Abstract—In this paper, we present a new security risk assessment method of website based on generalized fuzzy numbers. First, we present a new similarity measure between generalized fuzzy numbers. We also prove some properties of the proposed similarity measure and make an experiment to use 18 sets of generalized fuzzy numbers to compare the experimental results of the proposed method with the existing similarity measures. The proposed method can overcome the drawbacks of the existing similarity measures. Based on this method, we present a new security risk assessment algorithm to assess the website security risk. In the proposed method, the possibility of threat occurrence and the loss of the result led by the threat are selected as parameters to compute security risk, where the values of the parameters are represented by generalized fuzzy numbers. The proposed method provides a useful way to achieve the website security risk assessment.

Index Terms—website, security risk assessment, generalized fuzzy numbers, similarity measures

I. INTRODUCTION

In the past few years, social networking websites have become a popular networking culture. Increasing numbers of people are using the website for searching, communicating, on-line trading and so on [1]. Although people are fond of these applications, social websites also encounter a number of privacy threats. Due to the vulnerability of Internet, it is possible for hackers to attack these websites. Some hackers prefer to change the entire server. For example, some logos of the enterprises or partial important contents of the government homepages might be modified to mislead the users, which might lead to undesirable social impact, significant economic loss and even undermine the national security [2]. To avoid website security problems, firstly, we must make clear to trigger this issue. Sum up, mainly in the following four reasons: hacker attacks, the lack of management, network flows and loopholes in the software or the “back door” [3].

According to the report of CNCERT/CC in 2009 [4], the basic website condition in China is holistically fine. Nevertheless, during the past years, there is a number of significant nationwide and province-wide website security events occurred. The report also indicates that the number of website security events increases a lot compared to the last year. Furthermore, according to the report, there are 42,000 websites in China mainland tampered by hackers in 2009. Notably, the government websites occupied a higher proportion, up to 12.24% in December, 2009. Consequently, it can be seen that it’s both critical and important to guarantee the security of websites [5].

In this paper, we present a new security risk assessment method of website based on generalized fuzzy numbers. First, we present a new similarity measure between generalized fuzzy numbers. Some methods have been presented to calculate the degree of similarity between fuzzy numbers [8-14]. But they all have some shortages. The new similarity measure, which combines the concepts of the area, the perimeter, the height and the center of gravity of generalized fuzzy numbers, can overcome the drawbacks of these similarity measures. Second, based on the new method, we present a new security risk assessment algorithm for dealing with the website problems. In the proposed method, we choose the possibility of threat occurrence and loss of the result led by the threat as parameters to compute security risk [6], where the values of the parameters are represented by generalized fuzzy numbers. The proposed method provides us a useful way to achieve the website security risk assessment.

The rest of this paper is organized as follows. In Section II, we briefly review basic concepts of generalized fuzzy numbers and their arithmetic operations. In Section III, we briefly review some existing similarity measures of fuzzy numbers. In Section IV, we present a new similarity measure between generalized fuzzy numbers and give some properties of the method. In Section V, we make a comparison of the
calculated results of the proposed method with the existing similarity measures. In Section VI, we apply the proposed similarity measures to propose a fuzzy number algorithm to deal with the website security problems and use an example to illustrate the website security risk assessment process of the proposed method. The conclusions are discussed in Section VII.

II. BASIC CONCEPTS OF GENERALIZED FUZZY NUMBERS

In this Section we briefly describe the basic concepts of generalized fuzzy numbers. Chen (1985) proposed the concept of generalized fuzzy numbers [7]. Let \( \mathbf{A} \) be a generalized trapezoidal fuzzy number, 
\[
\mathbf{A} = (a_l, a_m, a_r, a_0; w \mathbf{j}) \text{ where } a_l, a_m, a_r, a_0 \text{ are real values},
\]
\( w \mathbf{j} \) denotes the height of the generalized fuzzy number \( \mathbf{A} \), and \( w \mathbf{j} \in [0,1] \). If \( 0 \leq a_l \leq a_m \leq a_r \leq a_0 \leq 1 \), then \( \mathbf{A} \) is called a standard generalized fuzzy number. If \( w \mathbf{j} = 1 \), then \( \mathbf{A} \) becomes a traditional fuzzy number and can be represented as \( \mathbf{A} = (a_l, a_m, a_r, a_0) \). If \( a_l = a_r \), then \( \mathbf{A} \) is a triangular fuzzy number. If \( a_l = a_r = a_m = a_0 \), then \( \mathbf{A} \) is a crisp value. Now we briefly describe some arithmetic operations between generalized fuzzy numbers (Chen, 1985; Chen & Chen, 2007) [7, 9].

