Analytical Design of PID Decoupling Control for TITO Processes with Time Delays

Zeng-Rong Hu
Engineering Research institute of USTB, Beijing, China; School of Machinery and Electronic Engineering, Tai Yuan University of Science and Technology, Taiyuan, China; Email:huzengrong@sina.com

Dong-Hai Li
State Key Lab of Power Systems, Dept of Thermal Engineering, Tsinghua University, Beijing, China. Email: lidongh@tsinghua.edu.cn

Jing Wang
Engineering Research institute of USTB, Beijing, China; Email: wangj@nercar.ustb.edu.cn

Feng Xue
Tongliao Power Supply Bureau, East Inner Mongolia Electric Power Company, Tongliao, China Email: tlxuefeng@163.com

Abstract—Based on unit feedback closed-loop control structure, a simple analytical design method of decoupling controller matrix is proposed in terms of idea of coupling matrix for two-input-two-output (TITO) processes with time delays in chemical and industrial practice. By means of powerful robustness of two degree-of-freedom PID Desired Dynamic Equation (DDE) method, PID decoupling controller is analytically designed. And the Monte-Carlo stochastic experiment is introduced to analyze performance robustness of the controller. The most important merit of the proposed method is that for the nominal system the output of each channel can be decoupled entirely. Moreover, the decoupling matrix is simple and easily realized. Finally, illustrative simulation examples are included to demonstrate the remarkable superiority of the proposed method.

Index Terms—TITO, time delays, decoupling, performance robustness, DDE

I. INTRODUCTION

TITO system commonly appears in industrial processes, and many multivariable problems can be treated as series of TITO system problems [1-2]. Due to the presence of coupling between two variables, however, many well-developed closed-loop control methods for single-input-single-output (SISO) system with time delay can hardly be extended to TITO system with time delay [1]. Because of the phase lag caused by time delay, closed-loop system can hardly keep stable even under a fairly small adjustable gain. This problem gets more complex as a result of the internal coupling in multivariable process with time delay. The existence of time delay seriously impacts the stability and leads to bigger overshoot, longer regulation time, and even oscillation and divergence [3].

How to realize the control of TITO processes with time delays is a hot and difficult spot in the field of process control. Decentralized control and decoupling control are the most common control strategies in practice. Chen and Hsu et al. studied the utility of decentralized control structure (also called multi-loop structure) for manipulation of variable decoupling regulation [4][5]. Xue optimized PID parameters using genetic algorithm on the basis of the former research [6]. Lee et al. extended the generalized IMC–PID method from SISO systems to MIMO systems [7]. Although the regulations of control systems were simplified to some extent, the achieving performance criteria are much lower than that of the current methods which adopt decoupling controller. In order to overcome the disadvantages of decentralized control structure and to improve system performance, decoupling control structure is usually the first choice.

Traditional PID controllers can be designed with respect to only one of two cases: tracking fixed value or suppressing disturbance. Hence, it is difficult for traditional PID controller to acquire the optimal control effect. In 1963, Horowitz introduced the concept of two-degree-of-freedom to PID control system. Åström and Panagopoulos [10][11] used a structure of two-degree of freedom PI controller, designed maximum sensitivity of closed-loop system and tuned parameters through optimization. Wang et al. carried out a structure analysis for a kind of nonlinear, robust controller (abbreviated to TC) [12]. They derived an equivalent form of PID in which the control requirements are reflected in controller parameters by designing the coefficients of desired
dynamic equation. They proposed a desired dynamic equation (DDE) method for two-degree-of-freedom PID [13]. The method is independent on accurate mathematical model and it is able to adapt to the unmodeled dynamics of controlled process by online regulation. What is more, it is of strong robustness.

For structure of decoupling controller, design of decoupling controller matrix is very important. Nordfeldt and Hägglund presented a general formula of decoupling controller matrix for given TITO process. In references [2][15][16], derived reversely analytical decoupling controller matrix through proposing a desired diagonal closed-loop response transfer matrix. However, the decoupling controller obtained is of complex form, thus can not be implemented physically. Under the hypothesis that transfer function is a first-order plus dead-time model, Wang et al. proposed a design method for simple decoupling controller matrix whose principal diagonal elements are proportional coefficients. In some special cases, the principal diagonal may degenerate into time delay element and is not proportional coefficients necessarily.

