Newman-Watts Particle Swarm Optimization with Group Decision

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Abstract—Particle swarm optimization (PSO) is a novel swarm intelligent algorithm inspired by fish schooling and birds flocking. Due to the complex nature of engineering optimization tasks, the standard version can not always meet the optimization requirements. Therefore, in this paper, a new group decision mechanism is introduced to PSO to enhance the escaping capability from local optimum. Furthermore, a Watts Strogatz small-world model is incorporated into PSO to increase the population diversity. Seven famous numerical benchmarks are used to testify the new algorithm. Simulation results show it achieves the best performance when compared with three other variants of particle swarm optimization especially for multi-modal problems.

Index Terms—social parameter, particle swarm optimization, non-linear Manner, exponential curve

I. INTRODUCTION

Inded Particle swarm optimization (PSO) [1][2] is a novel population-based swarm intelligent search methodology. It simulates the animal social behaviors such as birds flocking, fish schooling, and insects herding. Due to its simple concept and fast convergent speed, it has been widely applied to many areas such as Diophantine equation solver [3], reactive power optimization[4], layout optimization of satellite module [5], robust decoupling control[6], nonlinear system identification[7], magnetoencephalography [8], inference of genetic regulatory networks[9][10].

In PSO methodology, each individual (called particle) represents a potential solution, flies across the problem space to search food. In each iteration, each particle maintains three different kinds of information: current position, the velocity, and the best memory remembered by itself. The entire swarm can remember the best location with the most food according to the cooperation and competition among particles.

Suppose \( x^k_j(t) = (x_1^k_j(t), x_2^k_j(t), \ldots, x_n^k_j(t)) \) is the position of \( j^{th} \) particle at generation \( t \), then it will fly with the following manner:

\[
x^k_j(t + 1) = x^k_j(t) + v^k_j(t + 1), \quad k = 1, 2, \ldots, n
\]

Where the symbol \( v^k_j(t + 1) = (v_1^k_j(t + 1), v_2^k_j(t + 1), \ldots, v_n^k_j(t + 1)) \) represent the velocity of particle \( j \) at time \( t + 1 \), and is updated by:

\[
v^k_j(t + 1) = \omega v^k_j(t) + c_1 r_1 (p^k_j(t) - x^k_j(t)) + c_2 r_2 (p^g_j(t) - x^k_j(t)) \tag{2}
\]

Where \( p^k_j(t) = (p^1_j(t), p^2_j(t), \ldots, p^n_j(t)) \) is the location with the most food resource particle \( j \) has been visited, while \( p^g_j(t) = (p^1_g(t), p^2_g(t), \ldots, p^n_g(t)) \) denotes the best position found by the entire swarm. Inertia weight \( \omega \) is a positive number within 0 and 1. Cognitive learning factor \( c_1 \) and social learning factor \( c_2 \) are known as accelerator coefficients, \( r_1 \) and \( r_2 \) are two random numbers generated with uniform distribution within \((0, 1)\). To keep the stability of particle swarm optimization, a predefined velocity threshold \( v_{\text{max}} \) is used to limit the moving size by:

\[
|v^k_j(t + 1)| \leq v_{\text{max}} \tag{3}
\]

Due to its easy concepts and fast convergent speed, any variants have been proposed aiming to increase the convergence speed and the efficiency. Hybrid with mutation, cross-over and root mean square variants, Ramana Murthy et al. [11] proposed a hybrid particle swarm optimization to solve optimisation problems. With the same method, Monson [12] combined Kalman filter with PSO to improve the information utilization ratio. Parameter selection is an important problem to affect the performance. To emphasize the exploration in the first stage, Ratnaweera [13] introduced a linear manner for both cognitive and social learning factors. Furthermore, Cai [14] predefined a line guiding the choice of these two accelerator coefficients. There are still some other references on PSO [15][16][17], due to the page limitation, more details can be found in corresponding. There are still many other variants, due to the page limitation, please refer the corresponding references [18].

