A Fast and Efficient Algorithm for Finding Frequent Items over Data Stream

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Abstract—We investigate the problem of finding the frequent items in a continuous data stream. We present an algorithm called \( \lambda \)-Count for computing frequency counts over a user specified threshold on a data stream. To emphasize the importance of the more recent data items, a fading factor \( \lambda \) is used. Our algorithm can detect \( \varepsilon \)-approximate frequent items of a data stream using \( O(\log \varepsilon) \) memory space and \( O(1) \) time to process each data record. The computation time for answering each query is \( O(\log \varepsilon) \), and for answering a query about the frequentness of a given data item is \( O(1) \). Experimental study shows that \( \lambda \)-Count outperforms other methods in terms of accuracy, memory requirement, and processing speed.

Index Terms—Data mining, data stream, frequent items

I. INTRODUCTION

In recent years, researchers have paid more attention to mining stream data. Mining frequent item sets from stream data is an important task in stream data analysis. Frequency is a fundamental characteristic in many data mining tasks such as association rule mining and iceberg queries. It has applications in many areas such as sensor data mining, business decision support, analysis of web query logs, direct marketing, network measurement, and internet traffic analysis. Correspondingly, the input stream data could be stock tickers, bandwidth statistics for billing purposes, network traffic measurements, webserver click streams, and data feeds from sensor networks. Traditional mining algorithms assume a finite dataset and multiple scans on the data. For the stream data applications, the volume of data is usually too large to be stored in memory or to be scanned for more than once. Furthermore, for data streams, there can only be sequential but not random access. Therefore, traditional frequent item mining algorithms are not applicable to stream data.

The problem is difficult because of the high throughput of the data streams, possibly in the order of gigabytes per second. Any feasible algorithm for detecting frequent data item must perform data processing and query fast enough so as to match the speed of arriving data in the stream. In addition, the algorithm can use only limited memory space and store only the sketch or synopsis of the data items in the stream.

Several solutions for finding frequent items in stream data have been proposed. Several algorithms use random sampling [1,5,6,7,8,9,10,11,12,13,14] to estimate the frequencies of the data items. For example, the Sticky Sampling [1] algorithm is a sampling based algorithm for computing an \( \varepsilon \)-deficient synopsis over a data stream. It is a probabilistic one-pass algorithm that provides an accuracy guarantee on the set of frequent data items and their frequencies reported. The second class of algorithms are deterministic algorithms [2,3,4,15,16,17,18,25]. The MG algorithm by Misra and Gries [4] is a well-known deterministic algorithm to detect frequent stream data.

In many applications, recent data in the stream is more meaningful. For instance, in an athlete ranking system, more recent records typically should carry more weight. One way to handle such problem is to use a sliding window model [19-22,29,31]. In this model, only the most recent data items in a time period of a fixed length are stored and processed, and only the frequent data items in this period are detected. The advantage of this method is that it can get rid of the stale data and only consider the fresh data, which are meaningful in many cases. To emphasize the importance of the recent data, there is another model for frequency measures in data stream which is called time fading model [32]. In this model, data items in the entire stream is taken into account to compute the frequency of each data item, but more recent data items contribute more to the frequency than the older ones. This is achieved by introducing a fading factor \( 0<\lambda<1 \). A data item that is \( n \) time points in the past is weighted \( \lambda^n \). Thus, the weight is exponentially decreasing.
In general, the closer to 1 the fading factor \( \lambda \) is, the more important the history is taken into account. There are two advantages of the fading model over the sliding window model. One is that in the fading model, frequency takes into account the old data items in the history, while the sliding window model only observes within a limited time window and entirely ignores all the data items outside the window. This is undesirable in many real applications. The second is that in the fading model, when more data arrive continuously, the frequency changes smoothly without a sudden jump which may occur in the sliding window model.

In this paper, we propose an efficient frequency estimation algorithm based on the fading model which needs as little space and running time as possible. We propose an algorithm called \( \lambda \)-Count which can detect \( \varepsilon \)-approximate frequent items in data stream. The algorithm requires \( O(\log\frac{\varepsilon}{\delta}) \) memory space and \( O(1) \) time for processing one input data item. Moreover, the computation time for answering each query is \( O(\log\frac{1}{\delta}) \), and for answering query about the frequency of a given data item is \( O(1) \). Through extensive experiments, we show that \( \lambda \)-Count outperforms other methods such as LC and EC in terms of the accuracy, memory requirement, and processing speed.

