Fault-tolerant Stabilization for Linear System with Time Delay

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Abstract—In this note, the FTC problem of time-delay systems with the special sensor model of failure is investigated. Firstly, based on Lyapunov stability theorem, through constructing a proper LKF and using integral inequality, the stability condition of the closed-loop system is obtained. Secondly, by using the nonlinear transformation and the cone complementary linearization algorithm, the controller existence condition of time-delay system in terms of LMIs is obtained, which guarantee the asymptotically stable of the closed-loop systems even if the sensor faults occur, and the controller parameters are also given. Finally, an example is given to show the effectiveness of the proposed methods in this paper.

Index Terms—time delay, fault-tolerant control (FTC), the cone complementary linearization, linear matrix inequality (LMI)

I. INTRODUCTION

Faults of system may drastically change the system behavior ranging from performance degradation to instability. In order to maintain high levels of system survivability and performance, extensive researches have focused on robust fault-tolerant control of systems over the past few decades. As fault tolerance is taken into account, the principle goal is the maintenance of system stability under fault scenarios.

To stability analysis of time-delay systems, in order to reduce the conservatism of stability criteria, many researchers developed different approaches, such as model transformation[1], integral inequality[2, 3], free matrices[4, 5], matrices decomposition[6] and constructing proper L-K functional[7, 8]. However, It is well known that the controller design depend on the stability results, many stability results, which is better in reducing conservatism, cannot be used to controller design due to the complexity of the corresponding stability criteria. Therefore, to the controller design problem, getting the parameters of the controller is important as well as reducing the conservatism [9], which needs some special functional construction and model transformation. To the fault-tolerant control of time-delay systems, many efforts mainly focused on the state feedback methods [10]. When the state cannot be obtained, in [11-12], the static output feedback controllers were designed to solve the fault-tolerant control of time-delay systems. To the best of our knowledge, few results have been achieved on the fault-tolerant control for time-delay systems.

Motivated by above discussion, in this issue, the fault-tolerant control for time-delay systems with sensor failures is considered. Attention was focused on the design of output dynamical feedback controller which guaranteed the asymptotical stability of the closed-loop systems when the sensor failed. By using Lyapunov stability theorem, the stability condition of the closed-loop system is derived, and then in order to get the controller existence condition in terms of LMI, nonlinear transformation is used and the sufficient conditions for the existence of feedback controller are obtained. Moreover, the desired controller is obtained by using the cone complementary linearization iterative algorithm. Numerical example is given to show the effectiveness of the proposed method.

Notations: In this paper, For symmetric matrices $A$ and $B$, $\lambda_{\max}(A)$ and $\lambda_{\min}(A)$ denote the largest and smallest eigenvalue of $A$, respectively; the notation $A \geq B$ (respectively, $A > B$ ) means that the matrix $A-B$ is positive semi-definite (respectively, positive definite). $\| \|$ denotes the Euclidean norm for vector or the spectral norm of matrices. In the symmetric block matrices, $*$ denotes a term that is induced by symmetry. Every matrices, if not explicitly mentioned in the paper, are assumed to have compatible dimensions.

II. PROBLEM FORMULATION

Consider the following time-delay system:

\[ \]
\[ \dot{x}(t) = Ax(t) + A_\tau x(t - \tau) + Bu(t) \]
\[ y(t) = Cx(t), x(t) = \varphi(t), t \in [-\tau_m, 0] \]  
(1)

where \( x(t) \in \mathbb{R}^n \), \( u(t) \in \mathbb{R}^q \), \( y(t) \in \mathbb{R}^m \) are state, input and output of the system. \( A, A_\tau, B, B \) are known constant matrices, \( \tau \) is the constant time delay and satisfies \( 0 \leq \tau \leq \tau_m \).

For the fault of the sensors, we introduce the sensor failure matrix \( F_s \):
\[
F_s \in \Theta_s = \{ X \mid X = \text{diag}\{f_{s_1}, f_{s_2}, \cdots, f_{s_l} \}, 0 \leq f_{s_i} \leq \delta_i, \delta_i \geq 1, i = 1,2,\cdots, l \} \]  
(2)

\( F_s \) is the diagonal matrix, \( f_{s_i} = 1 \) denote the corresponding circuit is normal, otherwise the sensor failure happen; where \( f_{s_i} < 1 \) denote the signal is reduced, \( f_{s_i} > 1 \) denote the signal is amplified.