Assume that \( \mathbf{A} \) and \( \mathbf{B} \) are two trapezoidal generalized fuzzy numbers where 
\[
\mathbf{A} = (a_l, a_m, a_r, a_0; w \mathbf{j}) \quad \text{and} \quad \mathbf{B} = (b_l, b_m, b_r, b_0; w \mathbf{y})
\]
are two trapezoidal fuzzy numbers, where \( a_l, a_m, a_r, a_0, b_l, b_m, b_r, b_0 \) are real values, \( 0 \leq w \mathbf{j}, w \mathbf{y} \leq 1 \). Some arithmetic operations between the generalized fuzzy numbers \( \mathbf{A} \) and \( \mathbf{B} \) are shown as follows:

Generalized fuzzy numbers addition \( \oplus \):
\[
\mathbf{A} \oplus \mathbf{B} = (a_l, a_m, a_r, a_0; w \mathbf{j}) \oplus (b_l, b_m, b_r, b_0; w \mathbf{y}) = (a_l + b_l, a_m + b_m, a_r + b_r, a_0 + b_0; \min(w \mathbf{j}, w \mathbf{y})).
\]

Generalized fuzzy numbers multiplication \( \otimes \):
\[
\mathbf{A} \otimes \mathbf{B} = (a_l, a_m, a_r, a_0; w \mathbf{j}) \otimes (b_l, b_m, b_r, b_0; w \mathbf{y}) = (a_l \times b_l, a_m \times b_m, a_r \times b_r, a_0 \times b_0; \min(w \mathbf{j}, w \mathbf{y})).
\]

Generalized fuzzy numbers division \( \oslash \):
Loss is taken into account and if \( a_l, a_m, a_r, b_l, b_m, b_r, b_0 \) are all nonzero positive real numbers, and then we have:
\[
\mathbf{A} \oslash \mathbf{B} = (a_l, a_m, a_r, a_0; w \mathbf{j}) \oslash (b_l, b_m, b_r, b_0; w \mathbf{y}) = \frac{a_l - a_m - a_r + a_0 - b_l}{b_l - b_m - b_r + b_0; \min(w \mathbf{j}, w \mathbf{y})}.
\]

Generalized fuzzy number multiplied by the real number is defined as follows:
\[
m \times \mathbf{A} = m \times (a_l, a_m, a_r, a_0; w \mathbf{j}) = (m \times a_l, m \times a_m, m \times a_r, m \times a_0; w \mathbf{j}).
\]

III. A REVIEW OF THE EXISTING SIMILARITY MEASURES BETWEEN FUZZY NUMBERS

In this section, we briefly introduce some existing similarity measures between fuzzy numbers from Chen (1996) [8], Lee (2002) [14], Chen and Chen (2003) [11], Wei & Chen (2009) [12] and Hejazi, et al. (2011) [13].

Let \( \mathbf{A} \) and \( \mathbf{B} \) be two trapezoidal fuzzy numbers, where 
\[
\mathbf{A} = (a_l, a_m, a_r, a_0) \quad \text{and} \quad \mathbf{B} = (b_l, b_m, b_r, b_0),
\]
as shown in Fig. 1.

Chen (1996) presented a similarity measure between fuzzy numbers \( \mathbf{A} \) and \( \mathbf{B} \) based on the geometric distance, where the degree of similarity \( S(\mathbf{A}, \mathbf{B}) \) between the fuzzy numbers \( \mathbf{A} \) and \( \mathbf{B} \) is calculated as follows:
\[
S(\mathbf{A}, \mathbf{B}) = 1 - \frac{\sum_{i=1}^{4}|a_i - b_i|}{4}.
\]

Where \( S(\mathbf{A}, \mathbf{B}) \in [0,1] \). The larger the value of \( S(\mathbf{A}, \mathbf{B}) \), the more the similarity between the fuzzy numbers \( \mathbf{A} \) and \( \mathbf{B} \).

If \( \mathbf{A} \) and \( \mathbf{B} \) are two triangular fuzzy numbers, where 
\[
\mathbf{A} = (a_l, a_m, a_r) \quad \text{and} \quad \mathbf{B} = (b_l, b_m, b_r),
\]
as shown in Fig. 2.

The degree of similarity \( S(\mathbf{A}, \mathbf{B}) \) between the triangular fuzzy numbers \( \mathbf{A} \) and \( \mathbf{B} \) is calculated as follows [Chen (1996)]:
\[
S(\mathbf{A}, \mathbf{B}) = 1 - \frac{\sum_{i=1}^{3}|a_i - b_i|}{3}.
\]

Where \( S(\mathbf{A}, \mathbf{B}) \in [0,1] \). The larger the value of \( S(\mathbf{A}, \mathbf{B}) \), the more the similarity between the fuzzy numbers \( \mathbf{A} \) and \( \mathbf{B} \).

