Particles Swarm Optimization Based Parameter Identification of Nonlinear Viscoelastic-Plastic Constitutive Equation of Soybean and Cottonseed

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Abstract—By using of combining theoretical model with empirical model, the nonlinear viscoelastic-plastic constitutive equations of soybean and cottonseed were developed on the basis of the creep test. The parameter inverse problem of constitutive model was proposed using improved particles swarm optimization. Numerical simulations of creep of soybean and cottonseed were carried out. The definition of critical pressing time for oil extraction and the method for determining the critical pressing time using creep rate were proposed. The values of critical pressing time for extruded soybean and cottonseed under six applied pressures were evaluated. Results indicated that the nonlinear viscoelastic-plastic constitutive model could simulate rheological behaviors of soybean and cottonseed very well. Mean relative deviations between the experimental and predicted values of soybean and cottonseed were 1.55% and 1.93%, respectively. There were significant decreases in the values for the creep rates in the early stage of pressing, and decreases to an insignificant and stabilize value in the later stage.

Index Terms—parameter identification, particles swarm optimization, constitutive equation, soybean, cottonseed

I. INTRODUCTION

Mechanical pressing is the most common method for oil extraction from vegetable oils [1-5]. Oilseed expresses complex mechanics behavior during mechanical pressing. Pore space is gradually dwindled and gas is gradually vented inside oilseed bed with increasing pressing pressure. Oilseed becomes close granular materials due to elastic-plastic deformation. Then oilseed becomes fluid-solid coupling material owing to the broken cell wall of oilseed and the extracted oil. Last, oilseed becomes oilseed cake for broken oilseed granule bonds each other [3]. Hysteretic deformation inside oilseed takes place for oil flows sluggishly due to impedance. Vegetable oilseeds exhibit viscoelastic-plastic behavior during the mechanical pressing. They are viscoelastic-plastic materials, and the deformation of viscoelastic-plasticity is nonlinear and relates to both applied pressure and pressing time [2-3]. It indicates that the oilseeds volume varies with pressing time under constant applied pressure [2-3]. There is a limiting compression for the volume under fixed applied pressure. Therefore, there is a critical pressing time for oil extraction. The identification of the nonlinear rheological characteristics and the determination of the critical pressing time provide scientific basis for designing presser and optimization of pressing process [2-3].

There are three methods to develop model of rheology, which are model theory, regression analysis and combining model theory with empirical formula, respectively [2-3, 6-9]. Inverse method is much used for evaluating model parameters. Model parameters identification based on optimization method is commonly used to ensure that identified parameters should both have designated physical meaning and meet the requirement of simulation with high precision [2,6]. Under normal conditions, the nonlinear viscoelastic-plastic constitutive models are very complex due to many model parameters to be evaluated. Optimization algorithms have a significant influence on identified results. Particle swarm optimization (PSO) algorithm is easy to implement and fast in convergence, and has few parameters to adjust. In view of the above virtues, PSO has been widely used in parameters identification for complex nonlinear model.

Researches in rheological characteristics and model developed of rapeseed, sesame and peanut have been carried out [2-4]. But the Researches of soybean and cottonseed have not been reported as yet in the world up till now. The objectives of this study were to measure variation of strain with pressing time under various applied pressure, to develop the viscoelastic-plastic constitutive model to simulate the nonlinear rheological
characteristic, and to determine the critical pressing time for extruded soybean and cottonseed.

II. MODEL DEVELOPMENT

A method of combining model theory with empirical formula, also called semi-empirical theory, was used to develop the nonlinear models. The nonlinear deformation of oilseeds was divided into two parts, which were linear and nonlinear deformation. The linear and the nonlinear deformations represent the viscoelastic deformation and the viscoplastic deformation, respectively. The method of model theory was used to develop the constitutive equation of visco-elasticity, and the method of empirical formula was used to develop the constitutive equation of visco-elasticity. Constitutive equation of viscoelastic-plasticity was developed by combining the constitutive equation of viscoelasticity with the constitutive equation of viscoplasticity. The method overcomes the difficulty for analysis of nonlinear rheology by model theory alone, and make up insufficient for lack designated physical meaning and universality by empirical formula alone as well. Model theory makes use of generalized Kelvin model characterizing viscoelasticity, and generalized Maxwell model characterizing viscoplasticity. Since model theory has distinct concept and physical meaning, it has been priority adopted [2-6].

