) \tag{17}
\]

where \(\hat{\theta} = \theta' - \theta\) is the parameter estimation error vector.

According to the mean value theorem, there exist constant \(0 < \alpha < 1\), \(h(\xi, \eta, u)\) can be described as

\[
h(\xi, \eta, u) = h(\xi, \eta, u') + h_u(u(z) - u'(z)) \tag{18}
\]

where \(h_u = \partial h(\xi, \eta, u)/\partial u\bigg|_{u=u'}\), \(u_z = \alpha u(z) + (1-\alpha)u'(z)\)

By substituting (5) into the equation (18) and considering (7), we get

\[
\bar{e} = A_0\bar{e} - b\lambda\tanh\left(\frac{b^T P \bar{e}}{\Xi}\right) - b\bar{h}_u (u(z) - u'(z)) - \cdots
\]

\[
- b\bar{h}_u (u(z) - u'(z)) \tag{19}
\]

\[
= A_0\bar{e} - b\lambda\tanh\left(\frac{b^T P \bar{e}}{\Xi}\right) - b\bar{h}_u (u(z) - u'(z)) \tag{20}
\]

Considering \(A_0 = A_0 - bK^T\), then (19) can be rewritten as

\[
\bar{e} = A_0\bar{e} - b\lambda\tanh\left(\frac{b^T P \bar{e}}{\Xi}\right) - b\bar{h}_u (u'(z) - u(z)) = h_u e_v \tag{21}
\]

We notice here that \(u'(z)\) is an unknown quantity, so the signal \(e_v\) defined in (17) is not available. Eq. (20) will be used to overcome the difficulty. Indeed, from (20), we see that even if the signal \(e_v\) is not available for measurement, the quantity \(h_u e_v\) is measurable. This fact will be exploited in the design of the parameters adaptive law.

In order to obtain the update law of \(\theta\), we consider a quadratic cost function defined as

\[
J_\theta = \frac{1}{2} e_v^2 = \frac{1}{2} \left(u'(z) - \phi^*(z)\right)^2 \tag{22}
\]

By applying the gradient descent method, we obtain as an adaptive law for the parameters \(\theta\)

\[
\dot{\theta} = -\gamma N_\theta J(\theta) = \gamma \phi(z) e_v \tag{23}
\]

From (22), we know \(e_v\) is not available. The adaptive law (22) cannot be implemented. In order to render (22) computable, from Eq. (20), we select the design parameter \(\gamma = \gamma_0 h_u\), where \(\gamma_0\) is a positive constant. We have

\[
\dot{\theta} = \gamma_0 \phi(z) h_u e_v
\]

\[
= \gamma_0 \phi(z) \left\{\phi^* + K^T \bar{e} + \lambda \tanh\left(\frac{b^T P \bar{e}}{\Xi}\right)\right\} \tag{24}
\]

At the same time, in order to improve the robustness of adaptive law in the presence of the approximation error, we modify it by introducing a \(\sigma\)-modification term as follows:

\[
\dot{\theta} = \gamma_0 \phi(z) \left\{\phi^* + K^T \bar{e} + \lambda \tanh\left(\frac{b^T P \bar{e}}{\Xi}\right) - \gamma_0 \sigma \theta\right\} \tag{25}
\]

where \(\sigma\) is a small positive constant.

Since the function of the \(\sigma\)-modification adaptive law is to avoid parameter drift, it does not need to be active when the estimated parameters are within some acceptable bound. The system stability relies entirely on the neural network because the proposed adaptive controller in the paper is only composed of a neural network part without additional control terms. The term \(\lambda \tanh\left(\frac{b^T P \bar{e}}{\Xi}\right)\) in the parameter adaptive law (24) plays, in some way, the role of a robustifying control term. Thus by selecting a large positive value for the design parameter \(\lambda\) and a small positive value for the parameter \(\Xi\), the robustness of the controller can be improved.