Lee (2002) presented a similarity measure between trapezoidal fuzzy numbers, where the degree of similarity \( S(\mathbf{A}, \mathbf{B}) \) between the trapezoidal fuzzy numbers \( \mathbf{A} \) and \( \mathbf{B} \) is calculated as follows:

![Figure 1. Trapezoidal fuzzy numbers \( \mathbf{A} \) and \( \mathbf{B} \)]
where \[ B(S_A, S_B) = \begin{cases} 1 & S_A + S_B > 0 \\ 0 & S_A + S_B = 0 \end{cases} \] (13)

where \[ S_A = a_4 - a_1 \text{ and } S_B = b_4 - b_1 \] are the lengths of the generalized trapezoidal fuzzy numbers \( A \) and \( B \). The larger the value of \( S(A, B) \), the more the similarity measure between two fuzzy numbers.

Wei & Chen (2009) proposed a method for calculating the similarity of two fuzzy numbers \( A \) and \( B \), where \( A = (a_1, a_2, a_3, a_4; w_1) \) and \( B = (b_1, b_2, b_3, b_4; w_2) \). If \( 0 \leq a_1 \leq a_2 \leq a_3 \leq a_4 \leq 1 \) and \( 0 \leq b_1 \leq b_2 \leq b_3 \leq b_4 \leq 1 \), then the degree of similarity \( S(A, B) \) between the generalized trapezoidal fuzzy numbers \( A \) and \( B \) is calculated as follows:

\[
S(A, B) = \left[ 1 - \sum_{i=1}^{4} \frac{|a_i - b_i|}{4} \right] \times \left( 1 - \frac{x_a^* - x_b^*}{y_a^* - y_b^*} \right) \times \frac{\min(y_a^*, y_b^*)}{\max(y_a^*, y_b^*)}. \tag{14}
\]

Where \( B(S_A, S_B) \) are defined as follows:

\[
B(S_A, S_B) = \begin{cases} 1 & S_A + S_B > 0 \\ 0 & S_A + S_B = 0 \end{cases} \tag{13}
\]

where \( S_A = a_4 - a_1 \) and \( S_B = b_4 - b_1 \) are the lengths of the generalized trapezoidal fuzzy numbers \( A \) and \( B \). The larger the value of \( S(A, B) \), the more the similarity measure between two fuzzy numbers.

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\[
S(A, B) = \left[ 1 - \sum_{i=1}^{4} \frac{|a_i - b_i|}{4} \right] \times \frac{\min(P(A), P(B)) + \min(w_1, w_2)}{\max(P(A), P(B)) + \max(w_1, w_2)}. \tag{14}
\]

Where \( S(A, B) \in [0,1] \); \( P(A) \) and \( P(B) \) are defined as follows:

\[
P(A) = \sqrt{(a_4 - a_1)^2 + w_1^2 + \sqrt{(a_3 - a_2)^2 + w_2^2}} + (a_1 - a_2) + (a_4 - a_3). \tag{15}
\]

\[
P(B) = \sqrt{(b_4 - b_1)^2 + w_2^2 + \sqrt{(b_3 - b_2)^2 + w_1^2}} + (b_1 - b_2) + (b_4 - b_3). \tag{16}
\]

\( P(A) \) and \( P(B) \) are the perimeters of generalized trapezoidal fuzzy numbers \( A \) and \( B \), respectively. The larger the value of \( S(A, B) \), the more the similarity measure between two fuzzy numbers.

Hejazi, etc. (2011) presented an improved similarity measure between two fuzzy numbers \( A \) and \( B \) combining the concept of geometric distance, height, areas and perimeters of generalized fuzzy numbers. The degree of similarity \( S(A, B) \) between the generalized trapezoidal fuzzy numbers \( A \) and \( B \) is calculated as follows:

\[
S(A, B) = \left[ 1 - \sum_{i=1}^{4} \frac{|a_i - b_i|}{4} \right] \times \frac{\min(A(A), A(B)) + \min(w_1, w_2)}{\max(A(A), A(B)) + \max(w_1, w_2)}. \tag{17}
\]
The larger the value of $S(\tilde{A}, \tilde{B})$, the more the similarity measure between two fuzzy numbers.

IV. A NEW SIMILARITY MEASURE OF GENERALIZED FUZZY NUMBERS

Many similarity measures between fuzzy numbers have been proposed [9-12]. However, it has been found that the existing methods cannot correctly calculate the degree of similarity between two generalized fuzzy numbers in some situations. In this section, we present a new method to calculate the degree of similarity between generalized fuzzy numbers, which consider the horizontal center-of-gravity, the perimeter, the height and the area of the two fuzzy numbers. The proposed similarity measure can overcome the drawbacks of the existing methods.

