Mode III Stress Singularity Analysis of Isotropic and Orthotropic Bi-material near the Interface End

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Abstract—The stress singularity near the Mode III interface end of isotropic and orthotropic bi-material was studied. Based on arbitrary angle boundary conditions, by solving a class of harmonic equations we discussed the root of the characteristic equation with the help of complex function method of material fracture. The expression and variation of the singularity index $\lambda$ of the tip of interfacial crack and the symmetric interface end is obtained as well. The analytical expressions of the stress and displacements fields and the stress intensity factor around the interfacial crack are derived.

Index Terms—isotropic and orthotropic, bi-material, Mode III interface end, symmetric, asymmetric, singularity index, stress field

I. INTRODUCTION

Several methods of singularity analysis are available for the class of problems under consideration. The interfacial crack of an isotropic bi-material was studied, For Mode I, II, the stress field has oscillatory singularity. For Mode III the stress has singularity of $r^{-3/2}$ around the tip of interfacial crack without oscillatory [1]. An isotropic bi-material interface end was studied by the Mellin transform method and the stress field around the interface end shows the singularity [2]. However, based on the elastic equations and series expansion, the paper presents the general solution of anti-plane interface end. The research showed the relationship of singularity index and interface end angular [3]. A technique, applied to the bi-material wedge by Bogy [4] and others, employs a Mellin transform in the literature this last method is the only one to provide conditions for a logarithmic stress singularity. Used separation of variables on the Airy stress function, the usual determinant conditions for singularities of the form $O(r^{-\lambda})$ as $r \to 0$ are established and further conditions are derived for singularities of the form $O(r^{-4} \ln r)$ as $r \to 0$. The order of the determinant involved in these conditions depends upon the number of materials comprising the wedge. Two systematic methods of expanding the determinant for the $N$-material wedge are presented [5]. Orthotropic bi-materials anti-plane interface end of flap lap was studied by constructing new stress function and using composite complex function method of material fracture. The expression of stress fields, displacements fields and stress intensity factor around flap lap interface end are derived by solving a class of generalized bi-harmonic equations [6]. Two systems of non-homogeneous linear equations with 8 unknowns are obtained. By solving the above systems of non-homogeneous linear equations, the two real stress singularity exponents can be determined when the double material parameters meet certain conditions. The expression of the stress function and all coefficients are obtained based on the uniqueness theorem of limit [7].

In this article, by enjoying the composite complex function method of material fracture, the singularity index $\lambda$ of the tip of interfacial crack and the symmetric interface end were determined as well. The analytical expressions of the stress and displacement fields and the stress intensity factor around the interfacial crack are derived by translating the governing equation into the generalized harmonic equations. The corresponding results of asymmetric interface end can be obtained similarly. This paper is organized as follows. Section II reviews the Mechanical model. Section III gives the Stress function and then Section IV discusses the symmetric and asymmetric interface end, the analytical expressions of the stress and displacement fields are derived. Section V presents the conclusions.

II. MECHANICAL MODEL

An isotropic and orthotropic composite bi-material Mode III interface end with arbitrary angle is shown in Fig. 1. The plane is bonded at $x > 0$ and $y = 0$. The part of the plane with $y > 0$ is an isotropic material. Its material parameters are $(G_{23})_1 = (G_{31})_1 = \mu$. The part of the plane with $y < 0$ is an orthotropic material. The material parameters are $(G_{23})_2$ and $(G_{31})_2$. © 2011 ACADEMY PUBLISHER
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From the theory of elasticity, the governing equations can be expressed as

\[
\begin{align*}
\frac{\partial^2 w_1}{\partial x^2} + \frac{\partial^2 w_1}{\partial y^2} &= 0 \\
(Q_{ss})_2 \frac{\partial^2 w_2}{\partial x^2} + (Q_{ss})_2 \frac{\partial^2 w_2}{\partial y^2} &= 0
\end{align*}
\]

(1)