V. STABILITY AND CONVERGENCE ANALYSIS OF CONTROL SYSTEM

In order to analysis the convergence of neural network weights, we firstly consider the following positive function:

\[
V_\theta = \frac{1}{2\gamma_0} \|\dot{\theta}\|^2 \tag{26}
\]

Using (17), (20) and (24), the time derivative of (25) can be written as

\[
\dot{V}_\theta = \dot{\theta}^T (\phi(z) h_u e_v + \sigma \theta)
\]

\[
= -\phi^*(z) \dot{\theta} h_u e_v + \sigma \dot{\theta}^T \theta
\]

\[
= -h_u e_v^2 + h_u \delta(z) e_v + \sigma \dot{\theta}^T \theta
\]

Considering the inequalities

\[
\sigma \dot{\theta}^T \theta = -\frac{\sigma}{2} \|\dot{\theta}\|^2 + \frac{\sigma}{2} \|\dot{\theta} + \theta\|^2
\]

\[
\leq -\frac{\sigma}{2} \|\dot{\theta}\|^2 + \frac{\sigma}{2} \|\dot{\theta}\|^2
\]

\[
- e_v^2 + \delta(z) e_v = -\frac{1}{2} e_v^2 + \frac{1}{2} \delta^2(z) - \frac{1}{2} (e_v - \delta(z))^2
\]

\[
\leq -\frac{1}{2} e_v^2 + \frac{1}{2} \delta^2(z) \tag{27}
\]

Considering (27) and (28), Eq. (26) can be bounded as

\[
\dot{V}_\theta \leq -\frac{1}{2} h_u e_v^2 + h_u \delta^2(z) - \frac{\sigma}{2} \|\dot{\theta}\|^2 + \frac{\sigma}{2} \|\dot{\theta}\|^2 \tag{29}
\]

Because the functions \(\delta(z)\) and \(h_u\) are bounded in this paper, and the parameters \(\theta\) are constants, so we can define a positive constant bound \(\psi\) as

\[
\psi = \sup_e \left(\frac{1}{2} h_u \delta^2(z) + \frac{\sigma}{2} \|\dot{\theta}\|^2\right) \tag{30}
\]

Then
\[
\dot{V}_0 \leq -\frac{1}{2} \rho V_0 + \psi - \frac{1}{2} h_\epsilon e^2 \leq -\rho V_0 + \psi
\]  
(31)

where \(\rho = \sigma_\gamma \alpha\). Eq. (31) implies that for \(V_0 > \psi / \rho\), \(\dot{V}_0 < 0\) and, therefore, \(\dot{\theta}\) is bounded. By integrating (31), we can establish that:

\[
\|\dot{\theta}\|^2 \leq \|\dot{\theta}(0)\|^2 e^{-\lambda t} + 2 \gamma_0 \psi / \rho \]
(32)

From (32), we have:

\[
\|\dot{\theta}\| \leq \|\dot{\theta}(0)\| e^{-\lambda t} + \sqrt{2 \gamma_0 \psi / \rho} \]
(33)

Using (33) and the fact that \(\delta(z)\) and \(h_\epsilon\) are bounded, we can write:

\[
\beta(\xi, \eta) h_\epsilon \left(\dot{\theta}(z) + \delta(z)\right) \leq \beta(\xi, \eta) h_\epsilon \left[\|\dot{\theta}(z)\| + \|\delta(z)\|\right] + \gamma_0 \psi / \rho \leq \|\dot{\theta}(0)\| e^{-\lambda t} + \sqrt{2 \gamma_0 \psi / \rho} \leq \|\dot{\theta}(0)\| + \sqrt{2 \gamma_0 \psi / \rho} \]
(34)

where \(\psi, \psi_1\) are some finite positive constants.

**Lemma 1:** The following inequality holds for all \(\Xi > 0\) and \(\zeta \in R\) with \(K_\epsilon = 0.2785\).

\[
0 \leq |\zeta| - \zeta \cdot \tanh \left(\frac{\zeta}{\Xi}\right) \leq K_\Xi \Xi
\]  
(35)

**Theorem 1:** Suppose that Assumption 1-3 are satisfied for the system (1), then the neural network controller and adaptation law given by (24) guarantees the convergence of the neural network parameters and to be uniformly ultimately bounded of all the signal in the closed-loop system.