Assume there are two generalized trapezoidal fuzzy numbers $\tilde{A}$ and $\tilde{B}$, where $\tilde{A} = (a_1, a_2, a_3, a_4; w_1)$ and $\tilde{B} = (b_1, b_2, b_3, b_4; w_2)$, $0 \leq a_1 \leq a_2 \leq a_3 \leq a_4 \leq 1$ and $0 \leq b_1 \leq b_2 \leq b_3 \leq b_4 \leq 1$. Then the degree of similarity $S(\tilde{A}, \tilde{B})$ between the generalized trapezoidal fuzzy numbers $\tilde{A}$ and $\tilde{B}$ is calculated as follows:

$$S(\tilde{A}, \tilde{B}) = \left[1 - x_1^* - x_2^*\right] \times \left[1 - w_1 - w_2\right] \times \min(P(\tilde{A}), P(\tilde{B})) + \min(A(\tilde{A}), A(\tilde{B})) \times \max(P(\tilde{A}), P(\tilde{B})) + \max(A(\tilde{A}), A(\tilde{B}))$$

Where $x_1^*$ and $x_2^*$ are the horizontal center-of-gravity of the generalized trapezoidal fuzzy numbers $\tilde{A}$ and $\tilde{B}$, calculated as follows:

$$x_1^* = \frac{y_1^*(a_1 + a_2) + (a_4 + a_1)(w_1 - y_2^*)}{2w_1}.$$  

Proof

(i) If $\tilde{A}$ and $\tilde{B}$ are identical, $\min(P(\tilde{A}), P(\tilde{B})) = max(P(\tilde{A}), P(\tilde{B}))$, $\min(A(\tilde{A}), A(\tilde{B})) = max(A(\tilde{A}), A(\tilde{B}))$, $x_1^* = x_2^*$, $w_1 = w_2$. The degree of similarity between is calculated as follows:

$$S(\tilde{A}, \tilde{B}) = \left[1 - y_1^* - y_2^*\right] \times \left[1 - w_1 - w_2\right] \times \min(P(\tilde{A}), P(\tilde{B})) + \min(A(\tilde{A}), A(\tilde{B})) \times \max(P(\tilde{A}), P(\tilde{B})) + \max(A(\tilde{A}), A(\tilde{B}))$$

(ii) If $S(\tilde{A}, \tilde{B}) = 1$, then

$$S(\tilde{A}, \tilde{B}) = \left[1 - y_1^* - y_2^*\right] \times \left[1 - w_1 - w_2\right] \times \min(P(\tilde{A}, P(\tilde{B})) + \min(A(\tilde{A}), A(\tilde{B})) \max(P(\tilde{A}), P(\tilde{B})) + \max(A(\tilde{A}), A(\tilde{B}))$$

It implies that $x_1^* = x_2^*$, $w_1 = w_2$, $\min(P(\tilde{A}), P(\tilde{B})) = max(P(\tilde{A}), P(\tilde{B}))$ and $\min(A(\tilde{A}), A(\tilde{B})) = max(A(\tilde{A}), A(\tilde{B}))$. 

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Therefore, the generalized trapezoidal fuzzy numbers \( \tilde{A} \) and \( \tilde{B} \) are identical.

**Property 4.2.** \( S(\tilde{A}, \tilde{B}) = S(\tilde{B}, \tilde{A}) \).

**Proof.** Because

\[
S(\tilde{A}, \tilde{B}) = \left[ 1 - |x_a - x_b| \right] \times \left[ 1 - |w_a - w_b| \right]
\]

\[
\times \left( \min(P(\tilde{A}), P(\tilde{B}))) + \min(A(\tilde{A}), A(\tilde{B})) \right)
\]

\[
\times \left( \max(P(\tilde{A}), P(\tilde{B}))) + \max(A(\tilde{A}), A(\tilde{B})) \right)
\]

\[
S(\tilde{B}, \tilde{A}) = \left[ 1 - |x_a - x_b| \right] \times \left[ 1 - |w_a - w_b| \right]
\]

\[
\times \left( \min(P(\tilde{B}), P(\tilde{A}))) + \min(A(\tilde{B}), A(\tilde{A})) \right)
\]

\[
\times \left( \max(P(\tilde{B}), P(\tilde{A}))) + \max(A(\tilde{B}), A(\tilde{A})) \right)
\]

We can see that \( |x_a - x_b| = |x_a - x_b| \), \( |w_a - w_b| = |w_a - w_b| \), \( \min(P(\tilde{A}), P(\tilde{B})) = \min(P(\tilde{B}), P(\tilde{A})) \), \( \max(P(\tilde{A}), P(\tilde{B})) = \max(P(\tilde{B}), P(\tilde{A})) \), \( \min(A(\tilde{A}), A(\tilde{B})) = \min(A(\tilde{B}), A(\tilde{A})) \) and \( \max(A(\tilde{A}), A(\tilde{B})) = \max(A(\tilde{B}), A(\tilde{A})) \). Therefore, \( S(\tilde{A}, \tilde{B}) = S(\tilde{B}, \tilde{A}) \).

**Property 4.3.** If \( \tilde{A} = (a_1, a_2, a_3, a_4; w_a) \) and \( \tilde{B} = (b_1, b_2, b_3, b_4; w_b) \) are two generalized trapezoidal fuzzy numbers with the same geometric shape and the same height, then \( S(\tilde{A}, \tilde{B}) = 1 - d \), where \( d = |x_a - x_b| \) is the offset between \( \tilde{A} \) and \( \tilde{B} \).