As we known, the animal social behaviors are complex systems, and the communications are changed dynamically. Therefore, the fixed communication topology and decision capability do not describe the natural phenomenon, and will affect the performance significantly. Therefore, in this paper, a new variant of PSO, group-decided Watts-Strogatz particle swarm...
optimization (GWSPSO, in briefly), is designed, in which Watts Strogatz small-world model is incorporated into the methodology of PSO to simulate the dynamically changed communication, and group-decided mechanism is used to enhance the intelligent ability for each particle. Simulation results show it is more effective.

The rest of this paper was organized as follows: In section 2, the details of group-decided Watts-Strogatz particle swarm optimization are illustrated. In section 3, several famous benchmarks are used to testify the new algorithm. Simulation results show the proposed strategy outperforms three other previous variants of PSO significantly.

II. NEWMAN-WATTS PSO WITH GROUP DECISION

In this paper, only the unconstrained minimization problems are considered:

$$\min f(x), \quad x \in [U, D]^{n} \subseteq R^{n}$$  (4)

A. Group Decision Mechanism

For the standard version of particle swarm optimization, each particle is attracted by the historical best positions ($p_j(t)$ and $p_g(t)$) found by the neighborhood and itself, however, with this manner, other best positions found other particles in its neighborhood are neglected. Therefore, some useful information inside this positions are lost. Generally, a more precise model may provide a fast convergent speed. In this subsection, a group-decided method is designed to collect the useful information among all best positions.

In GWSPSO, we designed a new index by introducing the potential performance tendency, and the definition was provided as follows:

$$\text{Grade}_{j(t)} = \frac{F_{j(t)}}{\sum_{k=1}^{n} F_{k(t)}}$$  (5)

where $F_{j(t)}$ represents the convergent tendency of particle $j$ at time $t$, and is defined as

$$F_{j(t)} = \frac{f(x_j(t)) - f(x_j(t-1))}{\|x_j(t) - x_j(t-1)\|}$$  (6)

where $f(x_j(t))$ denotes the objective fitness value of particle $j$ for the position $x_j(t)$, $\|x_j(t) - x_j(t-1)\|$ represents the distance between positions $x_j(t)$ and $x_j(t-1)$.

As we known, if the fitness value of particle $u$ is better than which of particle $m$, the probability that global optima falls into $u$‘s neighborhood is larger than that of particle $m$. In this manner, the particle $u$ should pay more attentions to exploit its neighborhood. On the contrary, it may tend to explore other region with a larger probability than exploitation. Then, the group-decided position of particle $j$ with all best locations found by all particles in its neighborhood is showed as follows:

$$S_{j(t)} = \sum_{k=1}^{\text{neigh}(j(t))} \text{Grade}_{j(t)} \cdot p_k(t)$$  (7)

After determine the position $S_{j(t)}$, the velocity update equation is changed by

$$v_j^{(t+1)} = \omega v_j^{(t)} + c_1 r_1 (p_j^{(t)} - x_j^{(t)}) + c_2 r_2 (S_{j(t)} - x_j^{(t)})$$  (8)

B. PDF Creation

In 1998, Watts and Strogatz introduced a single-parameter small-world network model that bridges the gap between a regular network and a random graph [19]. Lately, Newman and Watts improved the original WS model [20], most of the recent works on small-world models were performed using the Newman and Watts (NW) model.

The NW model, instead of rewiring links between nodes, extra links called shortcuts are added between pairs of nodes chosen at random, but no links are removed from the existing network. Obviously, the NW model reduces to the originally nearest-neighbor coupled network for $p = 0$; while it becomes a globally coupled network for $p = 1$; reversely, the NW model is equivalent to the WS model for sufficiently small $p$ and large $N$.

The WS and NW models show a transition with an increasing number of nodes from “arge-world”regime in which the average distance between two nodes increases linearly with the system size, to a “mall-world”one in which it increases only logarithmically.

C. Threshold Selection

With new velocity update equation (8), due to the unexpected movement motion, the population diversity increases significantly while the convergent speed is decreased. Therefore, a predefined threshold $p_E$ should be needed to control the population diversity as follows:

If $t \leq \text{Generation} \cdot p_E$, the particle should obey the update (8) to escape from local optimum; otherwise, the particle should move with (2) to convergent as soon as possible.

