Observability Analysis and Simulation of Passive Gravity Navigation System

Fenglin Wang
Automation, Nanjing Institute of Technology, Nanjing 211167, China
Email: zdhxwfl@njit.edu.cn

Xiulan Wen and Danghong Sheng
Automation, Nanjing Institute of Technology, Nanjing 211167, China
Email: zdhxwfl@njit.edu.cn and zdhsdhl@njit.edu.cn

Abstract—A new simple and low cost passive navigation system can be composed of a rate azimuth inertial platform with a gravity sensor on it, a digital gravity abnormal map and a log. The system achieves the carrier’s true position by matching the gravity sensor measurements with the existing gravity maps, so the gravity field’s characteristics effects on the positioning accuracy greatly. The simplified error model of state variables and gravity observation equation of RAPINS/gravity matching integrated system are established in this paper. Based on the observability analysis theory of piece-wise constant system, the system observability matrix is established. By analyzing the singular of observation matrix, the influence of gravity field’s character to the navigation parameter accuracy is derived. The simulation of RAPINS/gravity matching navigation system is carried out. The results show that, with moderate precision inertial components, along the route with evident gravity anomaly and the suitable gravity gradient, the position error of this integrated system is less than one grid which equal to gravity anomaly map resolution, and platform angle error, azimuth angle error and velocity error are not big, which can ensure the underwater carrier long-term security hidden voyage achievable.

Index Terms—observability, passive navigation, gravity matching, rate azimuth inertial platform, kalman filter

I. INTRODUCTION

A new underwater passive gravity navigation system of RAPINS/log/gravity matching is composed of a rate azimuth platform inertial navigation system (RAPINS) [1], [2] with moderate precision accelerometers and gyroscopes, a gravity sensor on the rate azimuth platform, a digital gravity map, and a log [3], so it is simple and has low cost. This integrated system adopts kalman filter, which observation equation comes from comparing the measurement of gravity sensor with the gravity map, to estimate and correct navigation parameters error [4, 10, 11]. Because the navigation parameters of this integrated system is not directly observed, but estimated indirectly by the gravity observation data, the navigation precision is affected greatly by the gravity field’s character in the matching area. In this paper, the four dimensional state variables error equation of RAPINS/gravity matching integrated system, which only lacks of log, is given, gravity observation equation is set and linearized. Based on the local observability theory, the system observation matrix is established. By analyzing observation matrix singularity, the relation of gravity field character to observability is deduced, and then the influence of gravity field to the navigation accuracy, is achieved. With Matlab/Simulink, using gravity anomaly map whose resolution is 0.005°×0.005°, the simulation of RAPINS/gravity matching integrated system is carried out and simulation results show that, with moderate precision accelerometers and gyroscopes, along the route with appropriate gravity field, the positioning error of the RAPINS/gravity matching integrated navigation system, is less than one grid of gravity anomaly map resolution, velocity error, azimuth angle error and platform angle error are not large at the same time, which can meet up with the demands of underwater passive navigation.

II. ERROR EQUATIONS OF RAPINS/GRAVITY MATCHING

In the integrated system of RAPINS/log/gravity matching, the effect of the log is to damp velocity error, so as to calculate Eötvös effect, and the long-term positioning accuracy depends on the gravity matching. Therefore, the long-term positioning error of RAPINS/log/gravity matching is the same as RAPINS/gravity matching integrated system, which is only lack of log. To simplify the question, the error equations and observability of RAPINS/gravity matching integrated system are taken into account in this paper, and only four state vectors are considered to analyze observability.

A. State Equation

Under the local geodetic vertical (LGV) coordinate frame OENU, considering only four dimensional state variables of RAPINS/gravity matching integrated system, the basic error equations of the rate azimuth platform inertial navigation system are given as follows,
\[ \delta \phi = \frac{1}{R} \delta V_N \]
\[ \delta \dot{\lambda} = \frac{1}{R \cos \phi} \delta V_E + \frac{\sin \phi V_E}{R (\cos \phi)^2} \delta \phi \]  
\[ \delta \dot{V}_E = w_{ve} \]
\[ \delta \dot{V}_N = w_{vn} \]  
\[ (1) \]

*Where* \( \phi \) *is the geodetic latitude*, \( \lambda \) *is the geodetic longitude*, \( V_E \) *is east velocity*, \( V_N \) *is north velocity*, \( R \) *is radius of the earth*. \( w_{ve} \) *is east velocity random noise*, and \( w_{vn} \) *is north noise*.  

