Fuzzy Support Vector Machines Control for 6-DOF Parallel Robot

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Abstract—In order to realize the trajectory tracking control of six degrees of freedom parallel robot, the dynamics equation of six degrees of freedom parallel robot was established. The parallel robot has obvious nonlinear, uncertainty characteristics and external disturbance, so the sliding mode variable structure theory was introduced into the system control. A fuzzy support vector machines control strategy based on sliding mode control was designed to reduce the oscillation of the sliding mode control. Parameters of fuzzy support vector machines controller were optimized by hybrid learning algorithm, which combines least square algorithm with improved genetic algorithm, to get the optimal control performance for the controlled object. The controller designed consists of a fuzzy sliding mode controller and a fuzzy support vector machines controller, and the compensation controller is selected by comparing switching function with the thickness of boundary layer. Simulation results show that under the condition of model error and external disturbance, the control strategy designed gets tracking effect with high precision and speed.

Index Terms—parallel robot, fuzzy control, support vector machines, sliding mode control, dynamics equation

I. INTRODUCTION

The six degrees of freedom (6-DOF) Stewart platform parallel robot is a closed-loop mechanism in which the end-effector (mobile platform) is connected to the base by six extensible legs [1]. Compared with serial robot, the parallel robot has potential advantages in terms of compliance, accuracy, high speed and payload. Therefore, it has been used in precision lathe, assembly robotic manipulator and electronics manufacture [2,3]. Because of the complex condition and the uncertain object, the parallel robot is not only a complicated nonlinear multivariable and strong coupling system, but also a time-varying system. The parallel robot is not controlled accurately and its track is not kept better by the general model-based control method [4,5].

In the domain of artificial intelligence techniques for the system control, different control algorithms have been used to realize the trajectory tracking control for the parallel robot. The sliding mode control algorithm has complete adaptability for system disturbance and stirring, which is extensively applied in the control of the parallel robot [6,7]. In the fuzzy control, the mathematical model for the system does not be set up precisely and the joints of robots can be decoupled, but fuzzy control system is easily influenced by nonlinear, time-varying and random disturbance [8]. Neural network control algorithm has many advantages, such as self-learning, self-organizing, self-adaptive capacity, nonlinear and parallel distributed processing, and so on. However, it also has the congenital defects, such as it falls into local minimum easily, and it is weakly normalized for few samples [9-11]. These defects make it difficult to meet control precision for parallel robot.

Thus, some control methods are combined to realize the trajectory tracking control for the parallel robot. A cascade-control algorithm based on a sliding mode in the legspace was proposed by Hongbo Guo, Yongguan Luo, Guirong Liu and Hongren Li to realize the trajectory tracking control of hydraulically driven six degrees of freedom parallel robotic manipulator [6]. A control approach which is based on the coupling of sliding mode and multi-layers perceptron neural networks was proposed by Achili B, Daachi B, Amirat Y and Ali-cherif A to deal with the robust adaptive control tracking of a 6 degree of freedom parallel robot [9]. A sliding mode control with discontinuous projection-based adaptation laws was proposed by Yangjun Pi and XuanyinWang to improve the tracking performance of the parallel robot.
manipulator [10]. A robust neural-fuzzy-network control system was presented by Rongjong Wai and Pochen Chen to realize the joint position control of an n-rod robotic manipulator for periodic motion in order to deal with the uncertainties in application, such as friction forces, external disturbances, and parameter variations [12]. A new discrete sliding mode control approach for parallel robot was presented by Shaocheng Qu and Yongji Wang to achieve accurate servo tracking in the presence of load variations, parameter variations and nonlinear dynamic interactions [13]. A fuzzy-PI compound control system for three-cylinder hydraulic parallel robot was designed by Qidan Zhu, Xunyu Zhong and Bo Xu [14].

Support vector machines, which is based on the structural risk minimization rule, overcomes the shortcoming that neural network structure relies on the experience of designer. Its topology structure is decided by support vectors. It solves these problems well, such as high dimension, local minimum and small samples, and has advantages of both neural network and traditional model [15,16]. So the support vector machines is combined with fuzzy control to design a fuzzy support vector machines controller for parallel robot to reduce the chattering in sliding mode control. It is important to select the proper SVM parameters for improving the learning and generalizing capacity of the control system [16]. Thus, the fuzzy proportional coefficients were adjusted with the controlled object, and the parameters of the controller were optimized by least square (LS) learning algorithm and improved genetic algorithm (IGA) in order to improve control precision and working stability of the parallel robot.

