Research on Robust Control of Longitudinal Motion of SWATH

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Abstract—Taking the uncertainty of SWATH longitudinal motion control system into account, the parameter uncertainty model of SWATH was built, and the \( \mu \) -synthesis controller was designed to improve the longitudinal motion performance of SWATH. Then Matlab was utilized to simulate the controlled as well as uncontrolled longitudinal performance of SWATH. According to the simulation result, under different sailing conditions, the longitudinal motion performance of SWATH is still excellent. Therefore, the parameter uncertainty mathematical model built in this paper as well as the designed \( \mu \)-synthesis controller are effective and applicable to improve the longitudinal motion performance and the robust performance of SWATH, providing the theoretical basis for the research on SWATH with high performance.

Index Terms—SWATH, \( \mu \)-synthesis controller, longitudinal motion, fin stabilizer

I. INTRODUCTION

SWATH is a new ship with high-tech and high performance. It has the excellent seakeeping quality compared with the conventional mono-hull ship. However, it is easy for the longitudinal motion of SWATH to lose stability because of its little longitudinal restoring moment which results from the little waterplane. Currently, such problems are usually solved by installing the stabilizer fins. However, for improving the SWATH performance, it is still necessary to adopt some control methods for the more excellent longitudinal motion performance.[1]

Since the real longitudinal motion control system of SWATH not only has the parameter uncertainty and unmodel uncertainty but also undergoes many uncertainty external disturbances such as sea waves, it is tough to describe the longitudinal motion control system of SWATH with an exact mathematical model. To solve this problem this paper adopts the robust control methods to design controller for the system.

This paper is achieved with the following part. First, the parameter uncertainty model of the longitudinal motion control system of SWATH is built. Then, the \( \mu \)-synthesis controller for it is designed. Finally, the longitudinal motion performance under different sea states and sailing conditions is simulated to validate the control effect.

II. THE PARAMETER UNCERTAINTY MODEL OF LONGITUDINAL MOTION CONTROL SYSTEM

\[ E \dot{x}(t) = Ax(t) + Bu(t) + H_{\text{ext}}F(t) \]  

where \( x(t) = [\dot{h}, h, \dot{\theta}, \theta]^T \), \( h \) and \( \dot{h} \) indicate the heave displacement and velocity respectively, \( \theta \) and \( \dot{\theta} \) indicate the pitch angular displacement and angular velocity respectively; \( u(t) = [\alpha_1, \alpha_2]^T \), \( \alpha_i \) (\( i = 1, 2 \) denote the fore and aft fin respectively) is fin attack angle; \( F(t) = [F_{\text{ext}}, e^{-j\omega t}, M_{\text{ext}}, e^{-j\omega t}] \), \( \omega \), \( F_{\text{ext}} \) and \( M_{\text{ext}} \) indicate the encounter frequency, the complex amplitude of the wave-exciting force and moment respectively;
\[ H_1 = \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}. \]

Where \( U \) is the speed of SWATH; \( M \) is the mass of SWATH; \( I_1 \) is the mass moment of inertia; \( \theta, q \) and \( \dot{q} \) indicate the perturbation of the pitch angular displacement, angular velocity and angular acceleration respectively; \( Z, W \) and \( \dot{W} \) indicate the perturbation of the heave displacement, velocity and acceleration respectively; \( Z_1, Z_2, Z_3, Z_4, Z_5 \) and \( Z_6 \) indicate the hydrodynamic derivatives which is the force \( Z \) to the corresponding subscript respectively. \( M_1, M_2, M_3, M_4 \) and \( M_5 \) indicate the hydrodynamic derivatives which is the moment \( M \) to the corresponding subscript respectively; \( A_{\alpha_i}, C_{\alpha_i} \) and \( I_i \) (\( i = 1,2 \) denote the fore and aft fin respectively) indicate the projection area of the \( i \)th fins, the lift-curve slope and the longitudinal distance between the pressure centre of fin and the centre of gravity.

The hydrodynamic derivatives can be got by test, numerical calculation or empiric formula in equation(1). This paper uses the strip theory to calculate the hydrodynamic derivatives. According to the analysis, many of the hydrodynamic derivatives vary with the ship load, the wave frequency \( \omega \), the ship speed \( U \), the encounter angle \( \beta \), the hydrodynamic characteristic of fins and so on. Therefore, it is difficult to describe the longitudinal motion control system exactly with the mathematical model. Consequently, this paper builds the parameter uncertainty mathematical model by the following way[3].

(a) Ascertain the variable range of the ship speed \( U \), the wave frequency \( \omega \), the encounter angle \( \beta \), the ship load, the lift-curve slope of fins;
(b) Figure out the maximum perturbation bounds of the motion derivatives in equation(1) under different work conditions;
(c) Build the parameter uncertainty model.