The aim of this paper is to design the following dynamical feedback controller:
\[
\dot{x}_k(t) = A_k x_k(t) + B_k y(t)
\]
\[ u(t) = C_k \dot{x}(t), t \geq 0 \]  
(3)

for all \( t \in [0, \tau_m] \), when the sensor failure happen, such that the following closed-loop system is asymptotically stable.
\[ \ddot{x}(t) = \ddot{A}_\tau \ddot{x}(t) + \ddot{A}_\tau \ddot{x}(t - \tau) \]  
(4)

where
\[
\ddot{x}(t) = \begin{bmatrix} x(t) \\ \ddot{x}_k(t) \end{bmatrix}
\]
\[
\ddot{A} = \begin{bmatrix} A & BC_k \\ B_k F_s C & A_k \end{bmatrix}
\]
\[
\ddot{A}_\tau = \begin{bmatrix} A_\tau & 0 \\ 0 & 0 \end{bmatrix}
\]

**Lemma 1**\(^{[13]}\). Given any matrices \( X, Y \) and scalar \( \varepsilon > 0 \), then
\[
XY^T + YX^T \leq \varepsilon XX^T + \varepsilon^{-1} YY^T
\]

**Lemma 2**\(^{[14]}\). Assume that \( a(\cdot) \in \mathbb{R}^{n_0}, b(\cdot) \in \mathbb{R}^{n_0} \), and \( N(\cdot) \in \mathbb{R}^{n_0 \times n_0} \) are defined on the interval \( \Omega \). Then for any matrices \( X \in \mathbb{R}^{m_0 \times n_0}, Y \in \mathbb{R}^{m_0 \times n_0} \), \( Z \in \mathbb{R}^{m_0 \times n_0} \), the following holds:
\[
-2 \int_\Omega a^T(\alpha)N b(\alpha) d\alpha 
\]
\[ \leq \int_\Omega \begin{bmatrix} a(\alpha) \\ b(\alpha) \end{bmatrix}^T \begin{bmatrix} X & Y \\ * & Z \end{bmatrix} \begin{bmatrix} a(\alpha) \\ b(\alpha) \end{bmatrix} d\alpha
\]

III. MAIN RESULT

In this section, firstly, through constructing a LKF, we get the stability condition of system (4), then based on this condition and by using the cone complementary linearization iterative algorithm, the controller design condition is obtained.

Firstly, we consider the stability of the following system:
\[
\dot{x}(t) = Ax(t) + A_\tau x(t - \tau)
\]
\[ x(t) = \varphi(t), t \in [-\tau_m, 0] \]

Considering a LKF as follows:
\[
V(x_i) = \sum_{i=1}^3 V_i(x_i)
\]

where
\[
V_i(x_i) = x_i^T(t)P_i x_i(t)
\]
\[ V_2(x_i) = \int_{-\tau}^t \int_{-\beta}^\tau x_i^T(\alpha)Z_i x_i(\alpha)d\alpha d\beta
\]
\[ V_3(x_i) = \int_{-\tau}^\tau x_i^T(\alpha)Q x_i(\alpha) d\alpha
\]

The derivative of \( V_1(x_i) \) along the trajectory of system (1) are given by
\[
\dot{V}_i(x_i) = 2x_i^T(t)P_i x_i(t)
\]

since it holds that
\[
x(t) - x(t - \tau) = \int_{-\tau}^t [Ax(\alpha) + A_\tau (\alpha - \tau)]d\alpha
\]

then we can get
\[
\dot{x}(t) = (A + A_\tau)x(t) - A_\tau \int_{-\tau}^t [Ax(\alpha) + A_\tau (\alpha - \tau)]d\alpha
\]

and thus the following equation holds
\[
\dot{V}_1(x_i) = 2x_i^T(t)P(A + A_\tau)x(t)
\]
\[ - 2x_i^T(t)PA_\tau \int_{-\tau}^t \dot{x}(\alpha)d\alpha
\]