Where \( w_j \) \( (j = 1, 2) \) are the displacements. The expressions of boundary conditions for polar coordinates are

\[
\begin{align*}
\theta &= 0, \quad (r_{\theta_1})_1 = (r_{\theta_2})_1, w_1 = w_2 \\
\theta &= \theta_1, \quad (r_{\theta_2})_1 = 0 \\
\theta &= \theta_2, \quad (r_{\theta_2})_2 = 0
\end{align*}
\]

(2)

Based on the theory of elasticity, the formulae of stress components are given as

\[
\begin{align*}
(r_{\alpha_1})_1 &= \mu \cdot \frac{\partial w_1}{\partial r}, (r_{\alpha_2})_1 = (Q_{ss})_2 \cdot \frac{\partial w_1}{\partial r} \\
(r_{\alpha_2})_1 &= \mu \cdot \frac{\partial w_1}{r \partial \theta}, (r_{\alpha_2})_2 = (Q_{ss})_2 \cdot \frac{\partial w_2}{r \partial \theta}
\end{align*}
\]

(3)

Where the displacements \( u_j = v_j = 0, w_j = w_j(x, y) \) \( (j = 1, 2) \), \( r \) and \( \theta \) are polar coordinates defined in Fig. I. \( (G_{ss})_1 = (G_{ss})_1 = \mu \cdot (Q_{ss})_2 = (Q_{ss})_2 = (G_{ss})_2 \cdot \mu \cdot (G_{ss})_2 \) and \( (G_{ss})_2 \) are shear moduli.

### III. STRESS FUNCTION

Assume the displacement \( w_j = w_j(x+s_j y), (j = 1, 2) \), where \( w_j = w_j(x+s_j y) \) is an arbitrary complex function for a given \( j \). Substituting \( w_j = w_j(x+s_j y) \) into (1), we can obtain the characteristic equations

\[
\begin{align*}
1 + s_j^2 &= 0 \\
(Q_{ss})_2 + (Q_{ss})_2 \cdot s_j^2 &= 0
\end{align*}
\]

(4)

Equation (4) has a couple of conjugation imaginary roots. Choose the roots of the imaginary part to be greater than zero as follows:

\[
s_j = i \beta_j, (j = 1, 2), \quad \beta_1 = 1, \beta_2 = \sqrt{(Q_{ss})_2 /(Q_{ss})_2}
\]

(5)

By the complex function method in the \( z_j \) plane, the governing equations can be translating into the harmonic equations

\[
\begin{align*}
\nabla^2 w_1 &= \left( \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right) w_1 = 0 \\
\nabla^2 w_2 &= \left( \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right) w_2 = 0
\end{align*}
\]

(6)

From (1), the special stress functions are chosen as

\[
w_j(r, \theta) = \frac{1}{(\lambda + 1)\lambda} \left( \frac{z_j^{*\alpha_1} + z_j^{*\alpha_1}}{2} \cdot a_j + \frac{z_j^{*\alpha_1} - z_j^{*\alpha_1}}{2i} \cdot b_j \right) \quad (j = 1, 2)
\]

(7)

Where

\[
\begin{align*}
(z_j^{*\alpha_1})_1 &= x + s_j y = r(\cos \theta + s_j \sin \theta) \\
(z_j^{*\alpha_1})_2 &= x + s_j y = r(\cos \theta + s_j \sin \theta)
\end{align*}
\]

(8)

Let \( w_j(r, \theta) \) in (7) satisfy (1). Substituting (7) and (8) into (3), we can get

\[
(r_{\alpha_1})_1 = \mu \cdot \frac{\partial w_j}{\partial r} = \mu \cdot \frac{\partial w_j}{r \partial \theta}
\]

(9)

\[
(\cos \theta + s_j \sin \theta)^{*\alpha_1} \cdot a_j + \left( \cos \theta + s_j \sin \theta \right)^{*\alpha_1} \cdot b_j
\]