II. DYNAMICS MODEL OF 6-DOF STEWART PLATFORM PARALLEL ROBOT

A. Reference frame

The 6-DOF Stewart platform parallel robot is shown in Fig.1. It consists of mobile platform, base platform and six extensible leg, each of which is connected with the two platforms by spherical joints [2,3]. The legs are driven by six servo-electromotors. To describe the motion of the mobile platform, two reference frames are chosen: a fixed reference frame \(\{B, X_B, Y_B, Z_B\}\) attached to the base platform and a mobile reference frame \(\{P, X_P, Y_P, Z_P\}\) attached to the mobile platform, as shown in Fig.1. Six coordinates are used to further describe the position and the orientation of the mobile platform in detail. Three coordinates are the positional displacements \(\{X_P, Y_P, Z_P\}\), which describe the position of a fixed point in the mobile platform with respect to the fixed reference frame. The other three coordinates are the angular displacements, represented by Euler angles \(\{\gamma, \beta, \alpha\}\), which describe the orientation of the mobile platform with respect to the fixed reference frame. Therefore, the generalized coordinate vector, whose elements are the six variables chosen to describe the position and orientation of the mobile platform, can be defined as \(\{X_P, Y_P, Z_P, \gamma, \beta, \alpha\}\).

The rotation matrix from mobile reference frame to fixed reference frame can be described as follow:

\[
R^B_P = \begin{bmatrix} 
  c\alpha c\beta & c\alpha s\beta & -s\alpha & \epsilon \\
  s\alpha c\beta & s\alpha s\beta & c\alpha & \epsilon \\
  s\beta & -c\beta & 0 & \epsilon \\
  0 & 0 & 0 & 1 
\end{bmatrix} \tag{1}
\]

where \(c(\cdot)\) denotes \(\cos(\cdot)\); \(s(\cdot)\) denotes \(\sin(\cdot)\).

The \(i\)th leg vector \(\mathbf{l}_i\) with respect to the fixed reference frame can be described as

\[
\mathbf{l}_i = \mathbf{\bar{c}} + R^B_P \mathbf{b}_i - \mathbf{p}_i, \quad i=1,2,...,6 \tag{2}
\]

where \(\mathbf{\bar{c}}\) is the translation vector of the origin of the mobile reference frame with respect to the fixed reference frame; \(\mathbf{b}_i\) is the position vector of the \(i\)th joint point of base platform with respect to the fixed reference frame; \(\mathbf{p}_i\) is the position vector of the \(i\)th joint point of mobile platform with respect to the mobile reference frame.

The extended length of the \(i\)th leg is described as

\[
\Delta \mathbf{l}_i = \left| \mathbf{l}_i \right| - \mathbf{l}_{i0} \tag{3}
\]

where \(\mathbf{l}_{i0}\) is the original length of the \(i\)th leg.

Well-controlled lengths of six legs make the mobile platform follow the desired trajectory.

B. Dynamics equation

In order to solve the kinetic energy and the potential energy of parallel robot, the whole system is separated into the mobile platform and the six legs with the base platform.

Suppose the angle velocity of mobile platform is \(\mathbf{\omega}_k\), and then the kinetic energy of mobile platform \(KE_h\) which includes translational and rotating kinetic energy, can be described as

\[
KE_h = \frac{1}{2} \left( m_u (\dot{X}_P^2 + \dot{Y}_P^2 + \dot{Z}_P^2) + \mathbf{I}_{eh}^T \dot{\mathbf{\omega}}_h \right) \tag{4}
\]

where \(m_u\) is the mass of mobile platform; \(\mathbf{x}_P, \mathbf{y}_P, \mathbf{z}_P\) is the displacement about the axis \(X_P, Y_P, Z_P\) respectively; \(\mathbf{I}_{eh}\) is the inertia matrix of the mobile platform with respect to mobile reference frame. It can be computed by

\[
\mathbf{I}_{eh}^h = R^T \mathbf{R}^T \tag{5}
\]

where \(R\) is the corresponding rotating matrix, defined by the angle rotating rule of Roll-Pitch-Yaw; \(\mathbf{I}_{eh}^h\) is the rotating inertia with respect to the mobile reference frame. It is described as

\[
\mathbf{I}_{eh} = \begin{bmatrix} I_{X_P} & 0 & 0 \\
 0 & I_{Y_P} & 0 \\
 0 & 0 & I_{Z_P} \end{bmatrix} \tag{6}
\]
where $I_{x\beta}, I_{y\beta}, I_{z\beta}$ is the rotating inertia with respect to the axis $X_{\beta}, Y_{\beta}, Z_{\beta}$ respectively.