A. Viscoelastic constitutive equation

When applied pressure is lower than yield limit of oilseeds, the stress-strain relationships of oilseeds are linear and deformations are regarded as linear viscoelastic deformations. Choosing five-component Kelvin model (Figure 1), the linear viscoelastic constitutive equation is written as

$$e_k(\sigma, t) = \frac{\sigma}{E_k} + \frac{\sigma}{E_k} \left[ 1 - \exp\left( -\frac{E_k t}{\eta_k} \right) \right]$$

where $e_k(\sigma, t)$ is the linear viscoelastic strain (%), $E_k$, $E_k$, and $E_k$ are the elastic modulus of elastic components in Kelvin model (MPa), $\eta_k$, and $\eta_k$ are the viscosity coefficient (MPa • min), $t$ is the pressing time (min).

B. Viscoplastic constitutive equation

When applied pressure is higher than yield limit of oilseeds, oilseeds come into plastic deformation and their stress-strain relationships are nonlinear. The deformations are nonlinear viscoelastic deformations. In order to make full use of model theory, viscoplastic strains were divided into linear viscoelastic strains and nonlinear viscoelastic strains. Model theory and empirical formula were proposed to developed linear and nonlinear viscoelastic constitutive equation, respectively.

Choosing three-components Maxwell model (Figure 2), the linear viscoplastic constitutive equation is written as

$$e_{M1}(\sigma, t) = \frac{1}{E_M} + \frac{t}{\eta_M} (\sigma - \sigma_s)$$

where $e_{M1}(\sigma, t)$ is the linear viscoplastic strain (%), $E_M$ is the elastic modulus of elastic components in Maxwell model (MPa), $\eta_M$ is the viscosity coefficient of viscous components in Maxwell model (MPa • min), $\sigma_s$ is the yield limit (MPa).

Nonlinear viscoplastic strains are composed of instantaneous and non-instantaneous nonlinear viscoplastic strains, written as follows

$$e_{M2}(\sigma, t) = e_{M0}(\sigma) + e_{M}(\sigma, t)$$

where $e_{M0}(\sigma)$ is the instantaneous nonlinear viscoplastic strain (%), $e_{M}(\sigma, t)$ is the instantaneous nonlinear viscoplastic strain (%), $e_{M}(\sigma, t)$ is the instantaneous nonlinear viscoplastic strain (%).

Hyperbola faction was proposed to developed the instantaneous and non-instantaneous nonlinear viscoplastic constitutive equation

$$e_{M0}(\sigma) = \frac{A_1 A_3 (\sigma - \sigma_s)}{1 - A_2 (\sigma - \sigma_s)}$$

$$e_{M}(\sigma, t) = \frac{A_1 A_3 (\sigma - \sigma_s)}{1 - A_2 (\sigma - \sigma_s)} t$$

where $A_1, A_2, A_3$ and $A_3$ are the constants.

C. Nonlinear viscoelastic-plastic constitutive equation

According to superposition principle, viscoelastic-plastic strain is the superposition of viscoelastic strain and viscoplastic strain (Figure 3). Figure 4 shows the physical model of oilseeds, and equation (6) shows the superposed result.

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III. CREEP EXPERIMENT

A. Experiment Method

A visualization of test apparatus used for the experiment was specially designed. Its schematic diagram is shown in Figure 5.