(10)

\[
(\cos \theta + s_j \sin \theta)^{*\alpha_1}
\]
As (7) and (8) have

\[ \cos \theta + s \sin \theta = r \cos \theta + s \sin \theta, \]

we define \( \cos \theta + s \sin \theta = \rho e^{i\phi} \), where \( \rho = \sqrt{\cos^2 \theta + \beta^2 \sin^2 \theta} \), \( \tan \phi = \beta \tan \theta \) (j = 1, 2). Substituting the boundary conditions into (9), (10) and (11), (12) respectively, we can obtain the fourth-order homogeneous linear system about \((a_1, a_2, b_1, b_2)\) as follows

\[
\begin{aligned}
& a_1 - a_2 = 0 \\
& \mu b_1 -(Q_{uu})_2 \cdot b_1 - b_2 = 0 \\
& - \sin(\lambda + 1) \theta_1 \cdot a_1 + \cos(\lambda + 1) \theta_1 \cdot b_1 = 0 \\
& - \sin \theta_2 \cos \phi \lambda - \cos \phi \lambda \sin \theta_2 \sin \phi \lambda \cdot a_2 + \\
& ( \beta \cos \theta_2 \cos \phi \lambda - \sin \theta_2 \sin \phi \lambda ) \cdot b_2 = 0
\end{aligned}
\]

Simplify (13), we can obtain

\[
\begin{aligned}
& \cos(\lambda + 1) \theta_1 \left[ \sqrt{(Q_{ss})_2 (Q_{uu})_2} \sin \theta_1 \cos \phi \lambda \\
& \quad + (Q_{ss})_2 \cos \theta_2 \sin \phi \lambda \right] \\
& - \sin(\lambda + 1) \theta_1 \left[ \mu \sqrt{(Q_{ss})_2 (Q_{uu})_2} \cos \theta_2 \cos \phi \lambda \\
& \quad \mu \sin \theta_2 \sin \phi \lambda \right] = 0
\end{aligned}
\]

In order to make the homogeneous linear system have non-zero solutions, the determinant of the coefficient must be zero. Form the theory of trigonometric functions, by introducing the angle \( \phi \)

\[
\cos \phi = \frac{Q_{ss}}{\sqrt{(Q_{ss})_2 \cos^2 \theta_2 + (Q_{ss})_2 (Q_{uu})_2 \sin^2 \theta_2}}
\]

We have the characteristic equation as follows:

\[
1 - \sqrt{(Q_{ss})_2 (Q_{uu})_2} \left[ \sin(\lambda \theta_1) \lambda + \theta_1 + \phi \right] - \\
1 + \sqrt{(Q_{ss})_2 (Q_{uu})_2} \left[ \sin(\lambda \theta_1) \lambda + \theta_1 - \phi \right] = 0
\]

A. Interfacial crack

An isotropic and orthotropic composite bi-material Mode III interfacial crack is shown in Fig. 2.

\[
\begin{array}{c}
\text{material-1} \\
\begin{array}{c}
(\mathcal{G}_{32})_1, (\mathcal{G}_{33})_1 \\
(\mathcal{G}_{33})_2, (\mathcal{G}_{33})_2
\end{array}
\end{array}
\]

Figure 2. The model of an isotropic and orthotropic bi-material Mode III interfacial crack

Where \( \theta_1 = \pi, \theta_3 = -\pi, \phi = \arctan(\beta), \tan \theta_1 = -\pi, \phi = -\pi \).

Substituting these into the characteristic equation (16) we have

\[ \sin(2\pi \lambda) = 0 \]

Thus

\[ \lambda = \frac{k}{2}, \quad (k = -1, -2, \cdots) \]

If \( k \) is an even number, \( \lambda = n, (n = -1, -2, \cdots) \) and \( k \) is an odd number, \( \lambda = n - 1/2, (n = 0, -1, -2, \cdots) \) Then, for Mode III the stress has singularity of \( r^{-1/2} \) around the tip of interfacial crack without oscillatory.