The angle velocity of mobile platform is described as

$$\omega_h = \beta R_{x\beta} \alpha R_{y\beta} (\beta) \dot{X} + \beta R_{x\beta} \alpha \dot{Y} + \alpha \dot{Z}$$

(7)

where $R_{x\beta}, R_{y\beta}$ is the corresponding rotating matrix respectively, defined by the angle rotating rule of Roll-Pitch-Yaw; $c(\cdot)$ denotes $\cos(\cdot)$; $s(\cdot)$ denotes $\sin(\cdot)$.

The kinetic energy of mobile platform is rewritten as

$$KE_h = \frac{1}{2} \dot{q}^T M_h(q) \dot{q}$$

(8)

where $M_h(q)$ is defined as

$$M_h(q) = \begin{bmatrix} m_u & 0 & 0 & 0 & 0 & 0 \\ 0 & m_u & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & m_{h44} & -I_x \beta \\ 0 & 0 & 0 & m_{h44} & -I_x \beta \\ 0 & 0 & 0 & 0 & M_{h45} \\ 0 & 0 & 0 & 0 & M_{h45} \end{bmatrix}$$

(9)

where $M_{h44} = I_X \sin^2 \beta + I_Y \sin^2 \gamma \cos^2 \beta + I_Z \cos^2 \gamma \cos^2 \beta$, $M_{h45} = (I_x - I_z) \cos \gamma \sin \beta \cos \beta + M_{h45} = (I_y - I_z) \cos \gamma \sin \beta \sin \beta$, $M_{h45} = I_Y \cos^2 \gamma + I_Z \cos^2 \gamma$.

The potential energy of mobile platform is described as

$$P_h = m_g g z \dot{p} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \end{bmatrix}^T \ddot{q}$$

(10)

where $g$ is the gravity acceleration.

The extensible legs, which are driven by the servoelectromotors, are separated into the cylinders and the rods. They are regarded as the rigid parts with rotating inertia with respect to themselves. Each leg is represented by the centroid point of it.

The position of the $i$th leg centroid point $G_i$ is described as

$$B_iG_i = \begin{bmatrix} m_{i1} \delta_{i1} & + & m_{i2} \delta_{i2} & (L_i - l_{i2}) \delta_{i2} & \dot{\delta}_{i2} \\ m_{i1} \delta_{i1} & + & m_{i2} \delta_{i2} & (L_i) \delta_{i2} & \dot{\delta}_{i2} \end{bmatrix}$$

(11)

where $\delta_{i2} = l_{i2} - l_{i2} m_{i2}$, $l_{i2}$ is the distance between the center point of the $i$th base joint and the centroid point of cylinder of the $i$th leg; $l_{i2}$ is the distance between the $i$th upper joint and the rod of the $i$th leg; $m_{i1}$ is the mass of cylinder of the $i$th leg; $m_{i2}$ is the mass of rod of the $i$th leg; $L_i$ is the length of the $i$th leg; $u_i$ is the orientation of the $i$th leg. $u_i$ can be defined as

$$\dot{u}_i = \frac{B_i P_i}{L_i}$$

(12)

The velocity of centroid point of the $i$th leg $V_{G_i}$ is described as

$$V_{G_i} = \frac{\dot{\delta}_{i2}}{L_i} [\dot{V}_n - (\dot{V}_r \cdot \dot{u}_i) \dot{u}_i] + \frac{m_{i2}}{m_{i1} + m_{i2}} \ddot{V}_r$$

(13)

where $\dot{V}_r$ is the velocity vector of the centroid point of upper joint of the $i$th leg $P_r$.

The kinetic energy of the $i$th rod is described as

$$K_{L_i} = \frac{1}{2} (m_{i1} + m_{i2}) \dot{V}_{G_i}^2$$

(14)

Then, substituting equation (13) for equation (14), the kinetic energy of rod can be rewritten as

$$K_{L_i} = \frac{1}{2} (m_{i1} + m_{i2}) [h_i \dot{\delta}_{i2}^2 + k_i \dot{\delta}_{i2}^2 + (\hat{\delta}_{i2})^2]$$

(15)

where $h_i = \frac{l_{i2}}{l_{i2}} + \frac{m_{i2}}{m_{i1} + m_{i2}}$; $k_i = h_i - \frac{m_{i2}}{m_{i1} + m_{i2}}$.