![Figure 5. Schematic diagram of visualized compression cell](image)

It mainly consists of a loading piston, an outer cylinder, an inner cylinder, a sealing ring, a support plate, a porous stone and a base plate. The test apparatus is mounted in a universal hydraulic test machine capable of applying compressive loads of 300 KN. The pressing chamber is provided with a 44mm diameter×95mm deep bore through which the loading piston compresses sample. The visual cylinder is made of plexiglas. An outer cylinder made of mild steel is essential to visual cylinder in order to increase its strength and rigidity. The outer cylinder is provided with two observed windows with a 20mm width ×25mm height. The performance and phenomenon of compressive process of oilseed samples can be observed through the visual inner cylinder. Support plate made of stainless-steel with several 3 mm diameter traverse holes distributed uniformly is designed to prevent porous stone from breaking. In order to ensure uniform fluid pressure within oilseed cakes, both the bottom of loading piston and the top of base plate are provided with radial and circular grooves 5 mm width ×5 mm depth. The top and bottom of oilseed sample are respectively provided with a porous stone in order to expel liquid (including oil and water) and air from oilseed during compression.

Having been cleaned and sieved, the samples were chosen, which were made up from uniform granules. A 30g sample was chosen as the testing specimen for the experiment. After the specimen was poured into the compression-permeability cell, the cell was mounted in a computer-controlled precision universal test machine capable of applying compressive loads of 300 KN. A series of six experiments for soybean and cottonseed had been carried out respectively. Six applied pressures (10, 20, 30, 40, 50, and 60 MPa) were used for the specimens of soybean and cottonseed. The controlled pressure method was adopted, that was; the applied pressure was maintained for 30 minutes when desired pressure was reached [3].

B. Experiment Results

Defined the applied pressure $\sigma$ and the axial strain $\varepsilon$ are as follows

$$\sigma = \frac{F}{A} \quad (7)$$

$$\varepsilon = \frac{\Delta H}{H_0} \quad (8)$$

where $F$ is the pressure acting on the specimen surface, and $A$ is the area of section of the specimen, and $H_0$ is the initial height of the specimen, and $\Delta H$ is the displacement of the specimen. Figures 6 and 7 show the creep curves of soybean and cottonseed when the applied pressure is invariable, which indicated that the nonlinear rheological characteristic of soybean and cottonseed is obvious. Tables 1 and 2 show the measured creep of soybean and cottonseed at 20°C under six applied pressures, respectively. All the creep of soybean and cottonseed are include two stages, which are the attenuation creep and the constant speed creep, and the duration of attenuation creep is short while the constant speed creep is long.

![Figure 6. Measured creep of soybean at 20°C under six applied pressure](image)
IV. MODEL PARAMETERS IDENTIFICATION

There are $E_{1h}$, $E_{K1}$, $E_{K2}$, $\eta_{K1}$, $\eta_{K2}$, $E_M$, $\eta_M$, $\sigma_s$, $A_1$, $A_2$, $A_3$, $A_4$ and $A_5$ thirteen parameters in the model to be determined. Inversion algorithm was used to evaluate the thirteen parameters.

A. Parameter inversion model

An inverse method was used to identify the model parameters. It should both meet having designated physical meaning and high simulation precision. Therefore, the model parameters identification is thus changed to parameter optimization problem. In this paper, the thirteen parameters to be determined were chosen as the design variables, and the minimizing the sum of squares residuals of the strains by experiment and the strains calculated by equation (9) was chosen as objective function. For the purpose of improving precision of permeability model, the objective function should be determined according to the method of creep experiment.

The parameters optimization can be expressed as follows

$$
\min f(X) = \frac{1}{6} \sum_{i=1}^{6} \left( \sum_{j=1}^{11} \left( \frac{100 \epsilon_g - \epsilon_{gj}}{\epsilon_{gj}} \right)^2 \right)
$$

subject to $X=(X_1, X_2, ..., X_{13})$

Particle swarm optimization (PSO) algorithm was used in solving the optimization problems above.

B. Improved particle swarm optimization

PSO is one of the evolutionary computational techniques. It is motivated from the simulation of social behavior of animals such as bird flocking, fish schooling, and swarm. Mathematical description of Basic PSO can be made as follows [10-15].