In this paper, $p_E$ is set to 0.8. With this manner, the first 80% iteration will make an exploration, while the last 20% eration used to make the exploitation. This value is only coming from experiments.

D. The Steps of GWSPSO

The detail steps of GWSPSO are listed as follows:

Step1. Initializing each coordinate $x_j^0$ and $v_j^0$ sampling within $[x_{min}, x_{max}]$ and $[-v_{max}, v_{max}]$, respectively.

Step2. Computing the fitness value of each particle.
Step 3. For k-th dimensional value of j-th particle, the personal historical best position $p^k_j$ is updated as follows.

$$p^k_j = \begin{cases} x^k_j, & \text{if } f(x^k_j) < f(p^k_j), \\ p^k_j, & \text{otherwise.} \end{cases}$$

(9)

Step 4. For k-th dimensional value of j-th particle, the global best position $p^k_j$ is updated as follows.

$$p^k_j = \begin{cases} p^k_j, & \text{if } f(p^g_j) < f(p^k_j), \\ p^g_j, & \text{otherwise.} \end{cases}$$

(10)

Step 5. If $t \leq \text{Generation} \cdot p_E$, Updating the velocity and position vectors with equation (8) and (1); otherwise, updating the information with Eq. (1) and (2).

Step 6. If the criteria is satisfied, output the best solution; otherwise, goto step 2.

III. SIMULATION RESULTS

To testify the performance of proposed variant, seven typical unconstraint numerical benchmark functions are chosen to test the performance, and compared with standard PSO (SPSO), modified PSO with time-varying accelerator coefficients (MPSO-TVAC) [21] and Newman-Watts model-based particle swarm optimization (NWPSO) [22]. The corresponding formula can see as follows [23]:

**Rosenbrock:**

$$f_2(x) = -\sum_{i=1}^{n} \left(x_i \sin(\sqrt{|x_i|})\right)$$

while $-30 \leq x_i \leq 30$, and

$$\min(f_2) = f_2(1,...,1) = 0.$$  

**Schwefel 2.26:**

$$f_2(x) = -\sum_{i=1}^{n} \left(x_i \sin(\sqrt{|x_i|})\right),$$

while $-500 \leq x_i \leq 500$, and

$$\min(f_2) = f_2(420.9687,...,420.9687)$$

**Rastrigin:**

$$f_3(x) = \sum_{i=1}^{n} [x_i^2 - 10\cos(2\pi x_i) + 10]$$

while $-5.12 \leq x_i \leq 5.12$, and

$$\min(f_3) = f_3(0,...,0) = 0.$$  

**Ackley:**

$$f_4(x) = -20 \exp\left(-0.2 \sqrt{\frac{1}{n} \sum_{i=1}^{n} x_i^2}\right)$$

$$-\exp\left(\frac{1}{n} \sum_{i=1}^{n} \cos(2\pi x_i)\right) + 20 + e$$

while $-32 \leq x_i \leq 32$, and

$$\min(f_4) = f_4(0,...,0) = 0.$$  

**Griewank:**

$$f_5(x) = \frac{1}{4000} \sum_{i=1}^{n} x_i^2 - \prod_{i=1}^{n} \cos\left(\frac{x_i}{\sqrt{i}}\right) + 1$$

while $-600 \leq x_i \leq 600$, and

$$\min(f_5) = f_5(0,...,0) = 0.$$  

**Penalized Function 1:**

$$f_6(x) = \frac{\pi}{n} \left[10 \sin^2(\pi x_j) + \sum_{i=1}^{n} (y_i - 1)^2 \right]$$

$$[1 + 10 \sin^2(\pi y_{i+1})] + (y_n - 1)^2 \right] + \sum_{i=1}^{n} u(x_i,10,100,4)$$