The total mass of six legs can be written as

$$M_{legs} = \sum_{i=1}^{6} [M_{i1} + m_{i2}]$$

(16)

The context of kinetic energy of six legs may be expressed as

$$M_{legs} = \sum_{i=1}^{6} K_{L_i} = \frac{1}{2} \dot{q}^T M_h(q) \dot{q}$$

(18)

The potential energy of six legs may be written as

$$P_{legs} = \sum_{i=1}^{6} (m_{i1} + m_{i2}) \frac{l_{i2}}{l_{i2}} [\dot{\delta}_{i2}^2 - \dot{\delta}_{i2}^2]$$

(19)

Lagrange equation for parallel robot can be expressed as

$$\frac{d}{dt} \left[ \frac{\partial L(q, \dot{q})}{\partial \dot{q}_i} \right] - \frac{\partial L(q, \dot{q})}{\partial q_i} = \tau_i, \quad i = 1, 2, \ldots, n$$

(20)

where $q \in R^n$ is nominal coordinate; $L$ is the Lagrange function of mechanism system; $\tau_i$ is the force on the $i$th nominal coordinate.

The dynamic Lagrange equation of 6-DOF rigid parallel robot is described as

$$M(q) \ddot{q}(t) + V_m(q, \dot{q}, \ddot{q}) + G(q) + \tau_d = J^T \tau(t)$$

(21)

where $q, \dot{q}, \ddot{q} \in R^n$ is the position, the velocity and the acceleration of centroid point of mobile platform respectively; $M(q)$ is the mass of mobile platform and six legs; $V_m(q, \dot{q}, \ddot{q})$ is the velocity vector of mobile platform and six legs; $G(q) \in R^n$ is the gravity vector of mobile platform and six legs; $\tau_d \in R^n$ is the control force vector of mobile platform and six legs; $\tau_e \in R^n$ is the model error and external disturbance; $M(q), V_m(q, \dot{q}) \cdot G(q)$ can be computed by the equations of the kinetic energy and the potential energy of mobile platform and six legs.

If the model designed is precise, control law of robot is expressed as

$$\tau(t) = J^{-1} [M(q) \ddot{q} - k_e \dot{q} - k_e \dot{q} + V(q, \dot{q})]$$

(22)

where $q_d$ is expect angle, $e = q - q_d$; $\dot{e} = \ddot{q} - \ddot{q}_d$.

Then, substituting equation (22) for equation (21), the control equation of stable close-loop system is expressed as
\[ \ddot{e} + k \dot{e} + k_e e = 0 \]  \hspace{1cm} (23)

Because it is difficult to build the real model of the object precisely, desired model is only built. Its control law is expressed as

\[ \tau(t) = J^{-1} [M(q) \ddot{q} - k_e \dot{e} - k_p e] + V_o(q, \dot{q}) \dot{q} + G_o(q) \]  \hspace{1cm} (24)

Then, substituting equation (24) for equation (21), the below equation is established.

\[ M(q) \ddot{q} + C(q, \dot{q}) \dot{q} + G(q) = M_o(q) \ddot{q} - k_e \dot{e} - k_p e + V_o(q, \dot{q}) \dot{q} + G_o(q) \]  \hspace{1cm} (25)

If \( \Delta M = M_o - M \), \( \Delta V = V_o - V \), \( \Delta G = G_o - G \), the below equation is obtained.

\[ \ddot{e} + k_e \dot{e} + k_p e = M_o^{-1} (\Delta M \ddot{q} + \Delta V \dot{q} + \Delta G) \]  \hspace{1cm} (26)

From equation (26), the decline of the control performance is brought partly by the parameter and non-parameter uncertainty. Thus, the compensation for the uncertainty is needed for improving the control precision of the robot.

### III. STRUCTURE OF CONTROL SYSTEM OF PARALLEL ROBOT

According to dynamics equation of the robot and its control law, sliding mode control method and fuzzy support vector machines are used to compensate for the uncertainty. Structure of control system for robots is shown in Fig.2. In the figure, \( q \) denotes the real track of robot; \( q_d \) denotes the expect track; \( e \) denotes the error vector; FSMC denotes the fuzzy sliding mode controller; FSVMC denotes the fuzzy support vector machines controller; \( R(e) \) denotes the function of fuzzy rules, whose inputs are \( e \) and \( \dot{e} \).