Defining \( a(\alpha) = x(t), b(\alpha) = \dot{x}(\alpha), N = PA_\tau \) for all \( \alpha \in [t - \tau, t] \) and using Lemma 2, we can get

\[
\begin{bmatrix} X & Y \\ * & Z \end{bmatrix} \geq 0
\]
\[
\dot{V}_1(x_t) \leq 2x^T(t)P(A + A_t)x(t) + \tau \dot{x}^T(t)x(t) \\
+ 2x^T(t)(Y - PA_t)\int_{\tau_t}^{t} \dot{x}(\alpha)d\alpha \\
+ \int_{\tau_t}^{t} \dot{x}^T(\alpha)Z\dot{x}(\alpha)d\alpha \\
= x^T(t)\left(A^T + P + PA + \tau X + Y + Y^T\right)x(t) \\
+ 2x^T(t)(Y - PA_t)x(t - \tau) \\
+ \int_{\tau_t}^{t} \dot{x}^T(\alpha)Z\dot{x}(\alpha)d\alpha
\]

and \(V_2(x_t)\) is calculated as

\[
\dot{V}_2(x_t) = \tau \dot{x}^T(t)Z\dot{x}(t) - \int_{\tau_t}^{t} \dot{x}^T(\alpha)Z\dot{x}(\alpha)d\alpha \\
= \tau \left[ Ax(t) + A_t(t - \tau) \right]^T Z \left[ Ax(t) + A_t(t - \tau) \right] \\
- \int_{\tau_t}^{t} \dot{x}^T(\alpha)Z\dot{x}(\alpha)d\alpha
\]

Therefore, we can get the derivative of \(V(x_t)\) as:

\[
\dot{V}(x_t) = \sum_{i=1}^{3} \dot{V}_i(x_t) \\
\leq x^T(t)\left[A^T P + PA + \tau X + Y + Y^T\right]x(t) \\
+ 2x^T(t)(Y - PA_t)x(t - \tau) \\
+ \tau \left[ Ax(t) + A_t(t - \tau) \right]^T Z \left[ Ax(t) + A_t(t - \tau) \right] \\
+ x^T(t)Qx(t) - x^T(t - \tau)Qx(t - \tau) \\
\leq \begin{bmatrix} x(t) \\
 x(t - \tau) \end{bmatrix} \begin{bmatrix} \Delta & PA_t - Y + \tau A_t^T ZA_t \\
* & -Q + \tau A_t^T ZA_t \end{bmatrix} \begin{bmatrix} x(t) \\
 x(t - \tau) \end{bmatrix}
\]

where

\[
\Delta = A^T P + PA + \tau X + Y + Y^T + \tau A_t^T ZA_t + Q
\]

So using the Lyapunov-Krasovskii stability theorem and Schur complement, it is easy to know that, for \(\forall \tau \in [0, \tau_m]\), the system is asymptotically stable, if the following LMIs hold:

\[
\begin{bmatrix} \Delta & PA_t - Y + \tau A_t^T ZA_t \\
* & -Q + \tau A_t^T ZA_t \end{bmatrix} < 0
\]

and

\[
\begin{bmatrix} X & Y \\
* & Z \end{bmatrix} \geq 0
\]

From above derivation, we can see that, for \(\forall \tau \in [0, \tau_m]\), the time-delay system (4) is asymptotically stable, if there exist matrices \(\bar{P} > 0, \bar{Q} > 0\), and any matrices \(\bar{X}, \bar{Y}, \bar{Z}\), such that the following LMIs hold:

\[
\Sigma = \begin{bmatrix} \bar{P}A_t + \bar{A}_t^T \bar{P} + \tau_m \bar{X} & \bar{P}A_t - \bar{Y} & \tau_m \bar{A}_t^T \bar{Z} \\
* & -\bar{Q} & \tau_m \bar{A}_t^T \bar{Z} \\
* & * & -\tau_m \bar{Z} \end{bmatrix} < 0
\]

\[
\begin{bmatrix} \bar{X} & \bar{Y} \\
* & \bar{Z} \end{bmatrix} \geq 0
\]

In the following derivation, we study the controller design condition. In order to get the main result of this paper, we need the following transformation.

Here, we let

\[
\begin{align*}
\bar{A}_S &= \begin{bmatrix} A & BC_K \\
\delta_S B_K C & A_K \end{bmatrix} \\
B_S &= \begin{bmatrix} 0 \\
\delta_S B_K \end{bmatrix} \\
C_S &= \begin{bmatrix} C & 0 \end{bmatrix} \\
\bar{F} &= \delta_S^{-1}(F_S - \delta_S I) \\
\Sigma_S &= \begin{bmatrix} \bar{P}A_S + \bar{A}_S^T \bar{P} + \tau_m \bar{X} + \bar{Y} + \bar{Y}^T + \bar{Q} & \bar{P}A_t - \bar{Y} & \tau_m \bar{A}_t^T \bar{Z} \\
* & -\bar{Q} & \tau_m \bar{A}_t^T \bar{Z} \\
* & * & -\tau_m \bar{Z} \end{bmatrix} \\
\end{align*}
\]