**Proof.** Because \( w_a = w_b \), based on Eqs. (25) and (26), we can get \( \min(P(\tilde{A}), P(\tilde{B})) = \max(P(\tilde{A}), P(\tilde{B})) \) and \( \min(A(\tilde{A}), A(\tilde{B})) = \max(A(\tilde{A}), A(\tilde{B})) \), so due to Eq. (22) the degree of similarity between \( \tilde{A} \) and \( \tilde{B} \) is calculated as follows:

\[
S(\tilde{A}, \tilde{B}) = \left[ 1 - |x_a - x_b| \right] \times \left[ 1 - |w_a - w_b| \right]
\]

\[
\times \left( \min(P(\tilde{A}), P(\tilde{B}))) + \min(A(\tilde{A}), A(\tilde{B})) \right)
\]

\[
\times \left( \max(P(\tilde{A}), P(\tilde{B}))) + \max(A(\tilde{A}), A(\tilde{B})) \right)
\]

\[
= \left[ 1 - |x_a - x_b| \right] \times \left[ 1 - 0 \right] \times \left[ 1 \right] = 1 - d
\]

V. A COMPARISON OF THE SIMILARITY MEASURES

In this section, we extend 15 sets of fuzzy numbers presented in Wei & Chen (2009) into 18 sets of fuzzy numbers, shown in Fig. 3, and compare the calculation results of the proposed method with the results of the existing similarity measures, shown in Table I. From Fig. 3 and Table I, we can see the drawbacks of the existing similarity measures:

1. From Fig. 3, we can see that Set 3 and Set 4 are different sets of fuzzy numbers. However, from Table I, we can see that if we apply Chen’s method (Chen, 1996) and Lee’s method (Lee, 2002), Set 3 and Set 4 get the same degree of similarity, respectively.

2. From Set 5 of Fig. 3, we can see that \( \tilde{A} \) and \( \tilde{B} \) are different generalized fuzzy numbers. However, from Table I, we can see that if we apply Chen’s method (Chen, 1996) and Lee’s method (Lee, 2002), then their result is a degree of similarity equal to 1, respectively, which is an incorrect result.

3. From Set 6 of Fig. 3 and Table I, we can see that if we apply Lee’s method (Lee, 2002), we cannot calculate the degree of similarity between two identical real values due to the fact that the denominator will become zero, such that \( S(\tilde{A}, \tilde{B}) = \infty \), which is an incorrect result.

4. From Set 7 of Fig. 3 and Table I, we can see that if we apply Lee’s method (Lee, 2002), we can see that Lee’s method cannot correctly calculate the degree of similarity between two identical real values due to the fact that the degree of similarity of the real values become zero, which is an incorrect result.

5. From Set 8 and Set 9 of Fig. 3, we can see that they are two different sets of fuzzy numbers. However, from Table I, we can see that if we apply Chen’s method (Chen, 1996), they get the same degree of similarity, respectively.

6. From Set 10, Set 11 and Set 12 of Fig. 3, we can see that they are different sets of generalized fuzzy numbers. However, from Table I, we can see that if we apply Chen’s method (Chen, 1996), they get the same degree of similarity, respectively.

7. From Set 14 and Set 15 of Fig. 3, we can see that \( \tilde{A} \) and \( \tilde{B} \) have the same shape and the offset \( d = 0.1 \) in the X-axis, respectively. By applying the proposed method, we can see that the proposed method has the good property that the degree of similarity between \( \tilde{A} \) and \( \tilde{B} \) is equal to \( 1 - |d| = 1 - 0.1 = 0.9 \). However, from Table I, we can see that if we apply Chen-and-Chen’s method (Chen & Chen, 2003), the degree of similarity is equal to 0.81, which is an incorrect result.

8. From Set 14 of Fig. 3, using Chen’s Method (Chen, 1996) and Lee’s Method (Lee, 2002), the result is a degree of similarity equal to 1, respectively, which is an incorrect result.

9. From Set 14 and Set 15 of Fig. 3, we can see that Set 14 is more similar than Set 15 by the intuition of the human being. However, from Table I, we can see that if we apply Chen-and-Chen’s method (Chen & Chen, 2003), then we can see that it gets an incorrect result.

10. From Fig. 3, we can see that Set 10 and Set 16 are different sets of generalized fuzzy numbers and set 10 is more similar than set 16 by the intuition of human being. However, from Table I, we can see that if we apply the methods presented by Chen (1996), Lee (2002) and Hejazi et al. (2011), Set 10 and Set 16 get the same degree of similarity, respectively, and if we apply the method presented by Wei & Chen (2009), the result shows that set 16 is more similar than set 10 which is incorrect.