When \( X(t) = [\delta \phi \; \delta \lambda \; \delta V_E \; \delta V_N]^T \), the state equation is described as,
\[ \dot{X} = FX + W \]  
\[ (2) \]

*Where* \( F = [f_{ij}]_{4 \times 4} \), *the nonzero elements of* \( F \) *are*
\[ f_{11} = \frac{1}{R} \]
\[ f_{21} = \frac{\sin \phi V_E}{R (\cos \phi)^2} \]
\[ f_{23} = \frac{1}{R \cos \phi} \]

*and* \( W = [0 \; 0 \; w_{ve} \; w_{vn}]^T \). Because the observation period of gravimeter is 60 seconds, the advisable kalman filter period cycle \( \Delta T \) *of RAPINS/gravity matching system is 60 seconds*. The discrete state equation is,
\[ X_{k+1} = \Phi_{k+1 \times k-1} X_{k-1} + W_{k-1} \]  
\[ (3) \]

*Where* \( \Phi_{k+1 \times k-1} \) *is the state transferring matrix*, and,
\[ \Phi_{k+1 \times k-1} = I + F \Delta T + \frac{(F \Delta T)^2}{2!} + \frac{(F \Delta T)^3}{3!} \]

*Using* \( A_{k+1} = \left[ \frac{\sin \phi V_E}{R (\cos \phi)^2} \right]_{k \times 1} \) *and* \( B_{k+1} = \left[ \frac{1}{R \cos \phi} \right]_{k \times 1} \), *the state transferring matrix can be given as*
\[ \Phi_{k+1 \times k-1} = \begin{bmatrix} 1 & 0 & 0 & R^{-1} \Delta T \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \]  
\[ (4) \]

### B. Observational Equation

The rate azimuth platform inertial navigation system is independent; it can provide general navigation parameter, which includes calculated position parameter \( (\hat{\phi}, \hat{\lambda}) \). With the digital abnormal gravity map, the abnormal gravity \( g_i(\hat{\phi}, \hat{\lambda}) \) *of the position* \( (\hat{\phi}, \hat{\lambda}) \) *can be achieved*.  

*According to* WGS84 *reference ellipsoid surface standard gravity equation provided by U.S. defense SMC*, \( g_i(\bar{\phi}) \) *of the position* \( (\bar{\phi}, \bar{\lambda}) \) *can be calculated as,
\[ g_i(\phi) = a_0 (1 + a_1 \sin^2 \phi) \times (1 - a_2 \sin^2 \phi)^{-a_3} \]
\[ (5) \]

*Where* \( a_0 = 9.7803267715 \), \( a_1 = 0.001931851353 \), \( a_2 = 0.0066943800229 \).  

Then, the calculated gravity of the position \( (\bar{\phi}, \bar{\lambda}) \)  
\[ g(\phi, \lambda) = g_i(\phi) + g_i(\hat{\phi}, \hat{\lambda}) \]  
\[ (6) \]

In the dynamic gravity measurement, the gravity sensor outputs value \( g_i(\phi, \lambda) \) *can be given as,*  
\[ g_i(\phi, \lambda) = g(\phi, \lambda) - E + r_{g} \]  
\[ (7) \]

*Where* \( (\phi, \lambda) \) *is the true position of its carrier*. \( g(\phi, \lambda) \) *is the static gravity value*, \( E \) *is the Eötvös effect*, which is produced owing to velocity, \( r_{g} \) *is the other error in dynamic gravity measurement*.  

Comparing the measurement value \( g_i(\phi, \lambda) \) *from gravity sensor with the calculated gravity value* \( g(\hat{\phi}, \hat{\lambda}) \) *from the abnormal gravity map*, *the observation equation of RAPINS/gravity matching integrated system can be obtained as follows,*  
\[ Z_g = [g(\hat{\phi}, \hat{\lambda}) - g_c(\phi, \lambda)] - g_i(\phi, \lambda) \]  
\[ (8) \]

*Where* \( E_c \) *is the calculated Eötvös effect*, which is worked out by the parameter provide by the rate azimuth platform inertial navigation system*. Regarding (6) and (7), *the observation equation is described as,*  
\[ Z_g = [g(\hat{\phi}, \hat{\lambda}) - g_c(\phi, \lambda)] - r_k - r_{g} \]  
\[ (9) \]