![Figure 2. Structure of control system for robots](image)

Compensation control law for fuzzy sliding mode is defined as

\[ \tau_i(t) = M(q) \ddot{q}_d - k_e \dot{e}_i - k_p e_i \]  \hspace{1cm} (27)

where \( u_1 \) is the control compensation of fuzzy sliding mode controller; \( u_2 \) is the control compensation of fuzzy support vector machines controller.

Function \( R(e) \) is used to decided which controller as the compensation controller. Suppose the thickness of boundary layer is \( Q \); if \( R(e) > Q \), FSMC is used for control compensation; if \( R(e) < Q \), FSVMC is used for control compensation; if \( R(e) = Q \), the below sliding algorithm is used for control compensation.

\[ u(t) = (1 - d(e)) u_2(t) + d(e) u_1(t) + \tau \]  \hspace{1cm} (28)

where \( \tau \) is the control torque; \( u_1 \) and \( u_2 \) are the outputs of FSMC and FSVMC respectively; \( d(e) \) is the sliding function, whose function is to make FSMC and FSVMC switch smoothly.

\[ d(e) = \begin{cases} 0, & E(t) \in A_{FSMC} \\ 1, & A_{FSVMC} \end{cases} \]  \hspace{1cm} (29)

where \( A_{FSMC} \) is the control range of fuzzy support machines controller; \( A_{FSVMC} \) is the control range of sliding mode controller. They are defined as

\[ A_{FSMC} = \{ E \mid \| E \|_p \leq Q \} \]  \hspace{1cm} (30)

\[ A_{FSVMC} = \{ E \mid \| E \|_p \leq Q + \zeta \} \]  \hspace{1cm} (31)

where \( \zeta \) is the thickness of switch layer, \( 0 < \zeta < Q \). \( P \) norm is defined as

\[ \| E \|_p = \left( \sum_i | E_i |^p \right)^{1/p} \]  \hspace{1cm} (32)

According to \( P \) norm definition, the sliding function used is described as

\[ d(e) = \max (0, \text{sat}(R - Q) / \zeta) \]  \hspace{1cm} (33)

where \( \text{sat}(y) = \begin{cases} y, & |y| < 1, y = \| E \|_p \\
\text{sgn}(y), & \text{otherwise} \end{cases} \)

If \( R(e) < Q \), \( e \in A_{FSMC} \), and \( d(e) = 0 \); if \( R(e) > Q + \zeta \), \( e \in A_{FSVMC} \), and \( d(e) = 1 \). If \( A' = A - A_{FSMC} \), \( 0 < d(e) < 1 \).

### IV. FUZZY SLIDING MODE CONTROLLER

In order to control the position of the parallel robot effectively, the FSM control system is proposed in this section. By defining the sliding surface as the input variable of fuzzy rules, the number of fuzzy rules for FSMC is smaller than that of conventional fuzzy control, which usually uses the error and the change-of-error as the input variables.

Structure of fuzzy sliding mode control system is shown in Fig.3.

![Figure 3. Structure of fuzzy sliding mode control system](image)

Global sliding plane is defined as

\[ s = \dot{e} + ce \]  \hspace{1cm} (34)

where \( c \) is the constant matrix, \( c > 0 \); \( e \) is the position tracking error.
Suppose sliding mode plane $s=0$, the below equation is obtained.

$$ce + \dot{e} = 0$$

(35)

The control compensation of sliding mode may be defined as

$$u(t) = \tau - K \text{sign}(s)$$

(36)

where $K$ is the coefficient matrix, $K > M^{-1}(\Delta M + \Delta q \dot{q} + \Delta \tau)$. Lyapunov function is defined as

$$V = \frac{1}{2s^2}$$

(37)

The below equation is obtained by equation (1), (7) and (8).

$$V = s\dot{s} \leq -K(t)|s|<0$$

(38)

Under the new control law, sliding mode exists and may be attained. Sliding mode switches at the sliding mode plane $s=0$, which brings the strong oscillation.

In the sliding mode control law, the switching gain $K(t)$, which is used to compensate the uncertainty, easily arises the chattering. For reducing the chattering, $K(t)$ should be varied with the time.

Fuzzy rules may be expressed as:

- If $s>0$, $K(t)$ should be increased; If $s<0$, $K(t)$ should be decreased.