11. From Fig. 3, we can see that Set 11 and Set 17 are
different sets of generalized fuzzy numbers and set 11 is more similar than set 17 by the intuition of human being. However, from Table I, we can see that if we apply the methods presented by Chen (1996), Lee (2002) and Hejazi et al. (2011), Set 11 and Set 17 get the same degree of similarity, respectively, and if we apply the method presented by Chen & Chen (2003) and Wei & Chen (2009), the result shows that set 17 is more similar than set 11 which is incorrect.

(12) From Fig. 3, we can see that Set 11 and Set 18 are different sets of generalized fuzzy numbers. However, from Table I, we can see that if we apply the methods presented by Chen (1996), Lee (2002), Hejazi et al. (2011) and Wei & Chen (2009), Set 11 and Set 18 get the same degree of similarity, respectively.

In summary, from Fig. 3 and Table I, we can see that the proposed method can overcome the drawbacks of existing similarity measures.
TABLE I.
RESULTS OF COMPARISONS BETWEEN EXISTING AND THE PROPOSED METHOD.

<table>
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</tr>
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<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
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<td>0.9</td>
<td>0.9</td>
<td>0.9</td>
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<tr>
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<td>0.5411</td>
<td>0.7</td>
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<tr>
<td>Set 9</td>
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<td>0.9</td>
<td>0.9</td>
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<tr>
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<td>0.9</td>
<td>0.7833</td>
<td>0.6261</td>
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<td>Set 11</td>
<td>0.9</td>
<td>0.75</td>
<td>0.72</td>
<td>0.8003</td>
<td>0.6448</td>
<td>0.7938</td>
</tr>
<tr>
<td>Set 12</td>
<td>0.9</td>
<td>0.8</td>
<td>0.8325</td>
<td>0.8289</td>
<td>0.7361</td>
<td>0.7478</td>
</tr>
<tr>
<td>Set 13</td>
<td>0.9</td>
<td>0.75</td>
<td>0.81</td>
<td>0.9</td>
<td>0.9</td>
<td>0.9</td>
</tr>
<tr>
<td>Set 14</td>
<td>1</td>
<td>1</td>
<td>0.7</td>
<td>0.7209</td>
<td>0.5113</td>
<td>0.5104</td>
</tr>
<tr>
<td>Set 15</td>
<td>0.95</td>
<td>0.75</td>
<td>0.9048</td>
<td>0.6215</td>
<td>0.383</td>
<td>0.4242</td>
</tr>
<tr>
<td>Set 16</td>
<td>0.9</td>
<td>0.8333</td>
<td>0.7425</td>
<td>0.814</td>
<td>0.6261</td>
<td>0.7321</td>
</tr>
<tr>
<td>Set 17</td>
<td>0.9</td>
<td>0.75</td>
<td>0.8911</td>
<td>0.838</td>
<td>0.6448</td>
<td>0.7432</td>
</tr>
<tr>
<td>Set 18</td>
<td>0.9</td>
<td>0.75</td>
<td>0.6976</td>
<td>0.8003</td>
<td>0.6448</td>
<td>0.7144</td>
</tr>
</tbody>
</table>

VI. A NEW SECURITY RISK ASSESSMENT METHOD OF WEBSITE

In this section, we apply the proposed similarity measure of generalized fuzzy numbers to present a new fuzzy number algorithm to deal with website security problems. We choose the possibility of threat occurrence and the loss of the result led by the threat as parameters to compute security risk and use semantic description to indicate the severity of consequences led by threats and the probability of occurring threats. Zhang (1986) uses trapezoidal fuzzy numbers to represent linguistic term [15]. In this paper, a 9-member linguistic term set (Zhang, 1986) is used to represent the linguistic terms. Each linguistic term in the 9-member linguistic term set is corresponding to a generalized trapezoidal fuzzy number, as show in Table II.

The parameters used in this method are defined as follows.

TABLE II.
A 9-MEMBER LINGUISTIC TERM SET (SCHMUCKER, 1984)

<table>
<thead>
<tr>
<th>Linguistic</th>
<th>Generalized trapezoidal fuzzy numbers</th>
</tr>
</thead>
<tbody>
<tr>
<td>Absolutely-low</td>
<td>(0,0,0,0;1.0)</td>
</tr>
</tbody>
</table>
\[ UXR = \begin{bmatrix} r_{11} & r_{12} & r_{13} & r_{14} & r_{15} & r_{16} & r_{17} & r_{18} & r_{19} \\ r_{21} & r_{22} & r_{23} & r_{24} & r_{25} & r_{26} & r_{27} & r_{28} & r_{29} \\ r_{31} & r_{32} & r_{33} & r_{34} & r_{35} & r_{36} & r_{37} & r_{38} & r_{39} \\ r_{41} & r_{42} & r_{43} & r_{44} & r_{45} & r_{46} & r_{47} & r_{48} & r_{49} \end{bmatrix} \]

Here the position of \( r_{ij} \) is respectively corresponding to a fuzzy number as shown in the Table II. Such as the position of \( r_{ij} \) represents the linguistic term Absolutely-low, and so on. According to the degree of the effects of index sets on the consequences caused by the threat, the weight vector is defined as \( \alpha = (\alpha_1, \alpha_2, \alpha_3, \alpha_4) \). We use the arithmetic operations proposed in the Section II to comprehensive evaluate them, we can have the comprehensive evaluation result of the consequences caused by the threat:

\[ B_i = \alpha \cdot UXR = (h_{b1}, h_{b2}, h_{b3}, h_{b4}, h_{b5}, h_{b6}, h_{b7}, h_{b8}, h_{b9}) \] (29)

Here \( h_{bj} \) are all fuzzy numbers.