For given TITO process with time delays, a simple design method of decoupling controller matrix is proposed in this paper. The method breaks through the limit that transfer function is a first-order plus dead-time model and generates a simpler and more easily realizable decoupling controller matrix in which two elements are proportional coefficients. A PID decoupling controller is devised analytically by means of the characteristics of strong robustness and independence of accurate model of the Desired Dynamic Equation (DDE) method for the two-degree-of-freedom (2-DOF) PID. And the Monte-Carlo stochastic experiment is introduced to analyze performance robustness of the controllers. The simulation results show that the method not only decouples between each two channels in nominal system completely, but also has a strong robustness.

II. PROBLEM DESCRIPTION

Consider the unit feedback closed-loop control system shown in Figure 1.

![Figure 1. Two-input-two-output PID decoupling control system](image)

where \( r_i \) and \( y_i \) stand for the input and output respectively; \( v_i \) and \( d_i \) respectively denote the control signal and disturbance signal; and \( u_i \) and \( h_i \) respectively denote the output and coefficient of the two-degree-of-freedom PID controller, where \( h_i \neq 0 \).

The decoupled controller matrix should be in the form of \( D=\begin{bmatrix} d_{ij} \end{bmatrix}_{2 \times 2} \), and the model of the controlled plant has the form of \( G=\begin{bmatrix} g_{ij} \end{bmatrix}_{2 \times 2} \).

The procedure of the controller design is as follows:
1) Design decoupling controller matrix \( D \);
2) Tune the controller parameters properly based on DDE;

The goal of the controller design is to satisfy the following requirements:
1) To have good performance of the system, such as adjustment time and overshoot;
2) To possess good performance robustness.

III. PID DECOUPLING CONTROLLER DESIGN AND PERFORMANCE ROBUSTNESS ANALYSIS

A. Decoupling Controller Matrix Design

The transfer function of TITO system with time delay that commonly appears in industrial processes can be identified as:

\[
G(s) = \begin{bmatrix} g_{11}(s) & g_{12}(s) \\ g_{21}(s) & g_{22}(s) \end{bmatrix}
\]

where \( g_{ij} = k_g g_{ij}(s) e^{-\theta_i} \), \( i, j = 1,2 \) of which \( g_{ij}(s) \) is a delay-free, physically proper and stable transfer function, and \( k_g \) denotes the real coefficient of steady-state gain.

The decoupling controller matrix can be written as:

\[
D(s) = \begin{bmatrix} d_{11}(s) & d_{12}(s) \\ d_{21}(s) & d_{22}(s) \end{bmatrix}
\]

In the nominal system, transfer function \( H \) can be obtained through decoupling control as:

\[
H(s) = GD = \begin{bmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{bmatrix} \begin{bmatrix} d_{11} & d_{12} \\ d_{21} & d_{22} \end{bmatrix}
= \begin{bmatrix} g_{11}d_{11} + g_{12}d_{12} \\ g_{21}d_{11} + g_{22}d_{12} \end{bmatrix}
\]

The characteristic of the internal coupling can be found through Equation (3), and the decoupling matrix of the system \( K_{\text{dec}} \) can be defined as follow:

\[
K_{\text{dec}} = \begin{bmatrix} 0 & g_{11}d_{12} + g_{12}d_{22} \\ g_{21}d_{11} + g_{22}d_{21} & 0 \end{bmatrix}
\]

By setting \( K_{\text{dec}} = 0 \), a TITO system can be turned equivalently into two SISO control systems, and the output can be decoupled completely.

From \( K_{\text{dec}} = 0 \), the following two equations are determined:

\[
k_{11}G_{11}e^{-\theta_{11}}d_{12} + k_{12}G_{21}e^{-\theta_{21}}d_{22} = 0 \tag{5}
\]
\[
k_{21}G_{12}e^{-\theta_{21}}d_{12} + k_{22}G_{22}e^{-\theta_{22}}d_{21} = 0 \tag{6}
\]

To make the system implement physically, all elements of the controller should not include pre-evaluation item. Thus the elements of the TITO decoupling controller matrix can be designed as follows:
For modeling unknown system, a model can be built through mechanism analysis or system identification and then a simple decoupling controller matrix can be designed according to Equations (7) and (8).