The rest of the paper is organized as follows. Section 2 reviews related work. Section 3 formally defines the problem and describes a data fading model. Section 4 describes the framework of the \( \lambda \)-Count algorithm and analyzes its space and time complexity. Section 5 reports our experimental results and Section 6 gives conclusions.

II. RELATED WORK

Problems related to frequency estimate have been actively studied. Algorithms for identifying frequent items and other statistics in the entire data stream have been proposed.

**Lossy Counting** [1] was among the first algorithms for finding frequent items from a data stream. Lossy Counting is a one-pass algorithm that provides an accuracy guarantee on the set of frequent data items and their frequencies reported. Given a user-specified support threshold \( S \), and an error threshold \( \varepsilon \), Lossy Counting guarantees that: 1) All items whose true frequency exceeds \( SN \) are detected, where \( N \) is the total number of data items processed. Namely, there are no false negatives. 2) No item whose true frequency is less than \((S-\varepsilon)N\) is output. 3) The estimated frequency of any item is less than its true frequency by at most \( \varepsilon N \). Nuno Homem et al. [28] presented an algorithm for identifying the most \( k \) frequent elements by merging the commonly used counter-based and sketch-based techniques. The algorithm also provides guarantees on the error estimate, order of elements and the stochastic bounds on the error and expected error estimates. Karp et al. [2], and Demaine et al. [3] applied a deterministic \( MG \) algorithm [4] to detect frequent stream data. They reduced the processing time of \( MG \) algorithm to \( O(1) \) by managing all counters in a hash table. The algorithm can easily be adapted to find \( \varepsilon \)-approximate frequent items in the entire data stream without making any assumption on the distribution of the item frequencies. This algorithm needs \( 1/\varepsilon \) counters for the most frequent data items in the stream. Processing the arrival data items entails incrementing or decrementing some counters.

Many algorithms for frequent item counting use random sampling. They make assumptions on the distribution of the item frequencies and the quality of their results is guaranteed probabilistically. Flajolet and Martin [5] and Whang et al. [6] proposed probabilistic algorithms to estimate the number of distinct items in a large collection of data in a single pass. Golab et al. [7] gave an algorithm for the case when the item frequencies are multinomially-distributed. Gibbons and Matias [8] presented sampling algorithms to recognize top-\( k \) queries. H. Liu et al. [9] presented an error-adaptive and time-aware maintenance algorithm for frequency counts over data streams. G.S. Manku et al. [1] advanced a sampling based algorithm called sticky sampling for computing an \( \varepsilon \)-deficient synopsis over a data stream of singleton items. It scans the data in the stream and randomly samples the data items based on three user-specified parameters: support \( S \), error bound \( \varepsilon \), and probability of failure \( \delta \).

Many algorithms use hashing technique to map the data items in a stream to a hash table which can be stored in the main memory. Estan and Varghese [10] presented a sampling algorithm and a hash-based algorithm for frequent item counting. Based on the hashing technique, Charikar et al. presented an algorithm named Count Sketch [11], which requires \( O(k\varepsilon^2\log n) \) memory space and \( O(k\varepsilon^2\log n) \) computation time to process one data item. The algorithm can output the items with frequency larger than \( 1/(k+1) \) under the probability of 1-\( \delta \). Cormod et al. presented an algorithm called groupTest [12] which requires \( O(k(\log k + \log(1/\delta)))\log M) \) memory and \( O(\log k) \) time for each item. Jin et al. [13] advanced an algorithm \( hCount \) which uses \( O(\varepsilon^2(\log(M)\log\varepsilon)) \) memory and \( O(\log(\log(M)\log\varepsilon)) \) time for each data element. The algorithm can detect the \( \varepsilon \)-approximate results under the probability of 1-\( \delta \). Fang et al. [14] also advanced several algorithms based on hashing to compute iceberg queries, but each requires at least two passes over the data stream.