We can get:

\[
\bar{A} = \bar{A}_S + B_S \bar{F}C_S
\]

For \(\bar{F} \bar{F}^T \leq I\), from Lemma 1, we can obtain:

\[
\begin{align*}
\Sigma &= \Sigma_S + \begin{bmatrix} \bar{P}B_S & 0 & 0 \\
0 & \bar{F} & 0 \\
\tau_m \bar{Z}B_S & 0 & 0 \end{bmatrix} + \begin{bmatrix} \bar{P}B_S & 0 & 0 \\
0 & \bar{F} & 0 \\
\tau_m \bar{Z}B_S & 0 & 0 \end{bmatrix}^T \\
&\leq \Sigma_S + \delta_S^{-1} \begin{bmatrix} \bar{P}B_S & 0 & 0 \\
0 & \bar{F} & 0 \\
\tau_m \bar{Z}B_S & 0 & 0 \end{bmatrix} \begin{bmatrix} \bar{P}B_S & 0 & 0 \\
0 & \bar{F} & 0 \\
\tau_m \bar{Z}B_S & 0 & 0 \end{bmatrix}^T
\end{align*}
\]
So, the condition of (5a) is, there exists a scalar \( \varepsilon > 0 \), such that the (8) holds:

\[
\begin{bmatrix}
\Theta & \tilde{P} \tilde{A}_t - \tilde{Y} & \tau_m \tilde{A}_S^T \tilde{Z} & \tilde{P} B_S & \varepsilon C_S^T \\
\ast & -\tilde{Q} & \tau_m \tilde{A}_t^T \tilde{Z} & 0 & 0 \\
\ast & \ast & -\tau_m \tilde{Z} & \tau_m \tilde{Z} B_S & 0 \\
\ast & \ast & \ast & -\varepsilon I & 0 \\
\ast & \ast & \ast & \ast & -\varepsilon I
\end{bmatrix} < 0
\]

where

\[
\Theta = \tilde{P} \tilde{A} + \tilde{A}^T \tilde{P} + \tau_m \tilde{X} + \tilde{Y} + \tilde{Y}^T + \tilde{Q}
\]

Then we let

\[
\tilde{P} = \begin{bmatrix} P_{2} & P_0 \end{bmatrix}, P_0 \in R^{n \times n}, P_i \in R^{n \times n}, (i = 2, 3)
\]

and \( P_2 \) is nonsingular.

Construct the following transform matrices:

\[
T_1 = \begin{bmatrix} I & 0 \\ 0 & PP_2^{-T} \end{bmatrix}
\]

\[
T_2 = \text{diag} \{ T_1, T_1, T_1 \}
\]

after similitude transformation, we know that LMI (8) and (5b) are equivalent to:

\[
\begin{bmatrix}
P \tilde{A} + \tilde{A}^T P + \tau_m \tilde{X} + \tilde{Y} + \tilde{Y}^T + \tilde{Q} \\
+ \tilde{Y} + \tilde{Y}^T + \tilde{Q}
\end{bmatrix} < 0
\]

\[
\begin{bmatrix}
\tilde{X} & \tilde{Y} \\
\ast & \tilde{Z}
\end{bmatrix} \geq 0
\]

where

\[
\tilde{P} = \begin{bmatrix} P & P \\ P & PP_2^{-T} P_3 P_2^{-1} P_3 \end{bmatrix},
\]

\[
\tilde{A} = \begin{bmatrix} A & BC_K \\
\delta S \delta B_K C & \delta K \end{bmatrix}
\]

\[
\tilde{A}_t = \begin{bmatrix} A_t \\ 0 \end{bmatrix}
\]

\[
\tilde{B}_S = \begin{bmatrix} 0 \\ \delta S \delta B_K \end{bmatrix}
\]

\[
\tilde{Z} = T_1 \tilde{Z} T_1^T, \quad \tilde{Q} = T_1 \tilde{Q} T_1^T
\]

\[
\tilde{X} = T_1 \tilde{X} T_1^T, \quad \tilde{Y} = T_1 \tilde{Y} T_1^T
\]

\[
\bar{A}_K = P^{-1} P_2 A_K P_2^{-1} P,
\]

\[
\bar{C}_K = C_K P_2^{-1} P
\]

\[
\bar{B}_K = P^{-1} P_2 B_K
\]

from (5b) (8) (11), we can see that the system \((\bar{A}_s, \bar{A}_t)\) is algebra equivalent to \((\bar{A}, \bar{A}_t)\), and the difference is the matrices \( A_K, B_K, C_K \) and \( \bar{A}_K, \bar{B}_K, \bar{C}_K \). Therefore, we can deal with the system \((\bar{A}_s, \bar{A}_t)\) instead of system \((\bar{A}_s, \bar{A}_t)\).