*Where* \( r_k \) *is the Eötvös effect calculation error*, which is small if log is considered. To study the relation of gravity field to gravity matching, *the influence of Eötvös effect calculation error is neglected*. Linearizing the observation equation of (9), *then,*  
\[ Z = \begin{bmatrix} \hat{g}_c(\phi, \lambda) \; \hat{g}_c(\phi, \lambda) \; \hat{g}_c(\phi, \lambda) \; \hat{g}_c(\phi, \lambda) \end{bmatrix} \]  
\[ \begin{bmatrix} \frac{\partial \hat{g}_c(\phi, \lambda)}{\partial \phi} \\ \frac{\partial \hat{g}_c(\phi, \lambda)}{\partial \phi} \\ \frac{\partial \hat{g}_c(\phi, \lambda)}{\partial \phi} \\ \frac{\partial \hat{g}_c(\phi, \lambda)}{\partial \phi} \end{bmatrix} + r_k \]  
\[ (10) \]

*Where* \( \hat{g}_c(\phi, \lambda) \) *are gravity gradient in the latitude and the longitude direction severally*, *and* \( r_k \) *is the sum of the Eötvös effect calculation error, gravity sensor noise, gravity anomaly map noise and linear error*.  

*Then, the observation matrix* \( H \) *is,*  
\[ H = \begin{bmatrix} \frac{\partial g_c(\phi, \lambda)}{\partial \phi} \\ \frac{\partial g_c(\phi, \lambda)}{\partial \phi} \\ \frac{\partial g_c(\phi, \lambda)}{\partial \phi} \\ \frac{\partial g_c(\phi, \lambda)}{\partial \phi} \end{bmatrix} \]  
\[ (11) \]

*In view of* \( g(\phi, \lambda) = g_i(\phi, \lambda) + g_i(\phi) \) *the observation matrix* \( H \) *can also be described as,*  
\[ H = \begin{bmatrix} \frac{\partial g_c(\phi, \lambda)}{\partial \phi} \\ \frac{\partial g_i(\phi, \lambda)}{\partial \phi} \\ \frac{\partial g_c(\phi, \lambda)}{\partial \phi} \\ \frac{\partial g_i(\phi, \lambda)}{\partial \phi} \end{bmatrix} \]  
\[ (12) \]

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III. OBSERVABILITY OF RAPINS/GRAVITY MATCHING

On the basis of state transition matrix \( \phi_{k/k-1} \) and the observation matrix \( H_k \), the local observable matrix \([5, 6]\) \( M \) is inferred as,

\[
M = \begin{bmatrix}
H_k \\
H_{k+1} \phi_{k+1/k} \\
\vdots \\
H_{k+n-1} \phi_{k+n-2/k+n-1} \\
\end{bmatrix}.
\]

If the rank of the local observable matrix \( M \) is \( n \), the integrated navigation system is locally observable in the time from \( k \) to \( k+n-1 \) [7, 8].

A. Observability of Four Dimensional State Variables

In view of (4) and (11), to four dimensional state variables \( X(t) = [\delta \varphi \; \delta \lambda \; \delta V_x \; \delta V_y]^T \), the locally observable matrix \( M_{4x4} \) is as follows,

\[
M_{1,1} = \frac{\partial g}{\partial \varphi} (k), \quad M_{1,2} = \frac{\partial g}{\partial \varphi} (k+1) + \frac{\partial g}{\partial \lambda} (k+1) \times A_{1i} \Delta T,
\]

\[
M_{1,3} = \frac{\partial g}{\partial \varphi} (k+2) + \frac{\partial g}{\partial \lambda} (k+2) \times (A_{1i-1} + A_{1i}) \Delta T,
\]

\[
M_{1,4} = \frac{\partial g}{\partial \varphi} (k+3) + \frac{\partial g}{\partial \lambda} (k+3) \times (A_{1i+1} + A_{1i+i}) \Delta T,
\]

\[
M_{1,5} = \frac{\partial g}{\partial \varphi} (k+4) + \frac{\partial g}{\partial \lambda} (k+4) \times B_{1i} \Delta T,
\]

\[
M_{3,1} = \frac{\partial g}{\partial \varphi} (k+2) \times (B_{1i-1} + B_{1i}) \Delta T,
\]

\[
M_{3,2} = \frac{\partial g}{\partial \varphi} (k+3) \times (B_{1i+1} + B_{1i+i}) \Delta T,
\]