B. Desired Dynamic Equation

For single-input-single-output (SISO) system, the controlled object can be formulated approximately as

\[ G(s) = \frac{H}{a_0 + a_1 s + \cdots + a_{n-1} s^{n-2} + s^n} \] (9)

where \( n \) is the number of system poles, \( j \) is the relative order, \( H \) is system high-frequency gain. Due to modeling error and system uncertainty, \( H, a_i (i=0,\cdots,n-1) \) and \( b_j (i=0,\cdots,n-j-1) \) are all unknowns to be determined.

Tornambe designed a kind of controller (named TC) with respect to equation (9). Supposing that \((A,B,C)\) is the realization of minimum energy control of system (9), which through transform

\[
\begin{aligned}
\dot{z}_i &= C A^{-1} x_i, & i &= 1,\ldots,j \\
\dot{w}_i &= x_i, & i &= 1,\ldots,n-j
\end{aligned}
\] (10)

System may be converted into standard form

\[ \dot{z}_i = z_{i+1}, i = 1,\ldots,j-1 \] (11)

\[ \dot{z}_j = \sum_{i=0}^{j-1} c_i z_{i+1} - \sum_{j=0}^{n-j-1} d_j w_{i+1} + Hu \] (12)

\[ \dot{w}_i = w_{i+1}, i = 1,\ldots,n-j-1 \] (13)

\[ \hat{w}_{n-j} = c_{n-j} z_{n-j+1} + z_i \] (14)

\[ y = z_i \] (15)

where \( c_i (i=0,\cdots,j-1) \) and \( d_j (j=0,\cdots,n-j-1) \) are all unknowns to be determined.

All kinds of system uncertainty and disturbance are reduced to extended state variable

\[ f(z,w,u) = -\sum_{i=0}^{j-1} c_i z_{i+1} - \sum_{j=0}^{n-j-1} d_j w_{i+1} + (H-1)u \] (16)

Then equation (12) can be rewritten as

\[ \dot{z}_j = f(z,w,u) + u \] (17)

Tornambe controller is designed as follows

\[ \hat{f} = \xi + kz \] (18)

\[ \dot{\xi} = -k \xi - k^2 \dot{z}_j - ku \] (19)

\[ u = -\sum_{i=0}^{n-j} h_i z_{i+1} - \hat{f} \] (20)

where equations (10) and (11) are observers which observe the extended state variable \( f(z,w,u) \) in real time. Selection of suitable parameters may make the dynamic characteristics of closed-loop system transfer function satisfy the equation

\[ y/r = h_i/(h_0 + \cdots + h_j s^{n-j-1} + s^j) \]

When relative order \( j = 2 \), DDE for two-degree-of-freedom PID redefines the extended state variable in TC which represents system uncertainty and disturbance:

\[ f(z,w,u) = -c_i z_1 - c_{i+1} - \sum_{i=0}^{n-1} d_i w_{i+1} + (H-1)u \] (21)

where \( l \) is a proper positive number. Equations (17) and (19) are rewritten as

\[ \dot{z}_2 = f(z,w,u) + lu \] (22)

\[ \dot{\xi} = -k \xi - k^2 z_2 - kl \] (23)

Solving derivatives of both sides of equation (18), and substituting (22) and (23) into (10) so as to carry out Laplace transform, we get

\[ \hat{f} = \frac{k}{s+k} f \] (24)

where \( k \) is a parameter of the observer. If only closed-loop system meets the DDE

\[ \frac{y(s)}{r(s)} = h(s) = \frac{h_0}{h_0 + h_j s + s^2} \] (25)

the control law should be

\[ u = [-h_0(z_1 - r) - h_1 z_2 - f]/l \] (26)

Replace the extended state variable \( \hat{f} \) by the observer variable \( \hat{f} \), and carry out Laplace transform on (22) and substitute it into (26), the control law is changed as

\[ u = [-h_0(y - r) - h_1 y_2 - \frac{k}{s+k} (s^2 y - lu)]/l \] (27)