Proof: Consider the Lyapunov function candidate:

\[
V(\xi, \eta) = \xi^T P \xi + \mu \psi_1(\eta)
\]  
(36)

Where \(\mu > 0\) is the design parameter. Considering (4), (19), (20), (34) and lemma 1, differentiating \(V(\xi, \eta)\) with respect to time, we obtain

\[
\dot{V}(\xi, \eta) = \xi^T \left( A^T P + PA \right) \xi - 2 \beta_\epsilon P \eta \tanh \left( \frac{b^T \Phi \eta}{\Xi} \right) 
\]

\[
\ldots + 2 \beta_\epsilon P h_\epsilon \left( u' - u \right) + \mu \frac{dV_\epsilon(\eta)}{dt}
\]

\[
\ldots + \xi^T \left( \Phi^T - 2 \beta_\epsilon P \eta \tanh \left( \frac{b^T \Phi \eta}{\Xi} \right) + \mu \frac{dV_\epsilon(\eta)}{dt} \right)
\]

\[
\ldots + 2 \beta_\epsilon P h_\epsilon \left( \dot{\theta}(z) + \delta(z) \right) \leq -\xi^T \Phi \xi - 2 \beta_\epsilon P \eta \tanh \left( \frac{b^T \Phi \eta}{\Xi} \right) + \mu \frac{dV_\epsilon(\eta)}{dt} \]
(37)

\[
\ldots + 2 \beta_\epsilon P \eta \psi e^{-\lambda t} + 2 \psi_1 K_\Xi
\]

If the design parameter \(\lambda\) is large enough to make \(\xi \geq \psi_1\) and considering assumption 4, we have:

\[
V(\xi, \eta) \leq -\xi^T \Phi \xi - 2 \beta_\epsilon P \eta \psi e^{-\lambda t} + 2 \psi_1 K_\Xi + \ldots + \mu \frac{dV_\epsilon(\eta)}{dt}
\]

\[
\ldots + \mu \frac{dV_\epsilon(\eta)}{dt}
\]

\[
\leq -\xi^T \Phi \xi - 2 \beta_\epsilon P \eta \psi e^{-\lambda t} + 2 \psi_1 K_\Xi + \ldots + 2 \beta_\epsilon P \eta \psi e^{-\lambda t} + 2 \psi_1 K_\Xi
\]
(38)

Considering assumption 2 and

\[
2 \beta_\epsilon P \eta \psi e^{-\lambda t} \leq 0.5 \xi^T \xi + 2 \beta_\epsilon P \eta \psi e^{-\lambda t}
\]
(39)

Then:

\[
\dot{V}(\xi, \eta) \leq -\left( \lambda_\epsilon(\eta) - 0.5 \right) \xi^T \xi - \mu_\epsilon \eta \eta + \ldots + \mu_\epsilon \eta \eta + \frac{1}{2} \eta^T P \eta \psi^2 e^{-\lambda t} + 2 \psi_1 K_\Xi
\]

Using the inequality

\[
\mu_\epsilon \eta \eta + \frac{1}{2} \eta^T P \eta \psi^2 e^{-\lambda t} + 2 \psi_1 K_\Xi
\]
(40)

\[
\mu_\epsilon \left( L_\epsilon b_\epsilon + L_\eta \right) \eta \eta \leq 0.5 \mu_\epsilon \eta \eta \eta \eta^T + \frac{1}{2} \mu_\epsilon \eta \eta \eta \eta^T + \frac{1}{2} \mu_\epsilon \eta \eta \eta \eta^T
\]
(41)

Then (39) satisfies

\[
\dot{V}(\xi, \eta)
\]

\[
\leq -\left( \lambda_\epsilon(\eta) - 0.5 \right) \xi^T \xi + \ldots + \mu_\epsilon \eta \eta + \mu_\epsilon \left( L_\epsilon b_\epsilon + L_\eta \right) \eta \eta
\]
(42)

where \(\epsilon_1, \epsilon_2\) are suitable positive constants. We adjust \(\epsilon_1, \epsilon_2\) to make

\[
\sigma_1 - \frac{1}{2} \sigma_2 L_\epsilon \epsilon_1 - \mu \left( \sigma_2 \epsilon_2 \left( L_\epsilon b_\epsilon + L_\eta \right) \right)^2 > 0
\]
Supposing \( \varepsilon = \frac{1}{4\varepsilon^2} + 2\|b\|^TP|\psi\varepsilon^{-m} + 2\psi,\Xi, \) and

selecting \( \mu = \left( \frac{1}{2} \sigma_1 - \frac{1}{2} \sigma_i L_i e_i \right), \) then

\[
\dot{V}(\varepsilon, \eta) \leq -\left( \lambda_{\text{inf}}(Q) - 0.5 - \frac{1}{2\varepsilon^2} \mu \sigma_i L_i \right) \|\eta\|^2 + \ldots
\]

\[
-\frac{1}{4} \left( \sigma_1 - \frac{1}{2} \sigma_i L_i e_i \right)^2 \|\eta\|^2 + \varepsilon
\]

