Robust Blind Beamforming Technique Based on Third-Order Cumulants

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Abstract—The goal of blind beamforming is to recover source signals only from the output of array without any a priori information about the array manifold. For the separation of independent sources, Cardoso and Souloumiac proposed an effectively blind beamforming method (JADE) using fourth-order cumulant. However, the high computation complexity of this method limits its applicability in many practical problems. Due to the fact that many source signals have nonzero third-order cumulants, such as speech and sonar signals consisting of phase-coupled sinusoids, in this paper we present a new blind beamforming method based on third-order cumulant to separate independent sources. Comparing with classic JADE method, the proposed algorithm has much lower computation complexity and is more robust to the errors resulting from finite sample size. Simulation results demonstrate these performances of our algorithm.

Index Terms—blind beamforming, third-order cumulant, robust, computation complexity

I. INTRODUCTION

For recent decades, array signal processing techniques have been widely used in many practical applications, such as radar, sonar and wireless communication systems. The essential part for these techniques is array beamforming. Conventional beamforming methods usually rely on a priori knowledge of the directional vector associated with desired source signal and, thus, are quite sensitive to the vector errors that mainly result from the array deformation, pointing errors, etc... Consequently, this defect strictly limits the performance of conventional beamforming methods in practice.

On the other hand, blind beamforming approaches try to recover source signals only from the outputs of array without this a priori information [1-9]. The first blind beamforming methods proposed are based on the direction finding [1], where the direction of each incoming wavefront is estimated, at the same time, a beamforming is constructed to restore the source signal from that direction. This requires at least that the array is calibrated. Therefore, this method needs intensive computation, especially in the case of multiple sources and/or multipath propagation. Thus, the applicability of this approach highly depends on the channel conditions. More recently, new types of blind beamforming approaches have been proposed that are not based on special channel conditions, but instead exploit the properties of signals. A celebrated example is the constant modulus algorithm (CMA) [2-3], which utilizes the fact that the source signals possess constant amplitude that is often well-known at the receiver, such as is the case for the frequency or phrase modulated signals in digital communication. The major advantage of CMA algorithm is that it depends on neither the channel properties nor array calibration. However, sometimes the explicit signal properties cannot be obtained. Hence, some new approaches exploiting the second-order statistics of sources are presented to recover source signals in the absence of explicit source information, for instance, the SOBI method [4]. These second-order statistics approaches perform very well only when the source signals have different power spectrum densities; otherwise, the higher-order statistics algorithms [5-8] are required.

In the relaxation of the assumption for different source power spectrum densities, Cardoso and Souloumiac proposed the effective JADE algorithm [5] for blind beamforming, based on fourth-order cumulant. Although this method can effectively separate independent sources, the problem of high computation complexity arises as the method needs plenty of fourth-order cumulants, and thus the related applications are limited. In this paper, based on the fact that many source signals have nonzero third-order cumulants such as speech and sonar signals consisting of phase-coupled sinusoids, we proposed a new method using third-order cumulants for blind separation of independent sources to reduce the computation complexity. In addition, because the estimates of third-order cumulants exhibit less variance.
than that of fourth-order cumulants, our algorithm is more robust to the errors resulting from finite sample size than the JADE method.

The rest of this paper is organized as follows. The problem statement is presented in II. In the section III, we describe the proposed algorithm. Numerical simulations are provided in the section IV to illustrate the effectiveness of proposed method. Finally, section V arrives at a conclusion.

II. PROBLEM FORMULATION

Assume that $m$ signals impinge on an array of $n$ sensors, the array output $x(t)$, corrupted by additive noise, can be modeled as

$$x(t) = \sum_{i=1}^{m} a(\theta_i)s_i(t) + n(t) = As(t) + n(t),$$

(1)

where $x(t)$ $n \times 1$ array output vector, $A$ unknown $n \times m$ mixing matrix, $s(t)$ unknown $m \times 1$ source vector formed by $s_i(t)$, $a(\theta_i)$ steering vector of signal $s_i(t)$ from the direction $\theta_i$ corresponding the column of $A$, $n(t)$ $n \times 1$ vector of stationary noise.