In addition to randomized algorithms, many deterministic algorithms for detecting frequent item in data stream are also reported. Calders et al. [15] proposed an algorithm for mining frequent items in a data stream. They defined a new frequency measure such that the current frequency of a data item is its maximal frequency over all possible windows in the stream from any time point in the past until the current time. B. Lin [16] et al. proposed an adaptive frequency counting algorithm to handle bursty data in the stream. They used a feedback mechanism that dynamically adjusts mining speed to cope with the changing arrival rate. Greenwald and Khanna [17] considered the problem of \( \varepsilon \)-approximate quantitative summaries. Wang [18] et al. proposed an algorithm to find \( \varepsilon \)-approximate frequent items in a data stream, its space complexity is \( O(\varepsilon^2) \) and the processing time for each item is \( O(1) \) in average. Moreover, the
frequency error bound of the results returned by the proposed algorithm is \((1-S+e)\epsilon/\lambda N\).

In many applications, recent data in the stream is more meaningful. The algorithms mentioned above do not discount the effect of old data, all data items in the whole history of the data stream are given equal weights. This is undesirable in solving many application problems. One way to handle such problems is to use a sliding window model. Recently several data mining algorithms over sliding windows are proposed. Arasu and Manku [19] gave the first deterministic algorithm for finding \(\epsilon\)-approximate frequent items over a sliding window. It requires \(O((1/\epsilon)\log(1/\delta))\) time for each query/update and uses \(O((1/\epsilon)\log^2(1/\delta))\) space. Their algorithm divides the sliding window into several possibly overlapping sub-windows with different sizes. The algorithm applies the MG algorithm to each of these sub-windows to find the frequent items in these sub-windows. These sub-windows are organized into levels so that whenever there is a query on the frequent items, one can traverse these sub-windows efficiently to identify the frequent data items. In [30] Regant Y. S. Hung et al. studied the space complexity of the \(\epsilon\)-approximate quantizes problem, and proved that any comparison-based algorithm for finding \(\epsilon\)-approximate quantizes in a data stream needs an \(\Omega((1/\epsilon)\log(1/\delta))\) space. Golab et al. [20] gave some heuristic algorithms for identifying frequent items over a sliding window. Lee and Ting [21] proposed an approximate frequent stream data mining algorithm which requires \(O(1/\epsilon)\) space. Their algorithm needs \(O(1/\epsilon)\) processing time for update and query. L. Zhang and Y. Guan [22] proposed a stream data frequency estimation algorithm over sliding windows. Their algorithm requires \(O(1/\epsilon)\) memory space and \(O(1)\) time for query/update. Other recent works on mining frequent items in data stream have been surveyed in [23,24,26,27,37]. The major algorithms for mining approximate frequent items in data stream are listed in Table 1.

### III. CONCEPTS AND DEFINITIONS

In this section we describe a data fading model by using a fading factor \(\lambda\) to discount the frequencies of the old data in a stream. We also give a formal definition of our mining problem. In this paper, we use a standard stream model with discrete time steps labeled as 0, 1, 2, \(\ldots\), and only one data record arrives at each time step, where \(X=\{x_1, x_2, \ldots, x_m\}\) is a domain containing discrete values.

To emphasize the importance of recent data, we use a fading factor \(\lambda\in(0,1)\) in calculating the data items’ support counts. For each data item \(x\), its support count decreases as \(x\) ages. We call such modified support counts the density of the data item. In each time step, the density of a data item will be reduced by the fading factor \(\lambda\).

**Definition 1 (Density of a data item)** The density of a data item \(x\in X\) at time \(t\) is defined as

\[
D(x,t) = \begin{cases} 
0 & t = 0 \\
D(x,t-1)\lambda + \delta(t,x) & t = 1, 2, 3, \ldots
\end{cases}
\]

Here, \(\delta(t,x) = \begin{cases} 1 & a(t) = x \\
0 & \text{otherwise}
\end{cases}\), where \(a(t)\) is the data record received at time \(t\).

The density of a data item is constantly changing. However, we found that it is unnecessary to update the density values of all data items at every time step. Instead, it is possible to update the density of a data item only when this item is received from the data stream. For each item, the time when it was last received should be recorded. Suppose a new data item \(x\) is received at time \(t_o\), and suppose the last time \(x\) was received before \(t_o\) is \(t_i\), then the density of \(x\) can be updated as follows:

\[
D(x,t_o) = D(x,t_i)\lambda^{t_o-t_i} + 1
\]

**Lemma 1** Let \(X(t)\) be the set of all the data items that are received at least once from time 0 to \(t\), we have:

1) \(\sum_{x\in X(t)} D(x,t) \leq \frac{1}{1-\lambda}\), for any \(t=1, 2, \ldots\).