(43)

Adjusting \( Q \) to make

\( \lambda_{\text{inf}}(Q) - 0.5 - \frac{1}{2\varepsilon^2} \mu \sigma_i L_i > 0. \)

From the above equation, we can know that tracking error and internal states \( \eta \) are all uniformly ultimately bounded. Besides, since \( \|\| \leq \|a\| + \|y_a, y_a, \ldots, y_a^{r-1}\| \leq \|a\| + b_a \), then the state \( \eta \) is uniformly ultimately bounded too. This completes the proof.

VI. SIMULATION RESULTS

In this part, the following SISO nonaffine nonlinear system with zero dynamics is simulated to illustrate the effectiveness of the proposed adaptive neural network tracking controller. The nonaffine nonlinear system is described as follows:

\[
\begin{align*}
\dot{\xi}_1 &= \xi_2 \\
\dot{\xi}_2 &= -2\left( (\xi_1 - \eta_1)^2 - 1 \right)(\xi_2 - \eta_2) - \eta_i - 0.2\eta_i + \ldots \\
&\quad + 2\sin\left( (\xi_1 - \eta_1)(\xi_2 - \eta_2) \right) \left[ u + \frac{1}{3}u^3 + \sin(u) \right] \\
\eta_1 &= \eta_2 \\
\eta_2 &= -2\eta_i - 0.2\eta_i + \xi_1 \\
y &= \xi_1
\end{align*}
\]

(44)

The control objective is to force the system output \( y \) to track the desired trajectory \( y_d = 2\sin t + \cos(0.5t) \). The simulation parameters are selected as follows:

\[
Q = \text{diag}(10, 10), \quad P = \begin{bmatrix} 15 & 5 \\ 5 & 5 \end{bmatrix}, \quad K = \begin{bmatrix} 1, 2 \end{bmatrix}, \quad \Xi = 0.01, \quad \lambda = 10, \quad \gamma = 11, \quad \sigma = 0.02.
\]

The output of RBFNN controller is \( u(z) = \phi'(z)\theta \). The basis function vector is \( \phi(z) = (\phi_1(z) \cdots \phi_M(z))^T \), where \( \phi(z) = \exp \left[ -\frac{(z-\mu_i)^T(z-\mu_i)}{\sigma_i^2} \right], i = 1, \ldots, M \). \( M \) is the number of hidden layer nodes which is stable at the 33 nodes by training on-line using the GGAP-RBF algorithm.

According to (23), the control law is

\[
\begin{align*}
\dot{\theta} &= -11\phi(z) \left( \dot{e} + e + 2e + 10 \tanh (100 \times (5e + 5\dot{e})) \right) \\
&\quad - 11 \times 0.02\theta
\end{align*}
\]

The system initial conditions are \( \xi(0) = [1, 2]^T \). The simulation results using MATLAB are shown in Fig 1, 2, 3, 4.

Figure 1 shows the result of output tracking, and the control input signal is shown in Figure 2. The growing and pruning automatically of hidden layer nodes are shown in Figure 3.
A new adaptive neural network tracking control algorithm is presented for a class of SISO nonaffine nonlinear systems with zero dynamics in this paper. The method does not assume boundedness on the time derivative of a control effectiveness term, and only need sign known and boundedness of the control effectiveness term. The update law of neural network adjustable parameters is obtained by the gradient descent algorithm. The overall adaptive scheme guarantees that all signals involved are uniformly ultimately bounded and the output of the closed-loop system tracks the desired output trajectory. Simulation results demonstrate the feasibility of the proposed control scheme.

VII. CONCLUSIONS

A new adaptive neural network tracking control algorithm is presented for a class of SISO nonaffine nonlinear systems with zero dynamics in this paper. The method does not assume boundedness on the time derivative of a control effectiveness term, and only need sign known and boundedness of the control effectiveness term. The update law of neural network adjustable parameters is obtained by the gradient descent algorithm. The overall adaptive scheme guarantees that all signals involved are uniformly ultimately bounded and the output of the closed-loop system tracks the desired output trajectory. Simulation results demonstrate the feasibility of the proposed control scheme.

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