Multiplying both sides of Equation (19) by \((s+k)l\), we get

\[ (s+k)lu = -h_0(s+k)(y - r) - h_1(s+k)s y - k(s^2 y - lu) \]

Namely,

\[ slu = -h_0(s+k)(y - r) - h_1(s+k)s y - k(s^2 y - lu) \]

\[ = -h_0(s+k)(y - r) - h_1(s+k)s y - k(s^2 y - lu) \]

Equation (28) being divided by \( sl \), the two-degree-of-freedom PID control law becomes

\[ u = k_p(r - y) + \frac{k_i}{s}(r - y) + k_d(s(r - y) - br) \] (29)

© 2011 ACADEMY PUBLISHER
Similarly, when relative order \( j = 1 \), the two-degree-of-freedom PI control law is

\[
\begin{align*}
    u &= k_p (r - y) + \frac{k}{s} (r - y) - b r \\
    k_p &= (h_k + k) / l, \quad k_h = h_h / l, \quad b = k / l
\end{align*}
\] (31)

and overshoot (\( \sigma \)%);

3) Repeat the simulation experiment \( N \) times, and get a collection of two-dimensional performance \( \{t_s, \sigma\} \) including the adjustment time \( (t_s) \) and overshoot \( (\sigma\%) \);

4) Compare the effects of different tuning methods and the dispersion of the performance indexes from the results of the Monte-Carlo experiment. The smaller the range of the control system performance, the stronger the performance robustness of the control system is.

D. PID Decoupling Controller Design

In summary, the procedure of the controller design is as follows:

1) In terms of given model of controlled object, design decoupling controller matrix using equations (7) and (8);

2) In terms of objective function decoupled, tune the controller parameters properly based on DDE;

   a) Given regulation time \( (t_i) \) and overshoot \( (\sigma\%) \), determine DDE parameters \( (h_i, h_s) \) according to the analysis of second order system by classic control theory;

   b) From equation (24), only if \( \dot{f} \rightarrow f \) when \( k \rightarrow \infty \), actual dynamics will meet equation (25).

However, increment of \( k \) may make system unstable. On the other hand, increment of \( f \) favors the promotion of response speed of observer, at the same time widens frequency band and thus decreases resistance to high-frequency noise. In order to consider both response speed and resistance capacity, let \( k = 10 \times \text{sgn}(H) \) while tuning the parameters of PID controller using the desired dynamic equation (DDE), where \( H \) is the high-frequency gain of the system;

   c) Determine the value of \( I \) according to the stable region of the parameters of the PID controller;

   d) Determine the parameters of the PID controller according to Equation (30).

3) Examine the performance robustness of the system through Monte-Carlo stochastic experiment mentioned in Section III.C. Check performance robustness of decoupling controller under the condition of uncertainty of the controlled object.

IV. SIMULATION EXAMPLES

Investigate Wood-Berry distillation process studied extensively in chemical industry

\[
G_{mb}(s) = \frac{12.8e^{-s}}{16.7s+1} - \frac{18.9e^{-3s}}{21s+1} - \frac{6.6e^{-7s}}{-19.4e^{-3s}} - \frac{3}{10.9s+1} + \frac{14.4s+1}{1}
\]

The model is a typical TITO process with time delays. To verify the effectiveness of our method, we compare our method with Wang’s method and Liu’s method which showed excellent performance on the model. Comparison is carried out through simulation. For the sake of fair comparison, tune the adjustable parameters to make the three methods have the similar capable of response speed and overshoot set-point tracking.

\[
\begin{align*}
    h_{i1} &= 0.64, h_{i2} = 2.4, l_1 = 125, k_1 = 10 \\
    h_{i2} &= 0.2127, h_{i2} = 1.2632, l_2 = 55, k_2 = -10
\end{align*}
\]

Substituting them into Equation (30), the control parameters of DDE controller are

\[
\begin{align*}
    K_{p1} &= 0.1971, k_{i1} = 0.0512, k_{d1} = 0.0992, h_i = 0.192 \\
    K_{p2} &= -0.2258, k_{i2} = -0.0387, k_{d2} = -0.1589, h_s = -0.2297
\end{align*}
\]

Simulations are carried out using SIMULINK toolbox in MATLAB. Advantages and disadvantages of the three methods are compared on complexity of design, dynamic output responses of nominal systems and performance robustness of perturbation systems.