\[
M_{3,3} = \frac{\partial g}{\partial \varphi} (k+4) \times (B_{1i-1} + B_{1i+i}) \Delta T,
\]

\[
M_{4,1} = \frac{\partial g}{\partial \varphi} (k+3) \times (B_{1i+1} + B_{1i+i}) \Delta T,
\]

\[
M_{4,2} = \frac{\partial g}{\partial \varphi} (k+4) \times (B_{1i+1} + B_{1i+2}) \Delta T,
\]

\[
M_{4,3} = \frac{\partial g}{\partial \varphi} (k+5) \times (B_{1i+1} + B_{1i+2}) \Delta T,
\]

\[
M_{4,4} = \frac{\partial g}{\partial \varphi} (k+6) \times (B_{1i+1} + B_{1i+2}) \Delta T.
\]

It can be seen that the fourth column of matrix \( M_{4x4} \) is almost a singular matrix, and the four dimensional state variables \( M \) is far less than the second column. Therefore, the locally observable matrix \( M_{4x4} \) has bad observability. Because the velocity error \( \delta V_x \) and \( \delta V_y \) is not directly in observation equation of (10), but indirectly observed by the system matrix coupled to the state variables \( \delta \varphi \) and \( \delta \lambda \). Then the observability of \( \delta V_x \) and \( \delta V_y \) is relatively bad, which means the velocity error of RAPINS/gravity matching integrated system is relatively large.

B. Observability of Two Dimensional State Variables

If only two dimensional state variables \( \delta \varphi \) and \( \delta \lambda \) are considered, then \( X(t) = [\delta \varphi \; \delta \lambda \; \delta V_x \; \delta V_y]^T \), the derivation of corresponding locally observable matrix \( M_{2x2} \) is as follows,

\[
M_{2x2} = \begin{bmatrix}
\frac{\partial g}{\partial \varphi} (k) & \frac{\partial g}{\partial \lambda} (k) \\
\frac{\partial g}{\partial \varphi} (k+1) & \frac{\partial g}{\partial \lambda} (k+1) \\
\end{bmatrix}.
\]

When latitude is less than 60°, \( A_{4i} \leq 3.3983 \times 10^{-8} \), so \( A_{i} \Delta T \times \frac{\partial g}{\partial \lambda} (k+1) \) can be ignored generally. The observability of the integrated system is entirely depended on the gravity gradient \( \frac{\partial g}{\partial \varphi} (k) \) and \( \frac{\partial g}{\partial \lambda} (k) \).

According to the singularity of matrix \( M_{2x2} \) , some conclusions are obtained,

1. To acquire better observability, the gravity gradient \( \frac{\partial g}{\partial \varphi} (k) \) and \( \frac{\partial g}{\partial \lambda} (k) \) should not be too small, which means the gravity character is obvious. If \( \frac{\partial g}{\partial \varphi} (k) \) and \( \frac{\partial g}{\partial \lambda} (k) \) are close to zero, \( M_{2x2} \) is nearly a singular matrix.

2. To acquire better observation, \( \frac{\partial g}{\partial \varphi} (k) \) and \( \frac{\partial g}{\partial \lambda} (k) \) should not differ too much. In the general area of the
earth, the gravity contour varies greatly in latitude direction, and gently in longitude direction, which means that \( \frac{\partial g}{\partial \varphi} (k) \) is far greater than \( \frac{\partial g}{\partial \lambda} (k) \) generally, so the first column of matrix \( M_{22} \) is much greater than the second column, thus \( M_{22} \) is close to singular, then \( \varphi \) is observable, \( \lambda \) is unobservable, and the estimated parameter \( \varphi \) has higher precision than \( \lambda \). In the special area, which gravity anomaly vary significantly, \( \frac{\partial g}{\partial \varphi} (k) \) and \( \frac{\partial g}{\partial \lambda} (k) \) are big at the same time, so the estimated parameter \( \varphi \) and \( \lambda \) both have high precision.

(3) To acquire better observation, \( \frac{\partial g}{\partial \varphi} (k) \) and \( \frac{\partial g}{\partial \lambda} (k) \) must vary drastically along with \( k \). If \( \frac{\partial g}{\partial \varphi} (k+1) \) is close to \( \frac{\partial g}{\partial \varphi} (k) \), or \( \frac{\partial g}{\partial \lambda} (k+1) \) close to \( \frac{\partial g}{\partial \lambda} (k) \), \( M_{22} \) is nearly singular. When they vary more greatly, matrix \( M_{22} \) has more nonsingular, so \( \varphi \) and \( \lambda \) have better observability.