B. The Possibility of Threat Occurrence

The possibility of threat occurrence evaluation factors set \( UY = \{UY_1, UY_2, UY_3, UY_4\} = \{\text{Attractive of the assets}, \text{Difficulty of turning the assets to the remuneration}, \text{Technology of the threat}, \text{Degree of the vulnerability to be exploited}\} \). The possibility of threat occurrence indexes set \( UYP = \{UYP_1, UYP_2, UYP_3, UYP_4, UYP_5, UYP_6, UYP_7, UYP_8, UYP_9\} = \{\text{Absolutely-low, Very-low, Low, Fairly-low, Medium, Fairly-high, High, Very-high, Absolutely-high}\} \). To judge the single factor \( UY_1 \), we can have the membership matrix \( UYR \):

\[ UYR = \begin{bmatrix} s_{11} & s_{12} & s_{13} & s_{14} & s_{15} & s_{16} & s_{17} & s_{18} & s_{19} \\ s_{21} & s_{22} & s_{23} & s_{24} & s_{25} & s_{26} & s_{27} & s_{28} & s_{29} \\ s_{31} & s_{32} & s_{33} & s_{34} & s_{35} & s_{36} & s_{37} & s_{38} & s_{39} \\ s_{41} & s_{42} & s_{43} & s_{44} & s_{45} & s_{46} & s_{47} & s_{48} & s_{49} \end{bmatrix} \]

Here the \( s_{ij} \) is corresponding a fuzzy number as the \( r_{ij} \). According to the degree of the effects of index sets on the possibility of threat, the weight vector is defined as. We use the arithmetic operations proposed in the Section 2 to comprehensive evaluate them, we can have the comprehensive evaluation result of the possibility of threat:

\[ B_i = \beta \cdot UYR = (h_{b1}, h_{b2}, h_{b3}, h_{b4}, h_{b5}, h_{b6}, h_{b7}, h_{b8}, h_{b9}) \] (30)

Here \( h_{bji} \) are all fuzzy numbers.

C. Calculate the Risk Value of the Website Security

Threats sets to the system are defined as \( T = \{T_1, T_2, \cdots, T_s\} \), \( R_i \) is the risk value of the specific threat \( T_i \), \( B_{i1} \) is the comprehensive evaluation result of the consequence grade of the comprehensive evaluation result caused by the specific threat \( T_i \). \( B_{i2} \) is the comprehensive evaluation result of the possibility grade caused by the specific threat \( T_i \), then \( R_i \) is defined as follows:

\[ R_i = \left( \sum_{j=1}^{9} h_{b_{ij}} \otimes h_{b_{ji}} \right) \odot \left( \sum_{j=1}^{9} h_{b_{ij}} \right) = \left( r_{i1}, r_{i2}, r_{i3} ; w_{ri} \right) \] (31)

Based on the algorithm we proposed before, we can know that the result is a fuzzy number. Then we use the new similarity measure of generalized fuzzy numbers to calculate the similarity of \( R_i \) and each linguistic term shown in Table II. The risk faced to the website is equal to the linguistic term with the largest degree of similarity with respect to \( R_i \).

D. A Numerical Example

In this subsection, a numerical example is presented. We use the security risk calculations of some websites to illustrate the procedures of the proposed method for assessing the website security. According to the discussions of all of the experts, the main threats to the website are determined, as shown in Table III.

<table>
<thead>
<tr>
<th>The Main Threats Faced to the Website</th>
<th>Distributions</th>
</tr>
</thead>
<tbody>
<tr>
<td>( T_1 ) Malicious code and viruses</td>
<td></td>
</tr>
<tr>
<td>( T_2 ) Management not in place</td>
<td></td>
</tr>
<tr>
<td>( T_3 ) No action or Operational errors</td>
<td></td>
</tr>
<tr>
<td>( T_4 ) Hacker attacks</td>
<td></td>
</tr>
<tr>
<td>( T_5 ) Physical attacks</td>
<td></td>
</tr>
</tbody>
</table>

The membership matrix \( UXP \) of the consequence evaluation indexes set caused by threat \( T_1 \) by the Delphi technique is:

\[ UXR = \begin{bmatrix} 0.00 & 0.01 & 0.05 & 0.10 & 0.15 & 0.20 & 0.30 & 0.15 & 0.05 & 0.00 \\ 0.05 & 0.00 & 0.05 & 0.10 & 0.15 & 0.20 & 0.25 & 0.25 & 0.00 & 0.00 \\ 0.00 & 0.05 & 0.10 & 0.15 & 0.20 & 0.25 & 0.25 & 0.25 & 0.10 & 0.05 \end{bmatrix} \]