Each particle represents a potential solution to the optimization problem and it has a fitness value decided by optimal function. Supposing search space is D-dimensional, each individual is treated as a volume-less particle in the D-dimensional search space. The position and rate of position change for $i$-th particle can be represented by $X_i=(x_{i1}, x_{i2}, ..., x_{id})$ and $V_i=(v_{i1}, v_{i2}, ..., v_{id})$, respectively. The best position previously visited by the $i$-th particle is recorded and represented as $P_i=(p_{i1}, p_{i2}, ..., p_{id})$, called pbest. The swarm best position previously visited by all the particles in the population is represented as $P_g=(p_{g1}, p_{g2}, ..., p_{gd})$. Initialize a population of particles with random positions and velocities in search space, called gbest. Then

Table 1. Measured creep of soybean at 20°C under six applied pressure

<table>
<thead>
<tr>
<th>Applied pressure (MPa)</th>
<th>Pressing time(min)</th>
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<tr>
<td></td>
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<tr>
<td>10</td>
<td>50.38</td>
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<td>30</td>
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<td>60</td>
<td>64.22</td>
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</table>

Table 2. Measured creep of cottonseed at 20°C under six applied pressure

<table>
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<th>Applied pressure (MPa)</th>
<th>Pressing time(min)</th>
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<tbody>
<tr>
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<td>72.33</td>
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<tr>
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<td>73.84</td>
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</table>
particles search their best position, which are guided by swarm information \( P_g \) and their own information \( P_i \). Each particle position is evaluated by using fitness function and updates its position and velocity according to the following equations
\[
v^k_i = \omega v^k_{i-1} + c_1 r_1 x^k_{p_i} + c_2 r_2 (p^k_g - x^k_{i})
\]
where \( k \) is iteration number, \( \omega \) is inertia weight, \( c_1 \) and \( c_2 \) are two acceleration coefficients regulating the relative velocity toward global and local best position, \( r_1 \) and \( r_2 \) are two random numbers in [0,1].

Many effects have been made over the last decade to determine the inertia weight [10-15]. A larger inertia weight facilitates a global search, while a small inertia weight facilitates a local search. By linearly decreasing the inertia weight from a relatively large value to a small value through the course of the PSO performance, the PSO tends to have more global search ability at beginning of the performance, while having more local search ability near the end of the performance [10-11]. The inertia weight with nonlinear decreasing strategies was studied. It was found that the inertia weight with nonlinear decreasing with concave function had an advantage over with linear decreasing, while the inertia weight with linear decreasing had an advantage over with nonlinear decreasing with concave function [15]. In this paper an inertia weight with nonlinear decreasing with concave function in PSO is introduced as follow
\[
\omega = \left( \omega_{\text{max}} - \omega_{\text{min}} \right) \frac{k^2}{k^2_{\text{max}}} + \left( \omega_{\text{min}} - \omega_{\text{max}} \right) \frac{2k}{k^2_{\text{max}}} + \omega_{\text{max}} \tag{12}
\]
where \( \omega_{\text{max}} \) and \( \omega_{\text{min}} \) are maximal inertia weight and minimal inertia weight respectively, \( k_{\text{max}} \) is the maximal iteration number.

The values of upper and lower of bound limit of design variable should be estimated on the basis of priori knowledge to ensure that model parameters have designated physical meaning. Referring to the rheology researches of rapeseed, sesame and peanut [3], the parameters limits of soybean and cottonseed were evaluated in Table 3. The procedure of PSO is as follow

1. Initialize a population of particles with random positions \( x^0_i \) and velocities \( v^0_i \) in search space. Set the maximal iteration number \( k_{\text{max}} \). The population of particles and the maximal number of calculation are 40 and 1000 in this paper, respectively.
2. Calculate the fitness value using equation (9) and record the best previous solution of each particle.
3. Compare each particle’s fitness evaluation \( f \) with particle’s pbest \( p_i \). If current value \( f \) is better than pbest \( p_i \), that is \( f < p_i \), then the current value \( f \) replace the pbest value \( p_i \) and the current position replace the pbest position in the search space.
4. Compare particle’s pbest \( p_i \) with the population’s overall previous best \( p_{\text{gbest}} \). If current value \( p_i \) is better than gbest \( p_{\text{gbest}} \), then the current value \( p_i \) replace the gbest value \( p_{\text{gbest}} \) and the current position replace the gbest position.
5. Update the position and velocity of each particle according to equations (10) and (11), then go to step (2) and go on calculating until the prescribed error or the maximal number of calculation is met.