(4) To acquire better observability, \( \frac{\partial g}{\partial \varphi} (k) \) and \( \frac{\partial g}{\partial \lambda} (k) \) can’t vary with the same rules as \( k \). If \( \frac{\partial g}{\partial \varphi} (k+1)/\frac{\partial g}{\partial \varphi} (k) \) is the same as \( \frac{\partial g}{\partial \lambda} (k+1)/\frac{\partial g}{\partial \lambda} (k) \), \( M_{22} \) is singular.

IV. SIMULATION OF RAPINS/GRAVITY MATCHING

With Matlab/Simulink tools, the simulation of the RAPINS/gravity matching integrated navigation system is carried out. The simulation block diagram is shown as Fig. 1.

Based on true navigation parameter provided by ideal rate azimuth platform navigation system, which includes the true location \((\varphi, \lambda)\) and true velocity \((V_{e}, V_{c})\) of the carrier, the true Eötvös effect \( E \) and standard gravity \( g_{b} (\varphi) \) can be obtained. With digital gravity abnormal map, \( g_{a} (\varphi, \lambda) \) also can be found. The true static gravity \( g(\varphi, \lambda) \) is the sum of \( g_{b} (\varphi) \) and \( g_{a} (\varphi) \). After \( g(\varphi, \lambda) \) subtracting \( E \), plusing other interference \( r_{ga} \), gravity sensor output \( g_{i} (\varphi, \lambda) \) can be simulated.

Adding inertial components error, installation error, initial error and other noise to ideal rate azimuth platform system, the actual rate azimuth platform inertial navigation system can be simulated, which outputs calculated location parameter \((\hat{\varphi}, \hat{\lambda})\), velocity parameter \((\hat{V}_{e}, \hat{V}_{c})\), platform angle error \((\hat{\phi}_{e}, \hat{\phi}_{c}, \hat{\phi}_{o})\) and others.

On basis of (9), the observation equation of RAPINS/Gravity matching is obtained.

\[
\mathbf{X}(t) = F(t)\mathbf{X}(t) + G(t)\mathbf{W}(t) .
\]

Where the system state vector \( X(t) = [\phi_{e}, \phi_{c}, \phi_{o}, \Delta V_{e}, \Delta V_{c}, \Delta V_{e}, \Delta \phi_{e}, \Delta \lambda_{e}, \Delta \lambda_{c}, \Delta \lambda_{o}, \Delta V_{e}, \Delta \lambda_{o}, \Delta \lambda_{o}, \Delta \lambda_{o}] \cdot \phi_{e} \) and \( \phi_{c} \) are platform angle error, \( \phi_{o} \) is azimuth angle error, \( E_{e}, E_{c} \), and \( E_{o} \) are gyroscopic drifts, \( \Delta \lambda_{e} \), \( \Delta \lambda_{c} \), and \( \Delta \lambda_{o} \) are accelerometer drifts. The state transferring matrix \( F(t) \), noise matrix \( G(t) \) and noise vector \( W(t) \) are all obtained by state error equations, which are the same as in [9].

On basis of (9), the observation equation of RAPINS/gravity matching integrated system can be shown as:

\[
\mathbf{Z}(t) = H(t)\mathbf{X}(t) + \mathbf{V}(t) .
\]
Where $H(t) = [h]_{i=15}$, observation noise $V(t)$ and $W(t)$ are all be confirmed by the parameters of navigation initial error, instrument error and other error.

The nonzero elements of $H(t)$ are,

\[ [h]_{i} = \frac{\partial g_{i} (\boldsymbol{\hat{\phi}}, \boldsymbol{\hat{\lambda}})}{\partial \phi} , \quad [h]_{k} = \frac{\partial g_{k} (\boldsymbol{\hat{\phi}}, \boldsymbol{\hat{\lambda}})}{\partial \lambda} \]