Here the column correspond to the 9-member linguistic terms, such as the first column correspond to the first linguistic term Absolutely-low, the second column correspond to the second linguistic term Very-low, and so on. The numbers in the matrix mean that the values here are the generalized fuzzy number multiplied by the real number, such as the number 0.05 in the first row, second column means that the value here is the fuzzy number Very-low multiplied by the real number 0.05, the result is a fuzzy number. The weight vector of the consequence evaluation factors set \( \alpha = (0.2, 0.1, 0.2, 0.4) \). So due to Eq. (29), the comprehensive evaluation result of the consequences grade caused by the threat can be obtained as follows:

\[ h_{b1} = (0, 0, 0, 0, 1) \]


\[ h_2 = (0, 0, 0, 0, 1) \]
\[ h_3 = (0.0003, 0.0017, 0.0055, 0.0090, 1) \]
\[ h_4 = (0.0038, 0.0063, 0.0168, 0.0229, 1) \]
\[ h_5 = (0.0251, 0.0412, 0.0824, 0.1035, 1) \]
\[ h_6 = (0.0622, 0.0734, 0.1184, 0.1368, 1) \]
\[ h_7 = (0.0700, 0.0821, 0.1143, 0.1270, 1) \]
\[ h_8 = (0.0735, 0.0816, 0.0850, 0.0850, 1) \]
\[ h_9 = (0.0150, 0.0150, 0.0150, 0.0150, 1) \]

The membership matrix \( UYP \) of the possibility of threat occurrence evaluation index set caused by threat \( T_i \) by the Delphi technique is:

\[
UYP = \begin{bmatrix}
0.00 & 0.05 & 0.35 & 0.15 & 0.15 & 0.15 & 0.10 & 0.05 & 0.00 \\
0.05 & 0.00 & 0.15 & 0.30 & 0.15 & 0.05 & 0.20 & 0.10 & 0.00 \\
0.00 & 0.00 & 0.15 & 0.20 & 0.35 & 0.15 & 0.10 & 0.00 & 0.05 \\
0.00 & 0.05 & 0.30 & 0.20 & 0.05 & 0.20 & 0.15 & 0.05 & 0.00 
\end{bmatrix}
\]

Here the values of the matrix are defined as same as \( UXR \). The weight vector of the possibility evaluation factors set of the threat \( T_i \) is \( \beta = (0.2, 0.1, 0.3, 0.4) \). So due to Eq. (30), the comprehensive evaluation result of the possibility grade of the threat \( T_i \) can be obtained as follows:

\[ b_{11} = (0, 0, 0, 0, 1) \]
\[ b_{22} = (0, 0, 0, 0, 1) \]
\[ b_{33} = (0.0004, 0.0025, 0.0081, 0.0132, 1) \]
\[ b_{44} = (0.0058, 0.0097, 0.0259, 0.0353, 1) \]
\[ b_{55} = (0.0174, 0.0286, 0.0572, 0.0718, 1) \]
\[ b_{66} = (0.0538, 0.0635, 0.1024, 0.1183, 1) \]
\[ b_{77} = (0.0674, 0.0791, 0.1100, 0.1223, 1) \]
\[ b_{88} = (0.0346, 0.0384, 0.0040, 0.0040, 1) \]
\[ b_{99} = (0.0150, 0.0150, 0.0150, 0.0150, 1) \]

So the risk value \( R_i \) of the threat \( T_i \) is calculated as follows:

\[
R_i = \left( \sum_{i=1}^{9} b_{ij} \otimes b_{jk} \right) \otimes \left( \sum_{i=1}^{9} b_{lj} \right) = \left( r_1, r_2, r_3, r_4, w_R \right)
\]

\[
= (0.1923, 0.3249, 1.0000, 1.0000, 1.0000)
\]

Use the method proposed before, we can obtain the degree of similarity between the generalized trapezoidal fuzzy number \( R_i \) and each linguistic term, as shown in Table IV.

From Table IV, we can see that 0.5755 is the largest value. Therefore, the generalized trapezoidal fuzzy number \( R_i \) is translated into the linguistic term “Fairly-high”. It means that the risk of the threat \( T_i \) faced to the website is Fairly-high.

In the same way, we can obtain the risks of the other four threats: the threat \( T_2 \) is Medium, the threat \( T_3 \) is Fairly-low, the threat \( T_4 \) is Very-high, the threat \( T_5 \) is High. Descending order of the risk is: \{the threat \( T_1 \) (Very-high), the threat \( T_5 \) (High), the threat \( T_4 \) (Fairly-high), the threat \( T_2 \) (Medium), the threat \( T_3 \) (Fairly-low)\}.

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