2) \(\lim_{t\to\infty} \sum_{x\in X(t)} D(x,t) = \frac{1}{1-\lambda} - \frac{1}{1-\lambda^2}\).

**Proof:** For a given time \(t\), \(\sum_{x\in X(t)} D(x,t)\) is the sum of density of the \(t+1\) data records that arrive at time steps 0, 1, \(\ldots\), \(t\). For each time step \(t_i\), \(0\leq t_i\leq t\), the data record contributes \(\lambda^{t_i}\) to the total density. Therefore, we have

\[
\sum_{x\in X(t)} D(x,t) = \sum_{i=0}^{t} \lambda^{t_i} = \frac{1-\lambda^{t+1}}{1-\lambda} \leq \frac{1}{1-\lambda^2}.
\]

Also, it is clear that:

\[
\lim_{t\to\infty} \sum_{x\in X(t)} D(x,t) = \lim_{t\to\infty} \frac{1-\lambda^{t+1}}{1-\lambda} = \frac{1}{1-\lambda} - \frac{1}{1-\lambda^2}.
\]

Q.E.D.

Since a data stream may consist of potentially huge volume of data items, the number of the data items in the stream could become very large, and the count of each item could overflow. From Lemma 1, we can see when a fading factor is used, the summation of the densities of the data items is independent of the number of the data items in the stream, and the density of each data item is within the range of \([0, 1/(1-\lambda)]\) and never overflows.

Like most previous work, our \(\lambda\)-Count algorithm takes two user-specified parameters, a support threshold \(S\in(0,1)\), and an error parameter \(\epsilon\in(0,1)\) such that \(\epsilon<S\).

**Definition 2 (Frequent data item)** Let \(S\) be a user-specified threshold, at time \(t\), a data item \(x\) is a frequent item if its density \(D(x,t)\) satisfies \(D(x,t) \geq S/(1-\lambda)\).

Given \(\epsilon\) as a user specified relative error bound and \(\epsilon<S\), we are asked to maintain some data items with density at least \(S-\epsilon\), which are called \(\epsilon\)-approximate frequent items.

Our algorithm outputs a list of \(\epsilon\)-approximate frequent items along with their estimated densities. Similar to Lossy Counting, the answers produced by our algorithm have the following guarantees:

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1. All items whose densities exceed \( \frac{S}{1 - \lambda} \) will be found, which means there are no false negatives.

2. No item whose density is less than \( \frac{S - \varepsilon}{1 - \lambda} \) will be found.

3. The estimated density for each item is no more than its actual density. The difference between the estimated density and the actual density is no more than \( \frac{\varepsilon}{1 - \lambda} \).

IV. THE \( \lambda \)-COUNT ALGORITHM

We now present the proposed \( \lambda \)-Count algorithm. We first describe the algorithm and prove its optimality. Then, we discuss some key implementation details and analyze the complexity of the algorithm.

A. Description of the algorithm

In our \( \lambda \)-Count algorithm, for each data item, it suffices to store a characteristic vector which consists of the necessary information of the data item. The \( \lambda \)-Count algorithm processes the incoming data from stream and updates a summary structure called item_list which is a list of frequent data item candidates. Each entry of item_list is a characteristic vector of a data item \( x \) : \( C(x) = [x, D_i(x, t_i), t_i] \), where \( t_i \) is the last time when \( x \) was received, and \( D_i(x, t_i) \) is the estimated density of the data item \( x \) at time \( t_i \).

The item_list is organized in a queue structure and has a size limitation of \( L = \log_{\frac{1}{\varepsilon}} \varepsilon \). These entries in item_list are arranged in the descending order of their \( t_i \) values. The entry with the least \( t_i \) value is located at the head of the queue while the one with largest \( t_i \) value is at the tail of the queue. Whenever the size of item_list goes beyond \( L \), the entry at the head of item_list should be deleted. Whenever a new entry is going to be inserted to item_list, it should be placed at the tail of the queue.

When a new data record \( x \) is received from the stream, the algorithm creates or updates its characteristic vector in item_list. If the characteristic vector of \( x \) already exists in item_list, the algorithm modifies its density and \( t_i \) value before moving it to the tail of the queue; otherwise, the algorithm creates a new entry for \( x \) and inserts it to the tail of the queue.