which can be calculated by abnormal gravity random linearity technology and differential of standard gravity equation (5). Nine dots fitting method is used to calculate gravity abnormal gradient $\frac{\partial g_{i} (\boldsymbol{\hat{\phi}}, \boldsymbol{\hat{\lambda}})}{\partial \phi}$ and $\frac{\partial g_{k} (\boldsymbol{\hat{\phi}}, \boldsymbol{\hat{\lambda}})}{\partial \lambda}$ in this simulation. As shown in Fig. 2, it selects the dot on the grid, which is the nearest to the integrated system output position $(\boldsymbol{\hat{\phi}}, \boldsymbol{\hat{\lambda}})$, as the center, and selects other nearby eight dots on the gravity abnormal grid, $\sigma_{\phi}$ and $\sigma_{\lambda}$ are obtained by state mean square matrix $P$ of kalman filter. In the fitting area confirmed by the nine grid dots, least square method is used to calculate the plane fitting for the gravity abnormal curve $g_{j} (\boldsymbol{\hat{\phi}}, \boldsymbol{\hat{\lambda}})$, which is determined by this nine grid dots. Using $g_{yi}$ as the abnormal gravity of point $i$, gravity abnormal gradient of the fitting plane is calculated as,

\[ \frac{\partial g_{i} (\boldsymbol{\hat{\phi}}, \boldsymbol{\hat{\lambda}})}{\partial \phi} = \frac{(g_{yi} + g_{y2} + g_{y3}) - (g_{y1} + g_{y8} + g_{y9})}{6 Md} \]

\[ \frac{\partial g_{k} (\boldsymbol{\hat{\phi}}, \boldsymbol{\hat{\lambda}})}{\partial \lambda} = \frac{(g_{y3} + g_{y6} + g_{y9}) - (g_{y1} + g_{y4} + g_{y7})}{6 Nd} \]  

In the RAPINS/gravity matching integrated navigation system, the state updated frequency of rate azimuth platform inertial navigation system can be assumed as 0.1 second, the observation cycle of gravity sensor is assumed as 60 seconds, so the sampling period $\Delta t$ of kalman filter can be set as 0.1 second, the filter cycle $\Delta T$ can be set as 60 seconds. The discretized state equation and observation equation can be described as,

\[ X_{k} = \phi_{k/k-1} X_{k-1} + \Gamma_{k/k-1} W_{k-1} \]

\[ Z_{k} = H_{k} X_{k-1} + V_{k} \]

Where $X_{k}$ is the 15 dimensional state vector of time $k$, $Z_{k}$ is one dimensional observation vector of time $k$, $\phi_{k/k-1}$ is state transition matrix from time $k-1$ to $k$, $\Gamma_{k/k-1}$ is system noise transformation matrix from time $k-1$ to $k$, $W_{k}$ is system noise, $V_{k}$ is observation noise. $W_{k}$ and $V_{k}$ are irrelevant zero mean white noise sequences, and,

\[ E [W_{i} W_{j}^T] = Q_{k} \delta_{ij} \]

\[ E [V_{i} V_{j}^T] = R_{k} \delta_{ij} \]

Based on the state transformation matrix $F(t)$ in equation (12), the discretized state transformation matrix $\phi_{k/k-1}$ can be calculated as,

\[ \phi_{k/k-1} = e^{F(t_{k})T} = \sum_{n=0}^{\infty} \frac{(F(t_{k})T)^{n}}{n!} \]  

Where $T$ is the state updated calculation cycle, $n$ can choose bigger when filter cycle is short, and choose smaller when filter cycle is long. In the later kalman filter recursive equation, state prediction equation $\hat{X}_{k/k-1}$ and mean square error equation $P_{k/k-1}$ are both needed to calculate $\phi_{k/k-1}$. When calculating $P_{k/k-1}$, the updated cycle $T$ of $\phi_{k/k-1}$ in equation (20) can use 0.1 second, which is the same as the state updated frequency of rate azimuth platform inertial navigation system, and $n$ can use 3. When calculating $\hat{X}_{k/k-1}$, the updated cycle $T$ of $\phi_{k/k-1}$ in equation (20) can use 60 seconds, which is the same as the observation cycle of gravity sensor, and $n$ can use 5.