In the fuzzy system, $s$ is chosen as the input; $\Delta K(t)$ which is the varying value of $K(t)$, is chosen as the output. Fuzzy sets of input and output of system are defined as:

$$s = \{NB, NM, ZO, PM, PB\}; \Delta K = \{NB, NM, ZO, PM, PB\}$$

where $NB$ denotes negative big; $NM$ denotes negative middle; $ZO$ denotes zero; $PM$ denotes positive middle; $PB$ denotes positive big.

The triangular functions are used to define the membership functions. The defuzzification of the control output is accomplished by the method of center-of-gravity. Distribution of fuzzy membership function is shown in Fig.4.

![Fuzzy input and output](image)

Figure 4. Distribution of fuzzy membership function

Fuzzy rules are defined as

- Rule1: If $s$ is $PB$ Then $\Delta K(t)$ is $PB$;
- Rule2: If $s$ is $PM$ Then $\Delta K(t)$ is $PM$;
- Rule3: If $s$ is $ZO$ Then $\Delta K(t)$ is $ZO$;
- Rule4: If $s$ is $NM$ Then $\Delta K(t)$ is $NM$;
- Rule5: If $s$ is $NB$ Then $\Delta K(t)$ is $NB$.

By the integral method, the upper limit of $K(t)$ can be defined as

$$\dot{K}(t) = G \int \Delta K dt$$

(39)

where $G$ is the proportional coefficient, $G > 0$.

Substituting $\dot{K}(t)$ for $K(t)$ in equation (36), control law can be rewritten as

$$u_1 = -\dot{K} \times \text{sign}(s)$$

(40)

V. FUZZY SUPPORT VECTOR MACHINES CONTROLLER

A. Structure of FSVM controller

Structure of FSVM controller is shown in Fig.5. Inputs of FSVM system are $\{q, \dot{q}\}$; the output compensation of FSVM system for uncertainty is $u_x$; $q_d$ is the desired positions of two joints; $q$ is the real positions of two joints; $e$ is the position error of two joints; $\dot{e}$ is the varying rate of position error of two joints.

![Structure of FSVM control system](image)

Figure 5. Structure of FSVM control system

Inputs and outputs of control system are fuzzified; $\{e, \dot{e}, u_2\}$ are fuzzified respectively as $\{E, \dot{E}, U_2\}$; their fuzzy subsets are $\{NB, NM, NS, Z, PS, PM, PB\}$, which respectively denotes $\{negative \ big, negative \ middle, negative \ small, zero, positive \ small, positive \ middle, positive \ big\}$. Quantified grades of them are $\{-6, -5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5, 6\}$. Triangle distribution function is selected as their membership function. According to varying range of inputs and outputs of control system, the proportional coefficients of fuzzy control are $K_e, K_{\dot{e}}$ and $K_{u_2}$ respectively. Decision process of fuzzy control is described by the three-layer SVM model.

1) Input layer

The function of input layer is that input variables are fuzzified as the input of control system $x$.

$$\begin{align*}
E &= K_e e \\
\dot{E} &= K_{\dot{e}} \dot{e} \\
x &= (E, \dot{E})
\end{align*}$$

(41)

2) Hidden layer

The function of inner layer is to realize the kernel computation of four-dimension input $x$ and SVM. The radial basis function kernel function is expressed as

$$K(x, x_i) = \exp(-|x - x_i|^2 / 2\sigma^2)$$

(42)

where $\sigma$ is the kernel width, which reflects the radius of close boundary.

3) Output layer

The function of output layer is to obtain the real input control value of the controlled object by computing SVM regression value.
\[
\begin{align*}
    y(x_i) &= \sum_{i=1}^{N} a_i K(x_i,x_j) + b \quad i = 1,2 \\
    u_i &= K_{ij}y(x_i)
\end{align*}
\]  

The controller parameters are optimized by the hybrid learning algorithm. First, least square algorithm is used for off-line optimize the parameters of support vector machines. Then, improved genetic algorithm is used for on-line optimizing the parameters of support vector machines and fuzzy proportional coefficients.

**B. Parameter optimization of control system**

The parameters of affecting SVM performance are number of training sample set \(D\), penalty coefficient \(\gamma\), kernel width \(\sigma\) and insensitive coefficient \(\varepsilon\), and so on. The system performance is also affected by fuzzy proportional relations between real values and fuzzy values in decision-making process of control system. Only when these parameters are in finite range, the system has the better control performance. Optimal parameter combination varies with the object.

1) **Off-line optimization of \(\gamma\) and \(\sigma\)**

Because \(\varepsilon\) may be preset by the noise, which reflects the prediction of data noise level by support vector machines, least square algorithm was only used to off-line optimize \(\gamma\) and \(\sigma\).