The framework of the algorithm is given in Algorithm 1.

Algorithm 1: \( \lambda \)-Count

**input:** str: the data stream; 
\( \lambda \): the fading factor; 
\( \varepsilon \): the density error bound; 
\( S \): the density threshold; 

**output:** item_list: list of frequent data item candidates;

begin
1. set \( t = 0 \); \( L = \log_{\frac{1}{\varepsilon}} \varepsilon \);
2. while not terminate do
3. receive a data item \( x \) from the data stream str;
4. if \( x \) is not in item_list then
5. create a new entry \( [x, 1, t] \);
6. if [item_list] \( \geq L \) (item_list is full) then
7. delete the entry at the head of item_list;
8. endif
9. insert the new entry \( [x, 1, t] \) to the tail of item_list;
10. else
11. update the corresponding entry \( [x, D_i(x, t_i), t_i] \)
end
move the entry $[x, D_s(x,t)\lambda^{t_i-1}+1, t]$ to the tail of item_list;
13. end if;
14. $t = t + 1$
15. end while

Since the density of a data item may be deleted previously, the estimated density recorded in the entry in item_list may be less than the actual density of the data item. We will show that the error is within a bound $\epsilon/(1-\lambda)$. If a data item $x$ is not listed in item_list, it is possible that $x$ has been deleted from item_list several times, causing its historical density to be discarded. We will show that if $x$ is not listed in item_list, then it cannot be a frequent item even if it has never been deleted from item_list. In other words, all the frequent items will be kept in item_list and there is no false negative.

**Theorem 1** Suppose an entry $C(x)=[x, D_s(x,t_i), t_i]$ of data item $x$ is deleted from item_list at time $t_i$, then its actual density $D(x,t)$ at time $t$ satisfies $D(x,t) \leq \epsilon/(1-\lambda)$.

**Proof:** Suppose ever since the beginning of the data stream, the item $x$ has previously been deleted at time steps $t_1$, $t_2$, ..., $t_p$, and now is deleted at time $t$. We denote $t$ as $t_{p+1}$. The density of the $i$th deletion is acquired during the period of $(t_i', t_i')$, where $t_i'$ and $t_i'$ are the time steps when $x$ is first and last received in this period, respectively. It is obvious that $t_i' \leq t_i' < t_i$, for $i=1,2,...,p+1$, where, $t_{p+1}' = t$ and $t_{p+1}' = t_i$. Then the characteristic vector of $x$ at time $t_i$ is $C(x)=[x, D_s(x,t_i')]$, $t_i' \]$. Since we have $D(x,t_i') = D_e(x,t_i')\lambda^{t_i'}$, the density of $x$ at time $t = t_{p+1}$ should be

$$D(x,t) = \sum_{i=1}^{p+1} D_e(x,t_i')\lambda^{t_i'-t_i} \leq \sum_{i=1}^{p+1} (1 + \lambda + \lambda^2 + ... + \lambda^{t_i'-t_i})\lambda^{t_i'-t_i} \leq \sum_{i=1}^{p+1} \lambda^{t_i'-t_i} \leq \sum_{k=1}^{t_i'-t_i} \lambda^k = \lambda^{t_i'-1} \sum_{k=1}^{t_i'-1} \lambda^k \leq \lambda^{t_i'-1} \frac{1-\lambda^{t_i'-1}}{1-\lambda}. \quad (3)$$

When $x$ is deleted from item_list, it must locate at the head of item_list and has the least $t_i$, value in item_list. Since the length of item_list is $L = \log_\lambda \epsilon$, and every entry in item_list has a different $t_i$ value, we know $t_{p+1}' - t'_{p+1} = t - t_i > L$. Thus, we have

$$\lambda^{t_i'-1} < \lambda^L = \lambda^{\log_\lambda \epsilon} = \epsilon.$$

Therefore from (3) we have,

$$D(x,t) \leq \lambda^{t_i'-1} \frac{1-\lambda^{t_i'-1}}{1-\lambda} \leq \epsilon \frac{1-\lambda^{t_i'-1}}{1-\lambda} \leq \epsilon \frac{1-\lambda^{L+1}}{1-\lambda} \leq \epsilon \frac{1-\lambda^{\log_\lambda \epsilon}}{1-\lambda} \leq \epsilon \frac{1}{1-\lambda}.$$

Q.E.D.