Let

\[
S^{-1} = PP_2^{-T} P_3 P_2^{-1} P - P
\]

\[
= PP_2^{-T} (P_3 - P_2^T P_1^{-1} P_2) P_2^{-1} P
\]

then we get

\[
\bar{P} = \begin{bmatrix} P & P \\ P & S^{-1} + P \end{bmatrix}
\]

from

\[
P(S + P^{-1}) = (S^{-1} + P) S > 0,
\]

we know

\[
J = S + P^{-1} > 0.
\]

Construct the following transform matrices:

\[
T_3 = \begin{bmatrix} J & -S \\ I & 0 \end{bmatrix}
\]

\[
T_4 = \begin{bmatrix} I & -JS \\ I & 0 \end{bmatrix}
\]

\[
T_5 = T_3 \bar{P} \bar{Z}^{-1}
\]

and introduce the matrices \( Q_4 > 0, Q_6 > 0, Q_5 \) satisfying:

\[
\begin{bmatrix} Q_4 & Q_5 \\ Q_5^T & Q_6 \end{bmatrix} \leq \begin{bmatrix} J^{-1} & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} Q_4 & Q_6 \\ Q_6^T & Q_4 \end{bmatrix} J^{-1} \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}
\]

after similitude transformation, the inequation (11a) is equivalent to:
\[
\begin{bmatrix}
\Xi_{11} & \Xi_{12} & A_r - Y_1 J^{-1} & A_r - Y_2 & J A^T + W_C J T B^T \\
* & \Xi_{22} & P A_r - Y_3 J^{-1} & P A_r - Y_4 & \tau_m A^T \\
* & * & - Q_4 & - Q_5 & \tau_m A^T \\
* & * & * & - Q_6 & \tau_m A^T \\
* & * & * & * & \tau_m Z_1 \\
\end{bmatrix} = \\
\begin{bmatrix}
\tau_m L^T & 0 & \omega J C^T \\
A^T P + C^T W_B W^T & W_B & \omega C^T \\
\tau_m A^T P & 0 & 0 \\
\tau_m A^T P & 0 & 0 \\
- \tau_m Z_2 & 0 & 0 \\
- \tau_m Z_3 & \tau_m W_B & 0 \\
* & - \varepsilon & 0 \\
* & * & - \varepsilon \\
\end{bmatrix} < 0
\]

\text{(13b)}

where
\[
\Xi_{11} = AJ - BW_C + JA^T + W_C J T B^T + \tau_m X_1 + Y_1 + Y_1^T + Q_1 \\
\Xi_{12} = AJ + I^T + \tau_m X_2 + Y_2 + Y_3^T + Q_2, \\
\Xi_{22} = PA + W_B C + A^T P + C^T W_B + \tau_m X_3 + Y_4 + Y_4^T + Q_3
\]

\[
X = \begin{bmatrix} X_1 & X_2 \\ X_2^T & X_1 \end{bmatrix}, \\
Y = \begin{bmatrix} Y_1 & Y_2 \\ Y_3 & Y_4 \end{bmatrix}, \\
Q = \begin{bmatrix} Q_1 & Q_2 \\ Q_2^T & Q_3 \end{bmatrix}, \\
Z = \begin{bmatrix} Z_1 & Z_2 \\ Z_2^T & Z_1 \end{bmatrix}, \\
L = PAJ + P \delta S B_K C J - PB \bar{C}_K S - PA_K S, \\
W_B = \delta S B K, \\
W_C = \bar{C}_K S.
\]

and the (11b) is equivalent to:
\[
\begin{bmatrix}
T_3 \bar{X} T_3^T & T_3 \bar{Y} T_3^T \\
* & T_3 \bar{Z} T_3^T
\end{bmatrix} \geq 0
\]

\text{(14)}

denoting:
\[
T_3 \bar{X} T_3^T = \begin{bmatrix} J^T & -S \\ J & 0 \end{bmatrix} \begin{bmatrix} P & P \\ P & P + S^{-1} \end{bmatrix} \begin{bmatrix} J & I \\ I & P \end{bmatrix}
\]

and introduce the matrix
\[
W = \begin{bmatrix} W_1 & W_2 \\ W_2^T & W_3 \end{bmatrix} \geq 0
\]

If the following equations hold
\[
\begin{bmatrix} X & Y \\ * & W \end{bmatrix} \geq 0, \\
W \leq \begin{bmatrix} J & I \\ I & P \end{bmatrix} Z^{-1} \begin{bmatrix} J & I \\ I & P \end{bmatrix}
\]

then we can get the (14).