A. Comparison of Decoupling Control Methods

The decoupling matrices and controller parameters in the three methods are listed in Table 1. From forms of decoupling matrices, Wang’s method and our method are simple and easy to be realized whereas Liu’s method is complex and difficult to be realized. From scheme of tuning controller parameters, Wang’s method tunes PID controller parameters through solving crossover frequency and complicated computation; Liu’s method tunes PID/PI decoupling controller using linear Pade approximation; our method analyzes two-order system according to classical control theory. Our method does not need complicated computation and is easy to tune the parameters for the reason that all the parameters applied have obvious physical meanings.
B. Nominal System Dynamic Responses

For nominal system, two unit step input signals with given values are added at $0, 150$ s, respectively, and two load step disturbance signals with amplitude 0.1 are added to input at $300, 450$ s, respectively. The simulation results are shown as Figure 2.

From Figure 2, Wang’s method shows an obvious overshoot and longer regulation time and Liu’s method shows inferior decoupling effect and large load disturbance. However, the proposed method shows steady response within given value, nearly complete decoupling between two output responses and significantly superior load disturbance response over Wang’s method and Liu’s method which has shown excellent performance on distillation model. Through statistical analysis of the simulation results, the detailed system output response parameters are shown in Table 2, where $r_{ij}$ denotes the output of the $i$-th channel of controlled object with the addition of given unit step input signal.

<table>
<thead>
<tr>
<th>Method</th>
<th>$\sigma$</th>
<th>$t_s$</th>
<th>Max deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$(r_{1y_1})$</td>
<td>$(r_{2y_1})$</td>
<td>$(r_{1y_2})$</td>
</tr>
<tr>
<td>Proposed</td>
<td>3.3397</td>
<td>2.8018</td>
<td>12.63(s)</td>
</tr>
<tr>
<td>Wang</td>
<td>16.2150</td>
<td>10.5099</td>
<td>21.27(s)</td>
</tr>
<tr>
<td>Liu</td>
<td>0.8580</td>
<td>1.1762</td>
<td>12.17(s)</td>
</tr>
</tbody>
</table>

**Figure 2.** System output response

From Figure 2, Wang’s method shows an obvious overshoot and longer regulation time and Liu’s method shows inferior decoupling effect and large load disturbance. However, the proposed method shows steady response within given value, nearly complete decoupling between two output responses and significantly superior load disturbance response over Wang’s method and Liu’s method which has shown excellent performance on distillation model. Through statistical analysis of the simulation results, the detailed system output response parameters are shown in Table 2, where $r_{ij}$ denotes the output of the $i$-th channel of controlled object with the addition of given unit step input signal.

**Table I.** COMPARISON OF CONTROL SCHEMES

<table>
<thead>
<tr>
<th>Method</th>
<th>Decoupling matrix</th>
<th>Controller parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>Proposed</td>
<td>$\begin{bmatrix} 19.4 &amp; (16.7s+1)e^{-2} \ 6.6 &amp; 21s+1 \ (14.4s+1)e^{-3} &amp; 12.8 \ 10.9s+1 &amp; 18.9 \end{bmatrix}$</td>
<td>$k_{m1}=0.871k_{m2}=0.032k_{m3}=0.092$</td>
</tr>
<tr>
<td>Wang</td>
<td>$\begin{bmatrix} 1 &amp; (35.6s+18.9)e^{-3} \ 0 &amp; 208s+128 \ 950s+66e^{-4} &amp; 1 \ 211.4s+194 \end{bmatrix}$</td>
<td>$k_{m1}=0.216k_{m2}=0.0757k_{m3}=0.0174$</td>
</tr>
<tr>
<td>Liu</td>
<td>$\begin{bmatrix} -19.4 &amp; (14.4s+1)(2.5s+1)e^{-4} \ -6.6e^{-3} &amp; 21s+1(6s+1)e^{-4} \ (10.9s+1)(2.5s+1)e^{-4} &amp; 12.8 \ 228.9s^2+31.9s+1 &amp; 240.5s^2+31.1s+1 \end{bmatrix}$</td>
<td>$k_{m1}=0.4477k_{m2}=22.299T_{m1}=35.8366$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Proposed</th>
<th>Wang</th>
<th>Liu</th>
</tr>
</thead>
<tbody>
<tr>
<td>$k_{m1}$</td>
<td>$0.871$</td>
<td>$0.216$</td>
<td>$0.4477$</td>
</tr>
<tr>
<td>$k_{m2}$</td>
<td>$0.032$</td>
<td>$0.0757$</td>
<td>$0.157$</td>
</tr>
<tr>
<td>$k_{m3}$</td>
<td>$0.092$</td>
<td>$0.0174$</td>
<td>$0.250$</td>
</tr>
<tr>
<td>$h_{1}$</td>
<td>$0.809$</td>
<td>$0.229$</td>
<td>$0.1447$</td>
</tr>
<tr>
<td>$h_{2}$</td>
<td>$0.087$</td>
<td>$-0.0192$</td>
<td>$-0.9250$</td>
</tr>
<tr>
<td>$h_{3}$</td>
<td>$-0.159$</td>
<td>$-0.0634$</td>
<td>$-0.3843$</td>
</tr>
</tbody>
</table>