From Theorem 1 we know that if a data item does not appear in item_list at time $t$, then its actual density must satisfy $D(x,t) < \epsilon/(1-\lambda)$.

**Theorem 2.** At time $t$, for any entry $C(x)=[x, D_s(x,t_i), t_i]$ in item_list, we have

a) $D_s(x,t_i) \leq D(x,t_i) \leq D_s(x,t_i) + \epsilon \frac{1}{1-\lambda}$, \quad (4)

b) $D_s(x,t) \leq D(x,t) \leq D_s(x,t) + \epsilon \lambda^{-t_i}$, \quad (5)

Here, $D(x,t)$ is the actual density of data item $x$ at time $t$.

**Proof.** a) If $x$ has not been previously deleted, $D_s(x, t_i) = D_s(x, t_i)$. Otherwise, since $x$ has been previously deleted from item_list, we have $D_s(x, t_i) \leq D(x, t_i)$. From Theorem 1, we infer that the actual density of $x$ when it is last deleted is at most $\epsilon/(1-\lambda)$.

Therefore, we have

$$D_s(x,t_i) \leq D(x,t_i) \leq D_s(x,t_i) + \epsilon \frac{1}{1-\lambda}. \quad Q.E.D.$$

b) Since $D_s(x,t) = D_s(x,t)\lambda^{t_i-t_i}$, $D(x,t) = D(x,t)\lambda^{t_i-t_i}$, and $D_s(x,t) \leq D(x,t)$, it is obvious that $D_s(x,t) \leq D(x,t)$.

Since $D(x,t) = D(x,t)\lambda^{t_i-t_i}$, from (4) we have

$$D(x,t) \leq D_s(x,t) + \epsilon \lambda^{t_i-t_i} = D(x,t) + \epsilon \lambda^{t_i-t_i}.$$

Q.E.D.

From Theorem 2, we can see that $D(x,t)$ is always less than $D(x,t)$. The error of using $D_s(x,t)$ to approximate $D(x,t)$ is less than $\epsilon/(1-\lambda)\lambda^{t_i-t_i}$.

When item_list is full, it has $L$ entries, the errors of which are less than

$$\frac{\epsilon}{1-\lambda} + \frac{\epsilon}{1-\lambda} + ... + \frac{\epsilon}{1-\lambda} = \frac{\epsilon L}{1-\lambda}.$$  

respectively. The average error is less than

$$\epsilon \frac{1-\lambda^L}{L(1-\lambda)} \leq \frac{\epsilon}{L(1-\lambda)},$$

Based on the Theorems above, the algorithm for answering a query at time $t$ is as follows.

**Algorithm 2.** $\lambda$-Count-Query($t,S$)

**input:** item_list=[ $(C(1), C(2), ..., C(L)]$: list of frequent data item candidates;

$S$: the density threshold;  
$\lambda$: the fading factor;  
$\epsilon$: the density error bound;  

**output:** $F$: the set of $\epsilon$-frequent data items;  

**begin**
The following Theorem shows that algorithm \( \lambda \)-Count-Query can correctly detect \( \epsilon \)-approximate frequent items.

**Theorem 3.** The \( \lambda \)-Count-Query algorithm has the following guarantees:

a) All items whose densities exceed \( \frac{S}{1-\lambda} \) will be output; there are no false negatives.

b) No item whose density is less than \( \frac{S-\epsilon}{1-\lambda} \) will be output.

c) The estimated density for each item is no more than the actual density. The error of the estimated density is less than \( \frac{\epsilon}{1-\lambda} \).

**Proof:** From Theorem 2 we know

\[
D(x,t) \geq D_{\lambda}(x,t) \geq D(x,t) \cdot \frac{\epsilon}{1-\lambda}^{t-\lambda}.
\]

If density of a data item \( x \) satisfies \( D(x,t) > \frac{S}{1-\lambda} \), then

\[
D(x,t) \geq \frac{S}{1-\lambda} \cdot \frac{\epsilon}{1-\lambda}^{t-\lambda} \cdot \frac{S-\epsilon}{1-\lambda}.
\]

According to the \( \lambda \)-Count-Query algorithm, \( x \) will be output.

b) If an item \( x \)'s density is less than \( \frac{S-\epsilon}{1-\lambda} \), then from Theorem 2 we know \( D_{\lambda}(x,t) \leq D(x,t) < \frac{S-\epsilon}{1-\lambda} \).