From the mentioned above, we can get the main result of this paper.

\textbf{Theorem 1.} If there exist matrices \( P > 0, J > 0, Q_1 > 0, Q_2 > 0, Q_4 > 0, Q_6 > 0, Z_1 \geq 0, Z_2 \geq 0, W > 0, \) any matrices \( Q_2, Q_3, Z_2, Y_1, Y_2, Y_3, Y_4, W_B, W_C, L \) and a scalar \( \varepsilon > 0, \) such that the (13a) \( (13b) (14) (15) \) hold, then there exists the dynamical output feedback controller \( (3), \) for \( \forall t \in [-\tau_m, 0], \) guarantee the asymptotical stability of the closed-loop system \( (4), \) and the parameter of the controller is given as:
\[
\begin{align*}
\bar{A}_K &= P^{-1} \left( PAJ + P \delta S B_K C J - PB \bar{C}_K S - L \right) S^{-1} \\
\bar{B}_K &= \delta S^{-1} P^{-1} W_B \\
\bar{C}_K &= W_C S^{-1}
\end{align*}
\]

We can see that the (13a), (14), (15) are nonconvex conditions, so we let \( Y_1 = 0, Y_2 = 0, \) introduce parameter matrices, and use cone complementary linearization iterative algorithm \([15]\) to get the controller parameters.

Let
\[
V = \begin{bmatrix} V_1 & V_2 \end{bmatrix} \geq 0
\]

then (13a) is equivalent to:
\[
\begin{bmatrix} Q_1 & Q_2 \\ Q_2^T & Q_3 \end{bmatrix} \begin{bmatrix} J & 0 \\ 0 & I \end{bmatrix} \begin{bmatrix} V_1 & V_2 \\ V_2^T & V_3 \end{bmatrix} \geq 0
\]

\text{(17)}

Let
\[
U = \begin{bmatrix} U_1 & U_2 \\ U_2^T & U_3 \end{bmatrix} \geq 0
\]

then (14) is equivalent to:
\[
\begin{bmatrix}
U_1 & U_2 \\
U_2^T & U_3
\end{bmatrix}
\begin{bmatrix}
J & I \\
I & P
\end{bmatrix}^{-1}
\begin{bmatrix}
J & I \\
I & P
\end{bmatrix}
\geq 0
\]

Let
\[
\alpha = \begin{bmatrix}
\alpha_1 & \alpha_2 \\
\alpha_2^T & \alpha_3
\end{bmatrix} > 0
\]
\[
\beta = \begin{bmatrix}
\beta_1 & \beta_2 \\
\beta_2^T & \beta_3
\end{bmatrix} > 0
\]
then (15) is equivalent to:
\[
\begin{bmatrix}
U_1 & U_2 \\
U_2^T & U_3
\end{bmatrix}
\begin{bmatrix}
\alpha_1 & \alpha_2 \\
\alpha_2^T & \alpha_3
\end{bmatrix} \geq 0
\]
\[
\begin{bmatrix}
\alpha_1 & \alpha_2 \\
\alpha_2^T & \alpha_3
\end{bmatrix}
= \begin{bmatrix}
J & I \\
I & P
\end{bmatrix}
\]
\[
\begin{bmatrix}
W_1 & W_2 \\
W_2^T & W_3
\end{bmatrix}
\begin{bmatrix}
\beta_1 & \beta_2 \\
\beta_2^T & \beta_3
\end{bmatrix} = I
\]

Therefore, given scalar \( \varepsilon > 0 \), the problem of dynamical controller is transformed to the following cone complementary linearization.