According to the above analysis, there are many uncertainty parameters in system. So it is very difficult to consider all of the parameter uncertainty during the \( \mu \)-synthesis. Because the perturbation parameters are more, the \( \mu \)-synthesis is more conservative[4]. Therefore, the perturbation parameters which are not important to the control can be neglected. The basic principle is to make every parameter change independently, then, neglect the parameters which affect the low frequency gain little and reserve the others. According to the Bode Chart which changes with the independent variation of every parameter, \( Z_2, M_2, Z_3, M_3, UZ_2, UM_2 \) and \( U^2C_{\text{law}} \) (\( i = 1,2 \)) have great effect to the low frequency gain of the system. At the same time, according to the reference[2], the static derivatives play an important role in the longitudinal motion static stability and critical dynamic stability of the system only and the effect of all of the derivatives are not evident to the longitudinal motion performance. This reveals that it is reasonable to only consider this 8 uncertainty parameters. Otherwise, \( Z_2, M_2 \) and \( Z_3 \) varies slightly because the cross section area of the struts varies slightly from the top to the bottom and the SWATH sails under the small perturbation. So they can also be neglected. Finally, the parameter uncertainty model can be built in the following section.

\[ \begin{align*}
&M_1 = (M_1)_0 + \delta_1 \Delta(M_1) \\
&M_2 = (M_2)_0 + \delta_2 \Delta(M_2) \\
&M_3 = (M_3)_0 + \delta_3 \Delta(M_3) \\
&M_4 = (M_4)_0 + \delta_4 \Delta(M_4) \\
&M_5 = (M_5)_0 + \delta_5 \Delta(M_5) \\
&M_6 = (M_6)_0 + \delta_6 \Delta(M_6) \\
&M_7 = (M_7)_0 + \delta_7 \Delta(M_7) \\
\end{align*} \]

Substitute equation(2) into equation(1), the parameter uncertainty model of the system is

\[ (E_0 + \sum_{i=1}^{5} E_i \delta_i) \dot{x}(t) = (A_0 + \sum_{i=1}^{5} A_i \delta_i) x(t) + (B_0 + \sum_{i=1}^{5} B_i \delta_i) u(t) + H_1 F(t) \]

Where \( E_0, A_0 \) and \( B_0 \) indicate the coefficient matrix of the nominal system; the other coefficient matrices is

\[ \begin{align*}
&E_i = 0_{4 \times 4}, (i = 1 \sim 5) \\
&A_i = A_0 = 0_{4 \times 4} \\
&B_i = B_2 = B_3 = 0_{4 \times 4} \\
\end{align*} \]

\[ A_i = \begin{bmatrix}
0 & 0 & 0 & \Delta(UZ_2) \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
\end{bmatrix} \]
$A_2 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}; \\
A_3 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}; \\
B_4 = \begin{bmatrix} \frac{\Delta(U^2C_{i\omega i})}{2} \\ 0 \\ \frac{\Delta(U^2C_{i\omega i})}{2} \\ 0 \end{bmatrix}; \\
B_5 = \begin{bmatrix} \frac{\Delta(U^2C_{i\omega i})}{2} \\ 0 \\ \frac{\Delta(U^2C_{i\omega i})}{2} \\ 0 \end{bmatrix}; \\
$ 

III. THE $\mu$-SYNTHESIS CONTROLLER DESIGN

When the influence of measurement noise is considered, the evaluation output function and measurement output function of the system is

$$
\begin{align*}
    e(t) &= Mx(t) + Vu(t) \\
    y(t) &= Cx(t) + Qd(t)
\end{align*}
$$

where

$$
M = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix}; \\
V = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 \end{bmatrix}; \\
C = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}; \\
Q = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix};
$$

$$
d(t) = [F_\theta e^{j\omega t} M_\theta e^{j\omega t} \xi_1(t) \xi_2(t)]^T, \xi_i(t)
$$

and $\xi_i(t)$ is the measurement noise of $y_i(t)$ and $y_2(t)$ respectively.