Therefore, \( x \) will not be output by the \( \lambda \)-Count-Query algorithm.

c) From Theorem 2, it is obvious that \( D(x,t) \leq D(x,t) \) and

\[
D(x,t) - D_{\lambda}(x,t) \leq \frac{\epsilon \lambda^{t-\lambda}}{1-\lambda} \leq \frac{\epsilon}{1-\lambda}.
\]

Q.E.D.

For a given data item \( x \), the algorithm for answering the query of whether \( x \) is a frequent item is as follows.

**Algorithm 3** \( \lambda \)-Count-Item-Query(\( x,t,S \))

**input:** item_list=[\( C(1), C(2), \ldots, C(L) \)]; list of frequent data item candidates;

\( S \): the density threshold;

\( \lambda \): the fading factor;

\( \epsilon \): the density error bound;

**output:** \( f \): a flag indicating whether \( x \) is a frequent item;

begin

1. \( f=false \);
2. if \( x \) is in the item_list then
3. \( f=true \);
4. output \( f \);
5. end if

end

\[ f=false; \]
\[ \text{if } x \text{ is in the item_list then} \]
\[ \text{get } x, D(x,t_1), t_1 \text{ and compute} \]
\[ D(x,t) = D(x,t_1) \lambda^{t-t_1}; \]
\[ \text{if } D(x,t) > S - \epsilon \text{ then } f=true \text{ endif}; \]
\[ \text{output } f; \]

end

\[ f=false; \]
\[ \text{if } x \text{ is in the item_list then} \]
\[ \text{get } x, D(x,t_1), t_1 \text{ and compute} \]
\[ D(x,t) = D(x,t_1) \lambda^{t-t_1}; \]
\[ \text{if } D(x,t) > S - \epsilon \text{ then } f=true \text{ endif}; \]
\[ \text{output } f; \]

end

\[ f=false; \]
\[ \text{if } x \text{ is in the item_list then} \]
\[ \text{get } x, D(x,t_1), t_1 \text{ and compute} \]
\[ D(x,t) = D(x,t_1) \lambda^{t-t_1}; \]
\[ \text{if } D(x,t) > S - \epsilon \text{ then } f=true \text{ endif}; \]
\[ \text{output } f; \]

end

**B. Data structure and complexity**

In every time step, the \( \lambda \)-Count algorithm processes the income data and updates a summary structure item_list. Each entry of item_list is a vector \([x,D(x,t),t_0,p_{\text{succ}},p_{\text{pre}}]\), where \( D(x,t) \) is the estimated density of the data item \( x \) at time \( t_0 \). \( t_0 \) is the last time when item \( x \) was received, \( p_{\text{succ}} \) and \( p_{\text{pre}} \) are pointers to its successor and predecessor respectively. The maximal length of item_list is \( L = \log_{1-\theta} \epsilon \). To accelerate the process of updating item_list, it is organized as a hash table using a hash function \( H \). For a data item \( x \), its address is \( H(x) \). Entries in item_list are arranged in a queue structure in the descending order of their \( t_0 \) values. The queue is constructed as a doubly linked list, as shown in Figure 1.

![Figure 1 The data structure of item_list.](image-url)
Since \itemlist is a doubly linked queue, each of those operations costs \(O(1)\) time.

From the above, we conclude that \(\lambda\text{-Count}\) processes one incoming data record in \(O(1)\) time. Since the maximum size of \itemlist is \(L = \log_\lambda \epsilon\), algorithm \(\lambda\text{-Count}\) requires no more than \(O(\log_\lambda \epsilon)\) memory space. Moreover, from the Algorithm 2 and Algorithm 3 we can see that the computation time for answering each query using \(\lambda\text{-Count-Query}\) is \(O(\log_\lambda \epsilon)\), and the time for answering a query about the frequentness of a given data item is \(O(1)\) using algorithm \(\lambda\text{-Count-item-Query}\).

V. EXPERIMENTAL RESULTS

We evaluated our algorithm and compared its performance against the revised versions of \Lossy Counting (LC) [11] and \EC [21] on the time fading model. We focus on the algorithms’ computing time, memory requirement, recall and precision in handling data streams.