In this model, the sources in vector $s(t)$ are assumed to be mutually independent with zero mean. We also assume that the vector $n(t)$ is additive white Gaussian noise and independent from source signals. In addition, it is assumed that the source signals have nonzero third-order cumulants defined by

$$k_s = \text{Cum}(s(t), s_j^*(t), s_k^*(t)).$$

(2)

An example is the linear frequency-modulated signals in sonar system as follows

$$s(t) = Ae^{j\omega t + \frac{\tau t^2}{2}}.$$  

(3)

The objective is to build a blind beamformer $B$ of size $m \times n$ only based on observed array output $x(t)$ without any knowledge of array manifold, such that $e(t) = Bx(t) = BAs(t) = Ps(t)$, where $P$ is an arbitrary $m \times m$ permutation matrix and $e(t)$ is the estimate of the source signal vector $s(t)$. Equivalently, the task is to estimate the mixing matrix $A$.  

III. PROPOSED ALGORITHM

For the model in (1), the correlation matrix of array output $x(t)$ takes the following structure:

$$R_x = E(xx^H(t))$$

$$= AR_A^H + R_n$$

$$= AR_A^H + \sigma^2 I,$$  

(4)

where $\sigma^2$ is the variance of components of noise vector $n(t)$.

Thanks to the white noise assumption, an estimate $\hat{\sigma}^2$ of the noise variance is the average of $n - m$ smallest eigenvalues of $R_n$. Denote $\mu_1, \mu_2, \cdots, \mu_m$ the $m$ largest eigenvalues and $h_1, \cdots, h_m$ the corresponding eigenvectors of $R_n$. A whitener is

$$W = [(\mu_1 - \sigma^2)^{-1/2}h_1, \cdots, (\mu_m - \sigma^2)^{-1/2}h_m]^H.$$  

(5)

Thus, the whitened array output is as follows:

$$z(t) = Wx(t) = Us(t) + Wn(t),$$

(6)

where $U = WA$ is a unitary matrix, which reduces the $n$ dimensional array output vector $x(t)$ to an $m$ dimensional vector $z(t)$. Hence, the estimate of mixing matrix $A$ is equivalent to the estimate of unitary matrix $U$.

To any column vector $h$, the $(i,j)$ -element of the $m \times m$ matrix $Q_j(h)$ called ’cumulant matrix’ is defined by

$$n_o = \sum_{i=1}^{m} \sum_{j=1}^{m} \sum_{k=1}^{m} \text{Cum}(u_i, u_j^*, u_k^*) h_i$$

$$= \sum_{i=1}^{m} \sum_{j=1}^{m} \sum_{k=1}^{m} \text{Cum}(s_i, s_j^*, s_k^*) h_i.$$  

(7)

Using the properties of cumulant, namely, additivity, multilinearity and Gaussian rejection, one can obtain the following result:

$$n_o = \sum_{i=1}^{m} \left( \sum_{j=1}^{m} \sum_{k=1}^{m} u_i^* u_j u_k^* k \right) h_i$$

$$= \sum_{j=1}^{m} \sum_{k=1}^{m} u_j^* u_k^* \sum_{i=1}^{m} u_i h_i$$

$$= \sum_{j=1}^{m} \sum_{k=1}^{m} k_j^* y_j^* h_i.$$  

(8)

Therefore, it follows

$$Q_j(h) = \sum_{j=1}^{m} \sum_{k=1}^{m} k_j^* y_j^* h_i = \Lambda U^n,$$  

(9)

where $\Lambda = \text{diag}(k_1u_1^*, k_2u_2^*, \cdots, k_mu_m^*)$. Accordingly, the unitary matrix $U$ can be obtained by the eigendecomposition of $Q_j(h)$ for some vector $h$.