In order to design $\mu$-synthesis controller, the uncertainty system should be represented as the linear fractional transformation [4] about $G_p$, namely

$$
\begin{align*}
    [e(t)] &= F_s(G_p, \Delta) [d(t)] \\
    [y(t)] &= G_p [u(t)]
\end{align*}
$$

Where

$$
G_p = \begin{bmatrix} \bar{A} & \bar{L} & \bar{H} & \bar{B} \\ W - RA & -RL & -R\bar{H} & Z - RB \\ M & 0 & 0 & V \\ C & 0 & Q & 0 \end{bmatrix};
$$

$$
\begin{align*}
    \bar{A} &= E_0^{-1}A_0; \bar{B} = E_0^{-1}B_0; \bar{H} = E_0^{-1}H; \\
    H &= \begin{bmatrix} H_1 & 0 \end{bmatrix}; \\
    \Delta &= \text{diag}(\delta_1, \delta_2, \delta_3, \delta_4, \delta_5);
\end{align*}
$$

$L, R, W$ and $Z$ can be got by the maximum rank resolving $E_0^{-1}[E_i A_i B_i] = L_i R_i W_i Z_i$, $(i = 1 \sim 5)$, namely $L = [L_1 \cdots L_5], R = [R_1^T \cdots R_5^T]$, $W = [W_1^T \cdots W_5^T], Z = [Z_1^T \cdots Z_5^T]$.

The evaluation output weight function $W_e$ and the external disturbance weight function $W_d$ selected by this paper are

$$
W_e = \text{diag}(1, 1, 1, 1 \times 10^{-1}, 1 \times 10^{-3});
$$

$$
W_d = \text{diag}(0.31 \times 10^6, 0.22 \times 10^7, 0.05, 0.05).
$$

Where every elements of $W_e$ is the weight function of $h$, $h$, $\dot{h}$, $\alpha$, and $\alpha$, respectively; every elements of $W_d$ is the weight function $F_\omega e^{j\omega t}$, $M_\omega e^{j\omega t}$, $\xi_1(t)$ and $\xi_2(t)$ respectively.

Finally, use MATLAB $\mu$-Toolbox to design the $\mu$-synthesis controller. The controller can be got by the $D - K$ iteration.

IV. SIMULATION ANALYSIS

Take the SWATH-6A [5] as an example, this paper designs the $\mu$-synthesis controller to ensure the longitudinal motion stability and satisfying motion performance under 18 knot sailing speed, degree 5 sea state and the significant value 1/3 2.5. The perturbation bound of the speed $U$ is 15-20 knot. The perturbation of $C_{1\omega i}, (i = 1, 2)$ is 10%. The wave frequency range of interest is 0.5 1.8 rad/s. Then, owing to the head and following sea situation, this paper figures out the perturbation bound of the corresponding hydrodynamic coefficients. Since $M_\omega$ is relevant with $GM_L$ and the load change results in the change of $GM_L$, the perturbation of $GM_L$ is taken as 15%.

The perturbation bounds of $M_\omega, UZ_w, UM_w$ and $U^2C_{1\omega i}, (i = 1, 2)$ are

$$
\begin{align*}
    -2.0 \times 10^3 & \leq M_\omega \leq -1.43 \times 10^3; \\
    -1.85 \times 10^5 & \leq UZ_w \leq -8.14 \times 10^6; \\
    -9.48 \times 10^5 & \leq UM_w \leq 1.37 \times 10^5;
\end{align*}
$$
This paper uses D–K iteration to design the controller and selects the third iteration result as the controller. Then reduce the controller and get a 5 rank controller finally.

The simulation curves of longitudinal motion of SWATH mounted the controllable fore and aft fin are shown by Fig. 2-5 when it sails in the head and following sea (β = 180°, 0° indicate the head and following sea wave respectively) at the speed of 15 and 19 knot under the condition of degree 5 sea state and the significant value $h_{1/3} = 2.5$ meters. The real line is the longitudinal motion curves after the fin is controlled and the dashdotted is the longitudinal motion curves before the fin is controlled. In view of the limit of the executive unit and the hydrodynamic trait of fin etc, select the fin attack angle $20°, (i = 1, 2)$ The variation curves of the fore and aft fin attack angle are shown by fig. 6-7 after they are controlled. The real line and dashdotted are the variation curves of the fore fin attack angle and aft fin attack angle respectively.
According to fig.2-5, the SWATH is the longitudinal motion stability and has the excel motion performance when the sailing states such as the speed $U$ and the wave heading $\beta$ change. Therefore, the parameter uncertainty model which this paper built is reasonable and the $\mu$-synthesis controller designed based on this model not only ensures the longitudinal motion robust stability of SWATH but also makes the SWATH work with the excel robust performance. According to Fig.6-7, the saturation rate of the fin angles is low. Therefore, the design is reasonable.

V. CONCLUSION

According to the simulation and analysis, the parameter uncertainty model of the longitudinal motion control system of SWATH which was built in this paper is reasonable and applicable. The $\mu$-synthesis controller designed by this paper not only ensures the longitudinal motion robust stability of SWATH but also makes the SWATH sail with excellent robust motion performance when the sailing condition and marine environment change. However, the result is obtained by the theoretical calculation. Consequently, the result of this paper needs to be validated by further test.

REFERENCES


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