Minimize 
\[
\text{tr}(UZ) + \text{tr}
\begin{bmatrix}
V_1 & V_2 \\
V_2^T & V_3
\end{bmatrix}
\begin{bmatrix}
Q_4 & Q_5 \\
Q_5^T & Q_6
\end{bmatrix}
\]
\[
+ \text{tr}
\begin{bmatrix}
W_1 & W_2 \\
W_2^T & W_3
\end{bmatrix}
\begin{bmatrix}
\beta_1 & \beta_2 \\
\beta_2^T & \beta_3
\end{bmatrix} + \text{tr}
\begin{bmatrix}
J & I \\
I & P
\end{bmatrix}
\begin{bmatrix}
\alpha_1 & \alpha_2 \\
\alpha_2^T & \alpha_3
\end{bmatrix}
\]

subject to
(13b), (15), (17), (18) and
\[
J > 0, P > 0, \begin{bmatrix}
U & I \\
I & Z
\end{bmatrix} > 0
\]
\[
\begin{bmatrix}
V_1 & V_2 \\
V_2^T & V_3
\end{bmatrix}
\begin{bmatrix}
I & 0 \\
0 & I
\end{bmatrix} \geq 0
\]
\[
\begin{bmatrix}
I & 0 & Q_4 & Q_5 \\
0 & I & Q_4^T & Q_5
\end{bmatrix} \geq 0
\]
\[
\begin{bmatrix}
\alpha_1 & \alpha_2 & I \\
\alpha_2^T & \alpha_3 & 0 \\
I & 0 & J \\
0 & I & P
\end{bmatrix} \geq 0
\]
\[
\begin{bmatrix}
\beta_1 & \beta_2 & I & 0 \\
\beta_2^T & \beta_3 & 0 & I \\
I & 0 & W_1 & W_2 \\
0 & I & W_2^T & W_3
\end{bmatrix} \geq 0
\]

As we all known, system uncertainty is often encountered, so in the following, by using the same method mentioned above, we consider the robust fault-tolerant control of the following time-delay systems with norm-bounded uncertainty.

Consider the following time-delay system:
\[
\begin{align*}
\dot{x}(t) &= (A + \Delta A)x(t) + (A_r + \Delta A_r)x(t-\tau) + (B + \Delta B)y(t) \\
y(t) &= (C + \Delta C)y(t) \\
x(t) &= \phi(t), t \in [-\tau, 0]
\end{align*}
\]
where \( \Delta A, \Delta A_r, \Delta B, \Delta C \) are the unknown matrices, denoting the uncertainty of this system and satisfying the following:
\[
\begin{bmatrix}
\Delta A \\
\Delta A_r \\
\Delta B \\
\Delta C
\end{bmatrix} = \begin{bmatrix}
E_1 \\
E_2
\end{bmatrix} \begin{bmatrix}
F_1 & H_1 & H_2 \\
H_1 & H_2 & H_3
\end{bmatrix}
\]

The aim is to design the following dynamical feedback controller:
\[
\begin{align*}
\dot{x}_k(t) &= A_k x_k(t) + B_k y(t) \\
u(t) &= C_k x_k(t), t \geq 0
\end{align*}
\]
to \( \forall t \in [0, \tau_m] \), when the sensor failure happen, such that the following closed-loop system is asymptotically stable.
\[
\begin{align*}
\dot{x}(t) &= \begin{bmatrix}
A + \Delta A \\
A_r + \Delta A_r \\
B + \Delta B
\end{bmatrix} \begin{bmatrix}
x(t) \\
x(t-\tau) \\
y(t)
\end{bmatrix} + \begin{bmatrix}
0 \\
I \\
0
\end{bmatrix} \begin{bmatrix}
x(t) \\
x(t-\tau) \\
y(t)
\end{bmatrix}
\end{align*}
\]
(22)
where
\[
\begin{align*}
x_k(t) &= \begin{bmatrix}
x(t) \\
x(t-\tau) \\
y(t)
\end{bmatrix} \\
\tilde{x} &:= \begin{bmatrix}
x(t) \\
x(t-\tau) \\
y(t)
\end{bmatrix} \\
\tilde{x} &:= \begin{bmatrix}
x(t) \\
x(t-\tau) \\
y(t)
\end{bmatrix} \\
\Delta A &:= \begin{bmatrix}
A_k & 0 \\
0 & A_r
\end{bmatrix}
\]

Using the same method in Theorem 1, we can get the desired dynamical feedback controller, which guaranteed the asymptotical stability of the closed-loop system (22) when the sensor failure happened. For simplicity, we deleted the proof process and gave the result directly.