First, the method of rising exponent was used to search the proper \(\gamma\) set and \(\sigma\) set. For example, \(\gamma = 2^{-4}, 2^{-2}, \ldots, 2^{10}\); \(\sigma = 2^{-10}, 2^{-8}, \ldots, 2^{-2}\).

Second, using the method of net search, the parameter combination(\(\gamma, \sigma\)) was selected to verify it crossly. The training sample set was divided into \(S\) groups \((G1, G2, \ldots, Gs)\). Selecting randomly \(S-1\) groups as training set, and another as verifying set; generalization capability was evaluated with the following equation.

\[
P = \sum_{i=1}^{S} \sum_{x \in S_i} (y(x) - y(x^N | x_i))^2
\]  

where \(G_i\) is \(i\)th group verifying set; \(y^N\) is the sample of verifying set; \(x_i\) is the parameter vector \([a,b]\) where \(D - G_i\) was set as train sample; \(y(x|\theta)\) is the output of SVM system.

Final, the parameter combination was selected circularly to verify it crossly and the performance index \(P\) was computed until the optimal parameter combination (\(\gamma, \sigma\)) is obtained.

2) **On-line optimization of fuzzy SVM parameters**

**(a) Encoding**

Because of the complexity and continuity of optimizing process of SVM parameters, the coding method of float number is used to avoid the effect of binary coding on the evaluation of algorithm performance and computing precision.

**(b) Selection of fitness function**

In improved genetic algorithm, the individual evolution is decided by individual fitness value. Thus the individual fitness value need be computed. The individual is sequenced by fitness value and sequenced population is lined out by the upper limit and lower limit. Fitness function is used to evaluate SVM individual and fitness function designed influences directly the performance of genetic algorithm. According to the feature of robot system, fitness function was described by the sum of error among given system input and real output. It was expressed as

\[
F_i = E_{i_{\text{max}}} - \sum_{k=1}^{M} |S_i(k) - T_i(k)|
\]  

where \(i = 1,2,\ldots, N\) is the number of individual in population; \(k\) is the number of individual variable.

Mean error \(E_{\text{MSE}}\) of system track was expressed as

\[
E_{\text{MSE}} = \frac{\sum_{i=1}^{N} (T_i - f(x))^2}{N_i}
\]  

**(c) Genetic operation**

Genetic operations include selection, crossover and mutation. Its objective is to substitute the new generation population into next generation population. The procedure of operation is given as follows:

- **Step1:** Generation of initial population;
- **Step2:** Re-evaluation and adding age;
- **Step3:** Selection of parents: prior selection of elder individuals;
- **Step4:** Crossover and mutation: generation of new individuals;
- **Step5:** Evaluation: evaluation of new individuals;
- **Step6:** Natural selection: selection considering the diverseness of individuals;
- **Step7:** Steps 2 to 6 are repeated until the convergence is achieved.

In genetic operation, set population is 200; set crossover probability is 0.75; set mutation probability is 0.02. Each parameter was set by following:

\[
D \in [1,512], \quad \gamma \in \left[\frac{1}{256}, \frac{255}{256}\right], \quad \sigma \in \left[\frac{1}{128}, \frac{127}{128}\right], \quad \varepsilon \in \left[\frac{1}{64}, \frac{63}{64}\right], \quad K_e \in \left[\frac{1}{16}, \frac{15}{16}\right], \quad K_s \in \left[\frac{1}{256}, \frac{255}{256}\right],
\]

where \(D, \gamma, \sigma\) and \(\varepsilon\) are coded respectively by 8 bit (8 bit integer), 9 bit (9 bit integer), 14 bit (8 bit integer, 6 bit decimal), 16 bit (8 bit integer, 8 bit decimal) binary strings; fuzzy proportional coefficients \(K_e, K_s, K_a\) are coded respectively by 12 bit (8 bit integer, 4 bit decimal), 10 bit (4 bit integer, 6 bit decimal), 11 bit (4 bit integer, 7 bit decimal) binary strings. Thus, they are coded by 72 bit binary strings and their values are discrete. Their units are 1, 1/256, 1/64, 1/32, 1/16, 1/64, 1/128 respectively. After, the individual fitness function is computed using these parameters, the individuals in the new population are selected by the desired value.