while $-50 \leq x_i \leq 50$, and

$$\min(f_6) = f_6(1,...,1) = 0.$$  

**Penalized Function 2:**

$$f_7(x) = 0.1 \left[\sin^2(3\pi x_j) + \sum_{i=1}^{n} (x_i - 1)^2 [1 + \sin^2(3\pi y_{i+1})]\right]$$

$$+(x_n - 1)^2 [1 + \sin^2(2\pi y_n)] + \sum_{i=1}^{n} u(x_i,5,100,4)$$

while $-50 \leq x_i \leq 50$, and

$$\min(f_7) = f_7(1,...,1) = 0.$$  

$$u(x_i,a,k,m) = \begin{cases} k(x_i-a)^m, & x_i > a, \\ 0, & -a \leq x_i \leq a, \\ k(-x_i-a)^m, & x_i < -a. \end{cases}$$

$$y_i = 1 + \frac{1}{4} (x_i + 1)$$

The coefficients of SPSO, MPSO-TVAC, NWPSO and GNWPSO are set as follows:

The inertia weight $\omega$ is decreased linearly from 0.9 to 0.4 within SPSO, MPSO-TVAC, NWPSO and GNWPSO, accelerator coefficients $c_1$ and $c_2$ are set to 2.0 within SPSO, NWPSO and GNWPSO, as well as in MPSO-TVAC, $c_1$ decreases from 2.5 to 0.5, while $c_2$ increases from 0.5 to 2.5. Total individuals are 100, and the velocity threshold $v_{\text{max}}$ is set to the upper bound of the domain. The dimensionality is 30, 100 and 300. In each experiment, the simulation run 30 times, while each time the largest iteration is $50 \times$ dimension.

The comparison results of these seven famous benchmarks are listed as Table 1-7, in which Dim.
represents the dimension, Alg. represents the corresponding algorithm, Mean denotes the average mean value, while STD denotes the standard variance. Figures 1 to 7 verify the dynamic behavior and 20 sample points are selected within the same intervals. In these points, average best fitness of historical best position of the swarm of all 30 runs are computed and plotted.

Based on the above analysis, we can draw the following two conclusions:

GNWPSO is the most stable and effective among four the variants of particle swarm optimization. It is especially suit for multi-modal functions with many local optima especially.
Table 1: Comparison Results for Rosenbrock

<table>
<thead>
<tr>
<th>Dim</th>
<th>Alg</th>
<th>Mean</th>
<th>STD</th>
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<tbody>
<tr>
<td>30</td>
<td>SPSO</td>
<td>5.6170E+01</td>
<td>4.3584E+01</td>
</tr>
<tr>
<td></td>
<td>TVAC</td>
<td>3.3392E+01</td>
<td>4.1940E+01</td>
</tr>
<tr>
<td></td>
<td>NWPSO</td>
<td>2.2295E+01</td>
<td>2.1073E+01</td>
</tr>
<tr>
<td></td>
<td>GNWPSO</td>
<td>2.6827E+01</td>
<td>2.6525E+01</td>
</tr>
</tbody>
</table>

| 100 | SPSO    | 4.1064E+02| 1.0594E+02  |
|     | TVAC    | 2.8517E+02| 9.8129E+01  |
|     | NWPSO   | 1.3754E+02| 4.1893E+01  |
|     | GNWPSO  | 1.5259E+02| 1.5776E+01  |

| 300 | SPSO    | 2.3307E+04| 1.9726E+04  |
|     | TVAC    | 1.4921E+03| 3.4571E+02  |
|     | NWPSO   | 4.9359E+02| 1.1364E+02  |
|     | GNWPSO  | 3.7838E+02| 1.0799E+02  |