In order to improve the robustness and reduce computation at the same time, all of third-order cumulants are required and should only be used once. Denote the $m \times 1$ vector with 1 in $k$th position and 0 elsewhere by $b_k$; one can define the following set:

$$S = \{Q_j(b_k) | 1 \leq k \leq m\}.$$  

(10)
Consequently, the unitary matrix $U$ can be computed as the joint approximate diagonalizer of the matrices in the set $S$ using Jacobi technique [10].

Our proposed algorithm can be summarized as follows:

1) Collect the data $x(t)$ and then calculate the correlation matrix $R_x$ according to (4),
2) Obtain the whitening matrix $W$ from (5) by the eigencomposition of $R_x$,
3) Compute all cumulant matrices in the set $S$ according to (7) and (10),
4) Find the joint approximate diagonalization matrix $U$ of the matrices in the set $S$ using Jacobi technique,
5) Estimate the mixing matrix $\hat{A} = W^*U$.

From the above discussion, it can be seen that both of our algorithm and the JADE method jointly diagonalize $m$ matrices of size $m \times m$. However, in order to build the $m$ matrices, our algorithm only needs to calculate $m^3$ third-order cumulants, which means that it has much lower computation complexity than the JADE method for which $m^4$ fourth-order cumulants are required, which is illustrated in Fig.1, especially in large number of sensors. Instead of using fourth-order cumulant, proposed algorithm utilizes third-order cumulant and, thus, is more robust to the errors resulting from finite sample size than the JADE method.

IV. SIMULATION RESULTS

To examine the performance of our proposed method, we show several numerical experiments in comparison with JADE approach in this section. In order to measure the performance of our proposed algorithm, the following performance index (PI) is used [9]:

$$\text{PI} = \left[ \frac{\sum_{i=1}^{m} \sum_{j=1}^{m} |g_{ij}|}{\max_{i,j} g_{ij}} \right] - 1 + \left[ \frac{\sum_{i=1}^{m} \sum_{j=1}^{m} |g_{i1}|}{\max_{i,j} g_{i1}} \right] - 1 - \frac{1}{m(m-1)}$$

(11)

where $g_{ij}$ is the $(i,j)$-element of the matrix $G = \hat{A}^*A$.

Experiment 1: In our simulations, a five-element sensor array is used and both of the real and image part of its mixing coefficients are randomly chosen from Gaussian distribution of zero mean and unit variance. Two source signals are impinging on the array. One is normally distributed; another is discrete signal that takes its values in the set $\{-1,0,3\}$ with the respective probability $\{1/4,2/3,1/12\}$ . The background noise is stationary white Gaussian noise.

From Fig.2-4, it is seen that our proposed algorithm has better performance than the JADE method, especially in the case of small sample size. For both methods, it is also obvious that, as is to be expected, the greater the sample size, the smaller the performance index. It is also shown that, the two algorithms have almost the same performance for high signal-to-noise ratio (SNR) and large sample number.

Experiment 2: In this experiment, we illustrate the performance of two algorithms in the presence of non-Gaussian noise. Fig. 5-6 show simulation results for the two algorithms, where it has the same conditions as experiment 1 except that the background noise obeys Uniform distribution over the interval $[0, 1]$. Comparing Fig.5-6 with Fig. 3-4, it can be seen that, in the presence of non-Gaussian noise our algorithm has more obvious advantage than JADE method, which results from less estimate errors for proposed algorithm than JADE method under finite sample size.

Figure 1. Number of required cumulants versus number of sensors

Figure 2. PI versus SNR for 100 sample data in presence of Gaussian noise

Figure 3. PI versus SNR for 500 sample data in presence of Gaussian noise

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third-order cumulant to perform blind separation of independent sources. In comparison with the classical JADE approach, the proposed method has lower computation complexity and is more robust to the errors resulting from finite sample size. Simulation results demonstrate the effectiveness of our proposed algorithm.

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V. CONCLUSIONS

In this paper, we presented a new method based on

Figure 4. PI versus SNR for 1000 sample data in presence of Gaussian noise

Figure 5. PI versus SNR for 500 sample data in presence of Uniform distributed noise

Figure 6. PI versus SNR for 1000 sample data in presence of Uniform distributed noise