© 2011 ACADEMY PUBLISHER
C. Performance Robustness Analysis of Perturbation System

Under the condition of invariant controller and decoupling controller matrix, the controlled plant model $g_y = (k_y / T_y s + 1)e^{-\theta_i}$, $i, j = 1,2$ needs to be determined. Assume that the perturbation range of parameters $(k_y, T_y, \theta_i)$ of the plant models is $\pm 10\%$. The number of the stochastic experiments is $N=300$. Monte-Carlo experiment was conducted according to the steps described in Section III.C. Figure 3 shows the system performance robustness in the case of existence of uncertainty in the controlled plant.

From Figure 3, the proposed method shows the best performance robustness; Liu’s method shows inferior performance robustness; and Wang’s method shows the worst performance robustness. Table 3 provides detailed statistical parameters of system performance robustness.

```
<table>
<thead>
<tr>
<th>Method</th>
<th>$\sigma(%)$ Range</th>
<th>Mean</th>
<th>Variance</th>
<th>$t_s$ (s) Range</th>
<th>Mean</th>
<th>Variance</th>
</tr>
</thead>
<tbody>
<tr>
<td>Proposed $(r_1, y_1)$</td>
<td>1.4023–4.8651</td>
<td>3.2672</td>
<td>0.4115</td>
<td>10.07–12.94</td>
<td>11.6131</td>
<td>0.294</td>
</tr>
<tr>
<td>Liu $(r_1, y_1)$</td>
<td>0–2.3551</td>
<td>0.8927</td>
<td>0.2817</td>
<td>9.43–26.37</td>
<td>14.0578</td>
<td>23.9789</td>
</tr>
<tr>
<td>Proposed $(r_2, y_2)$</td>
<td>0.1124–6.2601</td>
<td>2.9602</td>
<td>1.2517</td>
<td>17.97–39.58</td>
<td>20.7242</td>
<td>8.7628</td>
</tr>
<tr>
<td>Wang $(r_2, y_2)$</td>
<td>1.8879–20.7087</td>
<td>10.6778</td>
<td>13.4962</td>
<td>9.44–47.43</td>
<td>32.4465</td>
<td>40.3749</td>
</tr>
<tr>
<td>Liu $(r_2, y_2)$</td>
<td>0–6.3266</td>
<td>1.4469</td>
<td>1.7064</td>
<td>10.91–57.58</td>
<td>28.9123</td>
<td>38.8638</td>
</tr>
</tbody>
</table>
```

V. CONCLUSION

In this paper, we studied TITO processes with time delays that commonly appear in chemical and industrial practice. We proposed a simple analytical design method of decoupling controller matrix which avoids complicated inverse operation of matrix. We devised a PID decoupling controller by means of the characteristics of
independence of accurate mathematical model and strong robustness of the Desired Dynamic Equation (DDE) method for two-degree-of-freedom (2-DOF) PID.

The proposed method is simple and practical; it is able to achieve complete decoupling of the output response for each channel of the nominal system; and it exhibits good performance robustness when there are load disturbance and perturbation in parameters of the plant model. The simulation results show the advantages of the proposed method.

REFERENCES