**Theorem 2.** If there exist matrices \( P > 0, J > 0, Q_l > 0, Q_r > 0, Q_s > 0, Q_t > 0, X_l \geq 0, X_r \geq 0, Z_l \geq 0, Z_r \geq 0, W \geq 0, Q_l, Q_r, Z_l, Z_r, Y_l, Y_r \), \( Y_s, W_B, W_C \), \( L \) and scalars \( \varepsilon_1 > 0, \varepsilon_2 > 0 \) such that the following inequalities hold, then there exists the dynamical output feedback controller, for \( \forall t \in [-\tau_m, 0] \),
guarantee the asymptotical stability of the closed-loop system (22)

\[
\begin{bmatrix}
Q_4 \\
Q_3 \\
Q_2 \\
\bar{Q}_8
\end{bmatrix} \leq 
\begin{bmatrix}
J^{-1} & 0 & Q_1 & Q_2 \\
0 & I & \bar{Q}_1 & \bar{Q}_2 \\
0 & 0 & 1 & 0
\end{bmatrix} 
\begin{bmatrix}
J^{-1} & 0 \\
0 & I \\
0 & 0 & I
\end{bmatrix}
\]  

(23)

\[
\begin{bmatrix}
X \\
Y \\
* \\
W
\end{bmatrix} \geq 0
\]

\[
W \leq 
\begin{bmatrix}
J & I \\
I & P
\end{bmatrix} Z^{-1} 
\begin{bmatrix}
J & I \\
I & P
\end{bmatrix}
\]  

(24)

Example. Consider the following time-delay system:

\[
\dot{x}(t) = A x(t) + A x(t - \tau) + B u(t) \\
y(t) = C x(t)
\]

Where

\[
A = \begin{bmatrix}
-0.3 & 0.4 \\
0 & -0.8
\end{bmatrix}
\]

\[
A_{\tau} = \begin{bmatrix}
0.2 & 0 \\
0 & 0.1
\end{bmatrix}
\]

\[
B = \begin{bmatrix}
0.1 \\
-0.1
\end{bmatrix}
\]

\[
C = \begin{bmatrix}
-0.1 \\
0.1
\end{bmatrix}
\]

Here we let \( \delta_{\tau} = 2 \), \( \varepsilon = 0.8 \), \( \tau_m = 1.5 \)

using the main result of this paper and Matlab LMI tools, we can get the following parameters:

\[
P = \begin{bmatrix}
2.9188 & 0.8709 \\
0.8709 & 26.5270
\end{bmatrix}
\]

\[
J = \begin{bmatrix}
1.4726 & -1.1826 \\
-1.1826 & 1.6558
\end{bmatrix}
\]

\[
L = \begin{bmatrix}
0.0999 & -0.1568 \\
-0.4642 & 0.1393
\end{bmatrix}
\]

\[
S = \begin{bmatrix}
1.1266 & -1.1713 \\
-1.1713 & 1.6178
\end{bmatrix}
\]

\[
W_B = \begin{bmatrix}
0.0527 \\
-0.0638
\end{bmatrix}
\]

\[
W_C = \begin{bmatrix}
21.423 \\
-21.984
\end{bmatrix}
\]

and by using (16), we can obtain the dynamical output feedback controller :

\[
A_K = \begin{bmatrix}
-2.6193 & 0.1291 \\
1.98 & -0.7491
\end{bmatrix}
\]

\[
B_K(t) = \begin{bmatrix}
0.0094 \\
-0.0015
\end{bmatrix}
\]

\[
C_K(t) = \begin{bmatrix}
19.7680 & 0.7228
\end{bmatrix}
\]

The state trajectory of the closed-loop system is given in Fig. 1 with initial condition \( x^0(t) = [1 -2 -1 2] \).

From this figure, it can be seen that the closed-loop system is asymptotically stable under the dynamical output feedback controller the sensor failure happen, which illustrate that the proposed method in this paper is correct and effective.

IV. SIMULATION

In this section, a numerical example is given to illustrate the effectiveness of the proposed method in this paper.

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In this paper, the fault-tolerant control of time-delay system via dynamical output feedback is investigated. Based on the special sensor model of failure and delay-dependent stability criterion for nominal time-delay systems, by introducing the nonlinear transformation, the sufficient condition for the existence of feedback controller are obtained. Moreover, the desired controller is given by using the LMI and the cone complementary linearization iterative algorithm. Simulation is given to illustrate the effectiveness of the proposed method.

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