Based on the state transformation matrix $G(t)$ in equation (12), the discretized noise transformation matrix $\Gamma_{k/k-1}$ can be calculated as,

\[ \Gamma_{k/k-1} = \phi_{k/k-1} G(t_{k-1}) \Delta t \]

The discretized state noise matrix $Q_{k}$ and observation noise matrix $R_{k}$ can be calculated as,

\[ Q_{k} = \frac{Q(t_{k})}{\Delta t} \]

\[ R_{k} = \frac{R(t_{k})}{\Delta T} \]

The standard closed loop kalman filter recursive equation is as follows,

\[ P_{k/k-1} = \phi_{k/k-1} P_{k/k-1} \phi_{k/k-1}^T + \Gamma_{k/k-1} Q_{k} \Gamma_{k/k-1}^T \]

\[ K_{k} = P_{k/k-1} H_{k}^T (H_{k} P_{k/k-1} H_{k}^T + R_{k})^{-1} \]

\[ P_{k} = (1 - K_{k} H_{k}) P_{k/k-1} \]
\[ \hat{X}_{k+1} = \phi_{k+1} \hat{X}_k \]  
(27) State estimation calculation equation with close loop controlling method  
\[ \hat{X}_k = K_k Z_k \]  
(28)

After the closed loop kalman filter recursive equation is performed, the navigation parameter, such as position error, velocity error, and platform angle error are all evaluated, and these errors are used to correct the parameter of pure rate azimuth platform inertial navigation system to improve navigation precision, which is called integrated navigation parameter precision. Subtracting the integrated navigation parameter with the ideal navigation parameter, the navigation errors of RAPINS/gravity matching integrated system are obtained.

The carrier with RAPINS/gravity matching integrated system has initial conditions and model parameters as follows, rate azimuth platform system initial horizontal attitude (pitch and roll) error angles is 45°, initial azimuth error angle 2°, initial position error 50m, velocity error 0.3m/s, integrated gyro bias drift and random walk are 0.01°/h and 0.005°/h, rate gyro bias drift and random walk are 0.05°/h and 0.03°/h respectively, three accelerometers bias drift is 50 μg, the gravity sensor noise is 0.05mGal, and the Eötvös effect calculation error is ignored. The digital gravity anomaly map with grid resolution of 0.005° × 0.005° is adopted. Fig. 3 is the gravity anomaly contour of matching area from the region of (138°,15°) to (146°,27°).

Fig.4 is the corresponding gravity contour. In this region, the gravity contour is dense, and gravity character is obvious. The unit of colorbar is m/s² in Fig.3 and Fig.4. When the carrier sails toward the north from position (16°,142.5°) at a constant speed of 20 m/s, the positioning error curve of RAPINS/gravity matching integrated navigation system in the process of 16 hours sailing is shown in Fig. 5.

The horizontal axis of Fig. 5 is the carrier’s latitude, longitude is 142.5° all the time. The vertical axis is the navigation positioning error. In this figure, it can be shown that RAPINS/gravity matching integrated system has different positioning accuracy along different region.

To study the relation of gravity field to position precision obviously, the gravity gradient curve in the process of navigation, which is calculated by using (14), (15) and differential of equation (5), is shown in Fig. 6.

From Fig. 6, it can be seen that, along the route from (142.5°,16.2°) to (142.5°,19.2°), the gravity gradient curve is gentle, that is \( \frac{\partial g}{\partial \phi} (k) \) and \( \frac{\partial g}{\partial \lambda} (k) \) vary very slowly, which means \( \frac{\partial g}{\partial \phi} (k+1) \) is close to \( \frac{\partial g}{\partial \phi} (k) \), \( \frac{\partial g}{\partial \lambda} (k+1) \) is close to \( \frac{\partial g}{\partial \lambda} (k) \), which leads to \( M_{22} \) close to singularity. At the same time, in this region, \( \frac{\partial g}{\partial \phi} (k) \) varies mostly according to the same regularity as
\[
\frac{\partial g}{\partial \lambda}(k), \text{ which means } \frac{\partial g}{\partial \phi}(k+1)/\frac{\partial g}{\partial \phi}(k) \text{ is the same as } \frac{\partial g}{\partial \lambda}(k+1)/\frac{\partial g}{\partial \lambda}(k). \text{ So the observability of matrix } M_{2,2}
\]
is almost singular in this region. Because of these two reasons, the observability is bad, then the navigation positioning error is very large. This can be proved from Fig. 4, which shows \(\Delta \phi\) is up to 2.5 times the grid, and \(\Delta \lambda\) is nearly up to 4 times the grid. Because the curve of \(\frac{\partial g}{\partial \lambda}(k)\) is gentler than \(\frac{\partial g}{\partial \phi}(k)\) in this region, so the observation of latitude is more bad than longitude, and \(\Delta \lambda\) is bigger than \(\Delta \phi\).