Table 2: Comparison Results for Schwefel Problem 2.26

<table>
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<th>Dim</th>
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<th>STD</th>
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<tbody>
<tr>
<td>30</td>
<td>SPSO</td>
<td>-6.2376E+03</td>
<td>1.1354E+03</td>
</tr>
<tr>
<td></td>
<td>TVAC</td>
<td>-6.7472E+03</td>
<td>5.7059E+03</td>
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<tr>
<td></td>
<td>NWPSO</td>
<td>-9.3843E+03</td>
<td>5.5667E+03</td>
</tr>
<tr>
<td></td>
<td>GNWPSO</td>
<td>-9.3066E+03</td>
<td>5.6431E+03</td>
</tr>
</tbody>
</table>

| 100 | SPSO    | -1.8147E+04| 2.2012E+03  |
|     | TVAC    | -1.7943E+04| 1.5061E+03  |
|     | NWPSO   | -3.0831E+04| 1.4496E+03  |
|     | GNWPSO  | -3.1802E+04| 1.1102E+03  |

| 300 | SPSO    | -4.6205E+04| 6.0735E+03  |
|     | TVAC    | -5.6873E+04| 3.5125E+03  |
|     | NWPSO   | -2.7609E+04| 1.0864E+03  |
|     | GNWPSO  | -9.6958E+04| 3.4442E+03  |

Table 3: Comparison Results for Rastrigin

<table>
<thead>
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<th>Dim</th>
<th>Alg</th>
<th>Mean</th>
<th>STD</th>
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<tbody>
<tr>
<td>30</td>
<td>SPSO</td>
<td>1.7961E+01</td>
<td>4.2276E+00</td>
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<tr>
<td></td>
<td>TVAC</td>
<td>1.5471E+01</td>
<td>4.2023E+00</td>
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<td></td>
<td>NWPSO</td>
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<td>1.6502E+00</td>
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<tr>
<td></td>
<td>GNWPSO</td>
<td>8.9546E-01</td>
<td>9.6303E-01</td>
</tr>
</tbody>
</table>

| 100 | SPSO    | 9.3679E+01| 9.9635E+00  |
|     | TVAC    | 8.4473E+00| 9.4528E+00  |
|     | NWPSO   | 3.1659E+01| 1.0554E+01  |
|     | GNWPSO  | 8.5558E+00| 4.3039E+00  |

| 300 | SPSO    | 3.5492E+02| 1.9825E+01  |
|     | TVAC    | 2.7093E+02| 3.7639E+01  |
|     | NWPSO   | 1.5720E+02| 3.7422E+01  |
|     | GNWPSO  | 3.4824E+00| 3.4863E+00  |

Table 4: Comparison Results for Ackley

<table>
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<th>Dim</th>
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<th>Mean</th>
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</thead>
<tbody>
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<tr>
<td></td>
<td>TVAC</td>
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<td>3.3711E-06</td>
</tr>
<tr>
<td></td>
<td>NWPSO</td>
<td>4.7962E-14</td>
<td>1.8865E-14</td>
</tr>
<tr>
<td></td>
<td>GNWPSO</td>
<td>5.1692E-14</td>
<td>2.1699E-14</td>
</tr>
</tbody>
</table>

| 100 | SPSO    | 3.3135E-01| 5.0103E-01  |
|     | TVAC    | 4.6924E-01| 1.9178E-01  |
|     | NWPSO   | 7.1672E-09| 5.8386E-09  |
|     | GNWPSO  | 2.5817E-09| 1.9455E-09  |

| 300 | SPSO    | 2.8393E+00| 3.1490E+00  |
|     | TVAC    | 7.6605E-01| 3.1660E-01  |
|     | NWPSO   | 9.8992E-07| 5.3376E-07  |
|     | GNWPSO  | 8.3802E-07| 8.6547E-07  |
IV. Conclusion

Newman-Watts particle swarm optimization with group decision is a new intelligent variant aiming to increase the population diversity. The first improvement is the introduction of group decision mechanism by simulating the animal decision. Generally, animal may adjust its behavior motion according to the different environment, and this mechanism can combine the obtained information, and analyze them to provide the useful information. Therefore, the group decision mechanism may provide a useful search guideline. The second improvement is the introduction of Newman-Watts small-world topology in which a dynamic changeable structure is incorporated to simulate the natural environment. Simulation results show it is effective especially for high dimensional multi-modal problems. Further research topic includes the discrete GNWPSO and applications.

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