All experiments were run on a PC with 1.0GHz Pentium III CPU running Windows 2000. In our experiments, we set \(S=0.01\) and \(\lambda=0.99\).

A. Synthetic data sets

We generate four datasets based on Zipf distributions, with parameters 0.5, 0.75, 1, 1.25, respectively. Each dataset contains one million data records. We compare \(\LC\), \EC and \(\lambda\text{-Count}\) with two error bound settings, \(\epsilon=0.0005\) and \(\epsilon=0.001\).

Figure 2 and Figure 3 compare the memory requirements of the three algorithms. We can see that \(\lambda\text{-Count}\) requires the least memory for all the different settings of Zipf parameter and \(\epsilon\). In fact, the memory size of \(\lambda\text{-Count}\) is always around \(\log_\lambda \epsilon\), while the sampling sizes of \EC and \LC are at least \(\frac{1}{\epsilon} \log_\epsilon N\), respectively. Since \(N\) is the number of data items received from the stream, \LC may require huge memory space with the lapse of time.

Figure 4 and Figure 5 compare the average time for processing 1M data records by the three algorithms. We see that \(\lambda\text{-Count}\) is the fastest. In fact, \(\lambda\text{-Count}\) requires only \(O(1)\) time for processing one data item, while \LC has a processing time of \(O(1/\epsilon)\). For \EC, although theoretically it has a processing time of \(O(1)\), it has a larger hidden constant than \(\lambda\text{-Count}\). This is because, to delete a data in the list, \(\lambda\text{-Count}\) only needs to delete the head of the queue, while \EC needs to do multiple decrement operations. Therefore, \(\lambda\text{-Count}\) is much faster than the other two algorithms.

We also test and compare the quality of the results by the three algorithms in terms of recall and precision defined as follows.

\[
\text{recall} = \frac{\text{number of the truly frequent items reported by the algorithm}}{\text{number of all the truly frequent items}}
\]

\[
\text{precision} = \frac{\text{number of the truly frequently items reported by the algorithm}}{\text{number of all the truly frequent items}}
\]

Since all the algorithms have no false negatives, all the frequent items can be detected, their recalls are all 100%. Figure 6 shows the precision of the three algorithms on synthetic data with Zipf parameter 1.25. From the figure we can see that all the algorithms can achieve high precision. But precisions of algorithms \LC and \EC decrease when the length of the stream increases, while \(\lambda\text{-Count}\) obtains high precision close to 100% regardless of the length of the data stream.

**Figure 2** Comparison of the memory requirement of the three algorithms on datasets with different distributions (1M, \(\epsilon=0.0005\)).

**Figure 3** Comparison of the memory requirement of the three algorithms on datasets with different distributions (1M, \(\epsilon=0.001\)).

**Figure 4** Comparison of time cost of the three algorithms on different distribution data (1M, \(\epsilon=0.0005\)).

**Figure 5** Comparison of time cost of the three algorithms on different distribution data (1M, \(\epsilon=0.001\)).

B. Real datasets

In this section, we adopt the log file data of visitors to the 1998 Soccer World Cup official website [33]. This log file records all of the visit requests for the World Cup official website during 1998 World Cup. Each request consists of 8 attributes including visit time, IP address, the ID of the visited web page and so on. We pick up all
Our experimental results show that the recalls of the three algorithms are all 100%. Furthermore, in Figure 11 we show precisions of \( \lambda\text{-Count} \) and other two algorithms on different sizes of World Cup 98 data sets. From the figure we can see that \( \lambda\text{-Count} \) has higher precision than the other two algorithms.

In many modern applications, data arrives at a system as a continuous stream of transactions. Traditional stream mining algorithms were generally designed to handle all data items in the streams with equal weights. To emphasis the importance of the more recent data items, we present an algorithm \( \lambda\text{-Count} \) for computing frequency counts based on a fading model with a fading factor \( \lambda \). Our algorithm can detect \( \varepsilon \)-approximate frequent items of a data stream using \( O(\log \varepsilon) \) memory space and the processing time for each data item is \( O(1) \). Moreover, the computing time for answering each query is \( O(\log \varepsilon) \), and for answering query about the frequentness of a given data item is \( O(1) \). Through extensive experiments on both real and synthetic data, we show that \( \lambda\text{-Count} \) outperforms other methods in terms of accuracy, memory requirement, and processing speed.

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