At the same time, Fig. 6 shows that, along the route from (142.5°, 19.2°) to (142.5°, 26.2°), the gravity gradient curve is violent ups and downs, which means \(\frac{\partial g}{\partial \phi}(k)\) and \(\frac{\partial g}{\partial \lambda}(k)\) vary very quickly, \(\frac{\partial g}{\partial \phi}(k+1)\) differs greatly with \(\frac{\partial g}{\partial \lambda}(k)\), \(\frac{\partial g}{\partial \lambda}(k+1)\) differs greatly with \(\frac{\partial g}{\partial \lambda}(k)\), which is advantage to \(M_{2,2}\) nonsingularity.

What’s more, in this region, \(\frac{\partial g}{\partial \phi}(k)\) varies much differently from \(\frac{\partial g}{\partial \lambda}(k)\), which means \(\frac{\partial g}{\partial \phi}(k+1)\), \(\frac{\partial g}{\partial \phi}(k)\) is very unequal to \(\frac{\partial g}{\partial \lambda}(k+1)\), \(\frac{\partial g}{\partial \lambda}(k)\). So the observation matrix \(M_{2,2}\) is nonsingular in this region, and the observability is good, then the navigation positioning error is very small. This can be testified by Fig. 5, it shows \(\Delta \phi\) and \(\Delta \lambda\) is nearly within one grid, which is just the resolution of gravity anomaly map.

Furthermore, because the longitude \(\lambda\) is only related to anomaly gravity, then \(\frac{\partial g}{\partial \lambda}(k)\) is very small in most matching region on the earth owing to unapparent gravity anomaly. At the same time, the longitude \(\phi\) is the function of anomaly gravity and standard gravity, \(\frac{\partial g}{\partial \phi}(k)\) is relatively big in most matching region, \(\frac{\partial g}{\partial \phi}(k)\) and \(\frac{\partial g}{\partial \lambda}(k)\) differs greatly on the earth generally. Then, in most navigation area, the longitude \(\lambda\) is unobservable, and the longitude error \(\Delta \lambda\) is very big. Meanwhile, the longitude \(\phi\) is observable, and the longitude error \(\Delta \phi\) is very small. This can be proved by many other simulations along different routes. Because positioning accuracy is significantly different with the matching area of variant gravity field, so it is very important to search suitable navigation route for RAPINS/gravity matching integrated navigation system.

In addition, the corresponding velocity error curve of RAPINS/gravity matching integrated navigation system is shown in Fig. 7, and the velocity errors in east and north are both less than 0.4 m/s in most navigation area.

It can be seen that, along the bad position matching route from (142.5°, 16.2°) to (142.5°, 19.2°), the integrated navigation system nearly has the equal velocity error as along the well position matching route from (142.5°, 19.2°) to (142.5°, 26.2°), which means the velocity error is not affected by matching region very much. This is because matrix \(M_{4,4}\) is singular, and the velocity error has bad observability.

The azimuth angle error of RAPINS/gravity matching integrated navigation system is shown in Fig. 8, which is mostly less than \(5.2°\). The horizontal attitude angles error,
referring to pitch $f_x$ and roll $f_y$, is usually less than $1^\circ$, which is shown in Fig. 9.

V. SUMMARY

The integrated navigation system of RAPINS/gravity matching corrects positioning error real time by measuring gravity indirectly, so the positioning precision is relevant to gravity field of matching area. Based on the local observability theory, the relationship between observability and the character of gravity field is analyzed in this paper, which mainly points out that, to acquire better observability, the gravity gradient in latitude direction and longitude direction should not be too small, should not differ too much, must vary drastically along with time, and they can't change according to the same rules with time. The simulation results show that, along the route with suitable gravity field which meets the requirements of observation matrix nonsingularity, the integrated system of RAPINS/gravity matching has good navigation accuracy, positioning error is less than one grid, velocity error, platform angle error and azimuth angle error are not big at the same time, which can meet up with the demands of underwater passive gravity navigation.

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Fenglin Wang received the MEng and PhD in measurement technology and instruments from Southeast University, Nanjing, China, in 2003 and 2006 respectively, received Bachelor in industrial automation from Wuhan University of Hydraulic and Electric Engineering.

She is currently an associate professor of mechanical engineering in the automation department of Nanjing Institute of Technology. She is mainly engaged in integrated navigation, information fusion, precision measuring technology research, and acts as the first author of nearly twenty technical papers in these areas.