B. Symmetric right-angled Interface end

An isotropic and orthotropic composite bi-material Mode III interface end with right angle is shown in Fig. 3.

\[
\begin{array}{c}
\text{material-1} \\
\begin{array}{c}
(\mathcal{G}_{33})_1, (\mathcal{G}_{33})_1 \\
(\mathcal{G}_{33})_2, (\mathcal{G}_{33})_2
\end{array}
\end{array}
\]

Figure 3. The model of an isotropic and orthotropic bi-material Mode III right-angled interface crack
Where \( \theta_1 = \pi/2, \theta_2 = -\pi/2, \phi_1 = \arctan(\beta_1 \tan \theta_2) = -\pi/2 \). Substituting these into the characteristic equation (16) we obtain
\[
\lambda = k. \quad (k = 0, 1, 2, \cdots)
\] (18)
From the analytical expressions of the stress field, we can obtain the stress fields have no singularity in this case.

C. Symmetric convex and concave angled Interface end
An isotropic and orthotropic composite bi-material Mode III with symmetric convex and concave angle are shown in Fig. 4 and Fig. 5.

Transpose and simplify (16), we have
\[
\sin \left[ \left( \theta_1 - \phi_2 \right) \lambda + \theta_1 + \phi \right] = \frac{1 - \mu_i}{1 + \mu_i} \sqrt{(Q_{55})_1 (Q_{44})_2} \quad \text{(19)}
\]
and discuss the following three cases
(a) \( 1 - \mu_i \sqrt{(Q_{55})_1 (Q_{44})_2} > 0 \), considering in one period, the angle \( (\theta_1 + \phi_2) \lambda + \theta_1 + \phi \) and \( (\theta_1 - \phi_2) \lambda + \theta_1 - \phi \) are both in x-axis upper half plane or both in x-axis lower half plane.
(b) \( 1 - \mu_i \sqrt{(Q_{55})_2 (Q_{44})_1} = 0 \cdot \lambda = \frac{k \pi - \theta_1 + \phi}{\theta_1 - \phi_2}, \quad k = 1, 2, \cdots
\)
(c) \( 1 - \mu_i \sqrt{(Q_{55})_2 (Q_{44})_1} < 0 \), considering in one period, the angle \( (\theta_1 + \phi_2) \lambda + \theta_1 + \phi \) and \( (\theta_1 - \phi_2) \lambda + \theta_1 - \phi \) are in x-axis upper half plane and x-axis lower half plane respectively.

We discuss the former case (a) for example

\[
\begin{align*}
0 &< (\theta_1 + \phi_2) \lambda + \theta_1 + \phi < \frac{\pi}{2} \\
0 &< (\theta_1 - \phi_2) \lambda + \theta_1 - \phi < \frac{\pi}{2}
\end{align*}
\]
the solution is \( -1 < \lambda < \frac{\pi}{2 \theta_1} - 1 \)
\[
\begin{align*}
\frac{\pi}{2} &< (\theta_1 + \phi_2) \lambda + \theta_1 + \phi < \pi \\
\frac{\pi}{2} &< (\theta_1 - \phi_2) \lambda + \theta_1 - \phi < \pi
\end{align*}
\]
the solution is \( -1 < \lambda < \frac{\pi}{2 \theta_1} - 1 \)
\[
\begin{align*}
0 &< (\theta_1 + \phi_2) \lambda + \theta_1 + \phi < \frac{\pi}{2} \\
\frac{\pi}{2} &< (\theta_1 - \phi_2) \lambda + \theta_1 - \phi < \pi
\end{align*}
\]
the solution is \( -1 < \lambda < \frac{3 \pi}{4 \theta_1} - 1 \)

Summarization above results, we have obtained
\[
\begin{align*}
0 &< \theta_1 = \theta_2 < \pi/2, \quad \lambda > 0. \\
\pi/2 &< \theta_1 = \theta_2 < \pi, \quad -1 < \lambda < 0.
\end{align*}
\]
Thus, the stress fields of isotropic and orthotropic bi-material Mode III symmetric convex angled interface end has no singularity, case when symmetric concave angled interface end the singularity still exist. From (16), taking two cases into consideration, the variation of the singularity index \( \lambda \) of symmetric concave angled interface end are presented. The results equates with (20).