VI. SIMULATION AND APPLICATION

To verify the effectiveness of the presented control strategy for the parallel robot, the comparative simulation experimental researches were carried out between the
designed control strategy and the fuzzy sliding mode control strategies using experimental simulation. In simulation experiment of control performance, the mobile platform is driven by six asymmetric cylinders with a cylinder diameter of 85mm and a rod diameter of 64mm, and a full stroke of 840mm, which are controlled by six servo-electromotors. The installed sensors measure the leg lengths and forces between the centroid point of rods and the heads of the cylinders. The radius of the base platform and the mobile platform are 1250 and 540mm respectively. The simulation experiments of parallel robot were conducted by the simulation software. In the simulation experiments, the experimental values (100, 15, 1.0, 0.01, 100, 1.0, 0.1) are set as the initial values of control parameter combination \((D, \gamma, \alpha, \varepsilon, K_e, K_r, K_u)\), trace error \(EMSE\) is 2.134; after hybrid optimization, optimal parameter combination \((D, \gamma, \alpha, \varepsilon, K_e, K_r, K_u)\) is \((1.6, 3.2, 0.2, 1.5, 65, 0.4, 0.07)\), trace error \(EMSE\) is 0.014. The experiments concerned position tracking of centroid point of mobile platform for the following reference trajectories \(q_d(t) = 1.0 + 0.20 \sin(2\pi t)\) mm and \(q_d(t) = 1.0 + 0.40 \sin(2\pi t)\) mm by the designed control system. The experimental results are shown in Fig.6 and Fig.7.

It can be seen that the designed control system performs much better than the fuzzy sliding mode control methods from Fig.6, Fig.7, Fig.8 and Fig.9. It can be obtained that position tracking error of centroid point of mobile platform for the following reference trajectory \(q_d(t) = 1.0 + 0.20 \sin(2\pi t)\) mm is smaller than that for the following reference trajectory \(q_d(t) = 1.0 + 0.40 \sin(2\pi t)\) mm by the designed control system. From above figures, using designed controller, track error is low. Considering uncertainty and complexity of the system, the track error may be permitted.

The experiments concern the position tracking error of centroid point of mobile platform for the following reference trajectories \(q_d(t) = 1.0 + 0.20 \sin(2\pi t)\) mm and \(q_d(t) = 1.0 + 0.40 \sin(2\pi t)\) mm by the designed control system. The experimental results are shown in Fig.8 and Fig.9.

The experiments concern the velocity tracking of centroid point of mobile platform for the following reference trajectories \(q_d(t) = 1.0 + 0.20 \sin(2\pi t)\) mm and \(q_d(t) = 1.0 + 0.40 \sin(2\pi t)\) mm by the designed control system. The experimental results are shown in Fig.10 and Fig.11.
The experiments concern the velocity tracking error of the following reference trajectories \( q_d(t) = 1.0 + 0.20 \sin(2\pi t) \) mm and \( q_d(t) = 1.0 + 0.40 \sin(2\pi t) \) mm by the designed control system. The experimental results are shown in Fig.12 and Fig.13. It can be seen that the control input of legs for the following reference trajectory \( q_d(t) = 1.0 + 0.40 \sin(2\pi t) \) mm is different from that for the following reference trajectory \( q_d(t) = 1.0 + 0.20 \sin(2\pi t) \) mm by the designed control system.

It can be seen that the designed control system performs much better than the fuzzy sliding mode control methods in the velocity tracking from Fig.10, Fig.11, Fig.12 and Fig.13. It can be obtained that velocity tracking error of centroid point of mobile platform for the following reference trajectory \( q_d(t) = 1.0 + 0.40 \sin(2\pi t) \) mm is smaller than that for the following reference trajectory \( q_d(t) = 1.0 + 0.20 \sin(2\pi t) \) mm by the designed control system.

In this paper, the dynamics equation of 6-DOF parallel robot was established. According to the dynamics equation, a fuzzy support vector machines control strategy based on the sliding mode control was proposed. The proposed controller consists of a fuzzy sliding mode controller and a fuzzy support vector machines controller. The compensation controller is decided by comparing the switching function with the thickness of boundary layer. Using improved GA and FL algorithm to optimize the performance parameters of support vector machines and...
the fuzzy proportional parameters, a better control system was obtained. The system uncertainty and external disturbance was compensated. Experimental simulation was carried out with 6-DOF parallel robot to investigate the effectiveness of the proposed control method. The simulation results show that the control method designed gets tracking effect with high precision and speed, as well as reduces the chattering under the condition of existing model error and external disturbance.

REFERENCES


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