C. Results and discussion

Table 4 shows identified results of model parameters. Figures 8 and 9 show the comparisons of measured and predicted creep of soybean and cottonseed at 20°C under six applied pressures, respectively. The mean relative deviations between the experimental values and predicted values of soybean and cottonseed were 1.55% and 1.93%, respectively. The creep curves indicated that the nonlinear rheological characteristics of soybean and cottonseed were obvious. All the creep curves of soybean and cottonseed included two stages. They were attenuation creep and constant speed creep. The duration of attenuation creep was short while the constant speed creep was long. Figures 8 and 9 also show that all of creep curves under different applied pressure tended towards horizontal asymptote finally. It is typical trend of creep curve.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>( E_{\text{t}} )</th>
<th>( E_{\text{d}} )</th>
<th>( \eta_{\text{c}} )</th>
<th>( \eta_{\text{d}} )</th>
<th>( \sigma_1 )</th>
<th>( A_1 )</th>
<th>( A_2 )</th>
<th>( A_3 )</th>
<th>( A_4 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Limits</td>
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<td>0-10^7</td>
<td>0-10^7</td>
<td>0-10^7</td>
<td>0-10^3</td>
<td>0-20</td>
<td>0-5</td>
<td>0-5</td>
<td>0-5</td>
</tr>
</tbody>
</table>

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V. CREEP RATE AND DETERMINATION OF CRITICAL PRESSING TIME

Taking the partial derivative of equation (6) with respect to $t$, yields an expression for the creep rate under constant applied pressure

$$\frac{de(\sigma, t)}{dt} = \frac{\sigma}{\eta_{s1}} \exp\left(\frac{E_{E1}}{\eta_{s1}} t\right) + \frac{\sigma - \sigma_s}{\eta_M} \exp\left(\frac{E_{E2}}{\eta_{s2}} t\right) + \frac{\sigma - \sigma_s}{A \lambda} A_2 \left(\sigma - \sigma_s\right) t^{-1}$$

Equation (13) describes variation of bed strain with time of compression under constant applied pressure. Figures 10 and 11 show the predicted creep rates of soybean and cottonseed respectively. Both of them expressed that the creep rates varied sharply and reduced rapidly along with time at the initial stage of oil extraction, and varied very little and reduced to approximate value of zero when pressing time reached a critical value. It indicates that the creep is insignificant, the oil volume tends towards stabilization, the value of oil extraction tends towards zero, and the oil extraction reaches balanced state at that time. It is clear from figures 10 and 11 that there is a typical trend that all of creep rate curves under different applied pressure tended towards horizontal asymptote finally. So, the creep rate and its curve can be used to determine the critical pressing time under different applied pressure. In this paper, an order of magnitude of 10^{-4} was chosen as the value of creep rate for determining critical pressing time of soybean and cottonseed. Table 5 shows the critical pressing time for the extrude soybean and cottonseed.

VI. CONCLUSIONS

Nonlinear viscoelastic-plastic constitutive equation presented in this paper was developed by using of combining theoretical model with empirical model. In contrast to method of empirical formula alone and model theory alone, the method in this study both overcame the difficulty for analysis of nonlinear rheology and made up insufficient for lack designated physical meaning and universality. The definition of critical pressing time for oil extraction was proposed, and the method determined the critical pressing time applied creep rate was also proposed in this study.

No more than 1.55% and 1.93% of the average relative deviations between the experimental values and predicted values of soybean and cottonseed respectively indicated that the model predicted the creeps of soybean quite well as well as cottonseed. The predicted results demonstrated
that the model characterized the nonlinear rheology of soybean and cottonseed developed in this study was satisfactory. There were significant decreases in the values for the creep rates with time over the first ten odd minutes of pressing, and decreases to an insignificant and stabilize value with time over thirty odd minutes of pressing.

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