(1) Case when the symmetric concave angle are \( \theta_1 = \theta_2 = 3 \pi/4 \) (data1), \( \theta_1 = \theta_2 = 4 \pi/5 \) (data2) and \( \theta_1 = \theta_2 = 5 \pi/6 \) (data3) respectively, from Ref.[9], \( \beta_1 = 1.1863 \) is chosen, the variation of singularity index \( \lambda \) with the ratio of material parameters \( \mu_i \sqrt{(Q_{55})_2 (Q_{44})_1} \) is shown in Fig. 6, Fig. 7 and Fig. 8 respectively.

![Figure 4. The model of an isotropic and orthotropic bi-material Mode III convex angled interface crack](image)

![Figure 5. The model of an isotropic and orthotropic bi-material Mode III concave angled interface crack](image)

![Figure 6. Variation chart of \( \lambda \) with \( \mu_i \sqrt{(Q_{55})_2 (Q_{44})_1} \) (data 1)](image)
(2) Based on Ref.[9], case when the ratio of material parameters are

\[ \mu_1 \sqrt{Q_{ss2}}(Q_{44})_2 = 12.1756 \text{(data1)}, \]

\[ \mu_2 \sqrt{Q_{ss2}}(Q_{44})_2 = 14.3610 \text{(data2)}, \]

\[ \mu_3 \sqrt{Q_{ss2}}(Q_{44})_2 = 24.6634 \text{(data3)} \]

respectively, \( \beta_2 = 1.1863 \) is chosen, the variation of singularity index \( \lambda \) with the symmetric concave angle \( \pi/2 < \theta < \pi \) is shown in the Fig. 9, Fig. 10 and Fig. 11.

That is, from Fig. 6, Fig. 7 and Fig. 8, case when the symmetric concave angle is determined, the stress fields singularity of the symmetric concave angled interface end is increscent with the increase of \( \mu_1 \sqrt{Q_{ss2}}(Q_{44})_2 \). From Fig. 9, Fig. 10 and Fig. 11, case when the ratio of material parameters is determined, the stress fields singularity of the symmetric concave angled interface end increases with the concave angle increases.

D. Asymmetric Interface end

From (16), singularity index, stress and displacements field and the stress intensity factor of Mode III isotropic and orthotropic bi-material asymmetric interface end such as the flat lap and others can be obtained similarly.

An isotropic and orthotropic Mode III composite bi-material of the flat lap interface end is shown in Fig. 12.

Figure 7. Variation chart of \( \lambda \) with \( \mu_1 \sqrt{Q_{ss2}}(Q_{44})_2 \) (data 2)

Figure 8. Variation chart of \( \lambda \) with \( \mu_1 \sqrt{Q_{ss2}}(Q_{44})_2 \) (data 3)

Figure 9. Variation chart of \( \lambda \) with \( \mu_1 \sqrt{Q_{ss2}}(Q_{44})_2 \) (data 1)

Figure 10. Variation chart of \( \lambda \) with \( \beta_2 \) (data 2)

Figure 11. Variation chart of \( \lambda \) with \( \beta_2 \) (data 3)

Figure 12. The model of an isotropic and orthotropic bi-material Mode III Flat lap interface crack

Where \( \theta_1 = \pi/2, \theta_2 = \pi \), \( \phi_1 = \arctan(\beta_1 \tan(\theta_2)) = -\pi, \phi = -\pi \)

Substituting these into the characteristic equation (16) we obtain

\[ \lambda = -1 + \frac{2}{\pi} \arcsin \left( \frac{2 \sqrt{Q_{ss2}}(Q_{44})_2 + \mu}{2(\sqrt{Q_{ss2}}(Q_{44})_2 + \mu)} \right) \]

Thus, the variation of singularity index \( \lambda \) with the growth of \( \mu_1 \sqrt{Q_{ss2}}(Q_{44})_2 \) is shown in the Fig. 13.
As is shown in Fig. 13, this kind of problem has one singularity which is power singularity. The singularity index $\lambda$ tends to $-1/2$ as $\mu(2\pi r)^{1/2} \cdot b_1$ increases.

IV. STRESS AND DISPLACEMENT FIELDS AND THE STRESS INTENSITY FACTOR

Taking the case of interfacial crack, the Mode III stress fields of isotropic and orthotropic bi-material are shown as follows, and other cases can be obtained similarly.

From (16), if $k$ is an even number, $b_j = 0$ and $k$ is an odd number, $a_j = 0$.

$$
(\tau_{r\theta})_1 = \mu \cdot \cos \theta \cdot a_1 + \mu \cdot r^{1/2} \cdot \sin \frac{\theta}{2} \cdot b_1
$$

$$
(\tau_{r\theta})_2 = (Q_{ss})_2 \cdot \rho_2 \cos \phi_2 \cdot a_2 + (Q_{ss})_2 \cdot r^{1/2} \cdot \rho_2^{1/2} \cdot \sin \frac{\phi_2}{2} \cdot b_2
$$

$$
(\tau_{r\phi})_1 = -\mu \cdot \sin \theta \cdot a_1 + \mu \cdot r^{1/2} \cdot \cos \frac{\theta}{2} \cdot b_1
$$

$$
(\tau_{r\phi})_2 = -(Q_{ss})_2 \cdot \sin \theta \cdot a_2 + (Q_{ss})_2 \cdot \rho_2^{1/2} \cdot \left[ \frac{\beta_2 + 1}{2} \cos \left( \theta - \frac{\phi_2}{2} \right) + \frac{\beta_2 - 1}{2} \cos \left( \theta + \frac{\phi_2}{2} \right) \right] \cdot b_2
$$

The displacement fields are shown as

$$
\begin{align*}
    w_1(r, \theta) &= r \cdot \cos \theta \cdot a_1 + 2r^{1/2} \sin \frac{\theta}{2} \cdot b_1 \\
    w_2(r, \theta) &= r \cdot \rho_2 \cos \phi_2 \cdot a_2 + 2r^{1/2} \cdot \rho_2^{1/2} \sin \frac{\phi_2}{2} \cdot b_2
\end{align*}
$$

Define the stress intensity factor,

$$
K_{III}^j = \lim_{r \to 0} \sqrt{2\pi r} (\tau_{r\theta})_j
$$

$$
= \begin{cases} 
\mu \cdot (2\pi)^{1/2} \cdot b_1, & j = 1 \\
(Q_{ss})_2 \cdot (2\pi r)^{1/2} \cdot b_2, & j = 2 
\end{cases}
$$

$$
(\tau_{r\phi})_1 = \frac{K_{III}}{2\pi r} \cdot \sin \frac{\theta}{2}
$$

$$
(\tau_{r\phi})_2 = \frac{K_{III}^2}{2\pi r} \cdot \sin \frac{\phi_2}{2}
$$

$$
(\tau_{r\theta})_1 = \frac{K_{III}}{2\pi r} \cdot \cos \frac{\theta}{2}
$$

$$
(\tau_{r\theta})_2 = \frac{K_{III}^2}{2\pi r} \cdot \cos \frac{\phi_2}{2}
$$

$$
\begin{align*}
    w_1(r, \theta) &= \frac{2K_{III}}{\mu} \sqrt{\frac{r}{2\pi}} \sin \frac{\theta}{2} \\
    w_2(r, \theta) &= \frac{2K_{III}^2}{(Q_{ss})_2} \sqrt{\frac{r}{2\pi}} \sin \frac{\phi_2}{2}
\end{align*}
$$

V. CONCLUSIONS

By the complex function method in the $z_j$ plane, the stress Singularity near the Mode III symmetric and asymmetric interface end of isotropic and orthotropic bi-material is studied. The results are as follows:(1)The stress has singularity of $r^{-1/2}$ around the tip of interfacial crack without oscillatory.(2)The singularity index $\lambda$ of symmetric right-angled and convex-angled interface end is greater than zero, so the stress fields have no singularity in this case.(3) The value range of singularity index $\lambda$ of symmetric concave angled interface end is from 0 to -1. (4)The corresponding conclusions of asymmetric interface end such as the flat lap and others can be obtained similarly.(5) Taking the case of interfacial crack, the analytical expressions of the stress and displacements fields and the stress intensity factor are derived.

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